Investigation on the Efficiency of Sun Visors

23 pages

Introduction

My investigation was inspired when I was travelling in the front seat of my parent's car. There are situations when sunlight shines directly into my eyes and sometimes it is hard to see directly in front. Personally, this posed great danger to my mom as she got into an accident when she couldn't see the streetlight color due to sunlight. Since then, one problem was given - whether to use sun visors or tint the front face of the car if the sun visors aren't effective.

Sun visors are small screens inside a car with one side attached to the ceiling in front of the driver. It hangs down to block sunlight shining directly into the driver's eye. Since only one side is attached, the angles are adjustable by drivers based on the angle of sunlight entering the car. However, even with sun visors, there are cases when sunlight is not blocked. Hence, it will be crucial to find the practicality of these sun visors depending on the sun's angle.

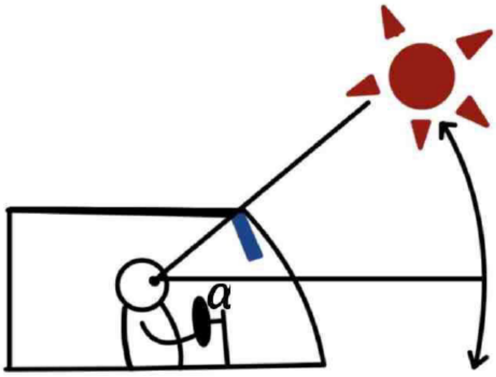
The main goal of this investigation is therefore to find the efficiency of a sun visor in a car. The efficiency will be defined as how much percentage of sunlight the sun visor can block entering a car's front window. It will be justified as "efficient" when at least 70% of sunlight is blocked.

A: An aim is clearly given and efficiency is defined.

In part I, I will find the equation of efficiency of sun visors from a two dimensional perspective. However, realistically, since our vehicles work in three dimensions, in part II, I will find the efficiency of sun visors from a three dimensional perspective.

I. Efficiency of sun visors in two dimensions

Figure 1 illustrates the various assumptions and set-up of the scenario. It will be assumed that a driver constantly drives their vehicle in the direction towards the sun because this is when sun directly enters through the front window. Since sun visors are minimum in length to prevent blocking the driver's vision, there is a limit to the amount of sunlight it can block. If the sun elevation angle, represented by α in the figure 1 below, is greater than the sun visor, it will block sunlight but if less, it would not block sunlight.



B: Clear diagram.

Figure 1. Self-drawn diagram of vehicle and sun elevation angle

Figure 2 below illustrates this situation. Origin O represents the driver's eye and r represents the length of the sun visor. Point C represents where the sun visor attaches to the ceiling of the car. As previously mentioned, the efficiency of sun visors depends on the sun elevation angle, therefore I will find the minimum angle at where sun visors will block sunlight, which is the tangent line with the sun visor, OB. Hence, a circle with center C and radius of r can be drawn around the sun visor, and the tangent line to point B (end point of sun visor), OB, will be set as the standard line. When sun elevation is higher than the angle this tangent makes with the horizontal line at the driver's eye level, the ray will be blocked but below, the ray will shine to the driver's eye, and sun visors will be considered not efficient.

E: The sun visor position is constrained. It seems that the sun visor position is fixed at its maximum position, i.e. CB seems to be fixed.

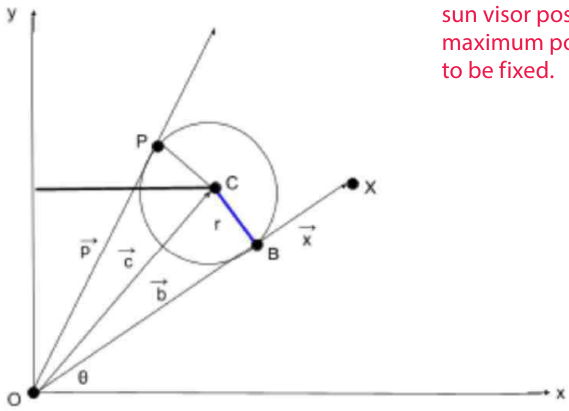


Figure 2.

First, to determine the standard at which sun visors will be efficient, I will find the expression for θ, the angle between the horizontal line at the driver's eye and tangent to point B.

B: Incorrect vector notation.

Vector c is the vector from the origin O to the center, point C. Point P is a point which lies on the circle around the sun visor. Vector p is the vector which intersects point P. Referring back to figure 2, if

$$\overrightarrow{OC} = \vec{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \overrightarrow{OP} = \vec{p} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

then,

$$\overrightarrow{CP} = \vec{p} - \vec{c}$$

Since $\vec{p} - \vec{c} = r$ ×B

$$|\overrightarrow{CP}|^2 = r^2$$

$$(\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) = r^2$$

Point B is also a point which lies on the circle around the sun visor. Vector OB is vector b which intersects point B, and is also tangent to the circle. Hence, referring back to figure 2,

$$\overrightarrow{OB} = \vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$(\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c}) = r^2$$

$$\begin{pmatrix} b_1 - c_1 \\ b_2 - c_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 - c_1 \\ b_2 - c_2 \end{pmatrix} = r^2$$

$$(b_1 - c_1)^2 + (b_2 - c_2)^2 = r^2$$

Considering vector b is the tangent to the end point of the sun visor, referring to figure 2, we can deduce that .

$$\overrightarrow{CB} \perp \overrightarrow{OB}$$

Hence,

$$\overrightarrow{CB} \cdot \overrightarrow{OB} = 0$$

$$(\vec{b} - \vec{c}) \cdot \vec{b} = 0$$

Referring back to the previously found equation $(\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c}) = r^2$,

$$(\vec{b} - \vec{c}) \cdot \vec{b} - (\vec{b} - \vec{c}) \cdot \vec{c} = r^2$$

Substituting $(\vec{b} - \vec{c}) \cdot \vec{b} = 0$,

$$0 - (\vec{b} - \vec{c}) \cdot \vec{c} = r^2$$

$$(\vec{b} - \vec{c}) \cdot \vec{c} = -r^2$$

$$\begin{pmatrix} b_1 - c_1 \\ b_2 - c_2 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = -r^2$$

Also from figure 3, we can deduce $\overrightarrow{CB} \perp \overrightarrow{BX}$.

Given that $\overrightarrow{OX} = \vec{x} = (x, y)$

$$\overrightarrow{CB} \cdot \overrightarrow{BX} = 0$$

$$(\vec{b} - \vec{c}) \cdot (\vec{x} - \vec{b}) = 0$$

$$(\vec{b} - \vec{c}) \cdot (\vec{x} - \vec{c} + \vec{c} - \vec{b}) = 0$$

$$(\vec{b} - \vec{c}) \cdot \left[(\vec{x} - \vec{c}) - (\vec{b} - \vec{c})\right] = 0$$

$$(\vec{b} - \vec{c}) \cdot (\vec{x} - \vec{c}) - (\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c}) = 0$$

$$(\vec{b} - \vec{c}) \cdot (\vec{x} - \vec{c}) = (\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c})$$

E: This is correct, but the candidate should have explained the rules being used in going from one step to the next.

Substituting the equation previously proven that $(\vec{b} - \vec{c}) \cdot (\vec{b} - \vec{c}) = r^2$, we can find:

$$(\vec{b}-\vec{c})\cdot(\vec{x}-\vec{c})=r^2$$

$$\begin{pmatrix} b_1 - c_1 \\ b_2 - c_2 \end{pmatrix}\cdot\begin{pmatrix} x - c_1 \\ y - c_2 \end{pmatrix} = r^2$$

$$\left(b_1-c_1\right)\left(x-c_1\right)+\left(b_2-c_2\right)\left(y-c_2\right) = r^2$$

Expressing this equation in terms of x and y,

$$\left(b_1-c_1\right)x + \left(b_2-c_2\right)y = r^2+c_1\left(b_1-c_1\right)+c_2\left(b_2-c_2\right)$$

Since it has been shown previously that $\begin{pmatrix} b_1 - c_1 \\ b_2 - c_2 \end{pmatrix}\cdot\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = -r^2$

$$\left(b_1-c_1\right)x + \left(b_2-c_2\right)y = r^2 - r^2$$

$$\left(b_1-c_1\right)x + \left(b_2-c_2\right)y = 0$$

$$y = -\frac{\left(b_1-c_1\right)}{\left(b_2-c_2\right)}x$$

Hence, we have found the equation for line OX, the tangent to the end point of the sun visor, B.

In figure 3 below, we can visually see line OX, and that it is perpendicular to vector CB, the sun visor.

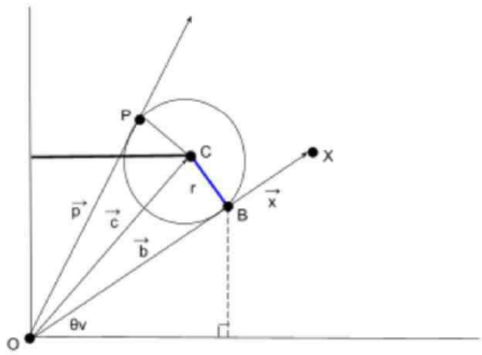


Figure 3.

B: The diagram is fine, but the axes are not labelled.

I will now find θv, the angle in between the tangent to the sun visor and the horizontal surface at eye level.

Referring to figure 3, since $tan(\theta v)$= slope,

$$\tan(\theta v) = -\left(\frac{b_1 - c_1}{b_2 - c_2}\right)$$

This should be true, since line OX is perpendicular to vector CB. The slope of vector CB should be $\frac{b_2 - c_2}{b_1 - c_1}$.

Since $\overrightarrow{OX} \perp \overrightarrow{CB}$, the multiple of the slope of vector CB and slope of line OX should yield -1. Hence, since

$$m_{\overrightarrow{OX}} \times m_{\overrightarrow{CB}} = -1$$

$$\frac{b_2 - c_2}{b_1 - c_1} \times \left(-\frac{b_1 - c_1}{b_2 - c_2}\right) = -1$$

the expression for tan (θv) is shown to be consistent with previous results.

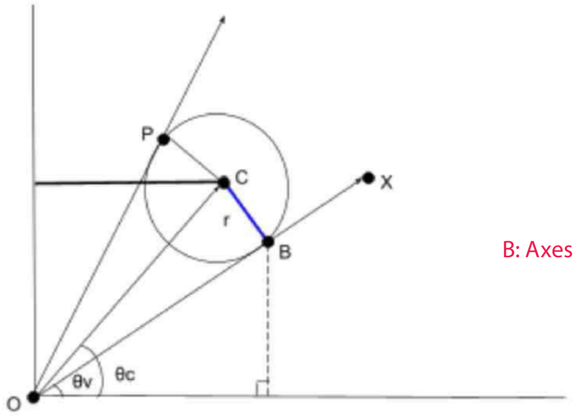
Hence, the angle between line OX and the horizontal surface from the driver's eye level is

$$\theta_v = arc\tan\left(-\left(\frac{b_1 - c_1}{b_2 - c_2}\right)\right)$$

Therefore, I have found the expression for the angle which is the standard, since when elevation angles are greater than this angle sun visors will be efficient but at angle less than this angle, sun visors wouldn't work. Hence, using this angle, I will now find the expression for the efficiency of sun visors using its angles based on time.

A: Clearly explained to address target audience.

As shown below in figure 4, θc is the angle between point C, the ceiling, and the horizontal surface from the driver's sight. This represents the maximum angle at which the sun will directly hit the driver's eye without a sun visor. θv, the angle between point B and the horizontal surface from the driver's sight, represents the maximum angle at which the sun will directly hit the driver's eye with a sun visor. Hence, to find the efficiency of the sun visor we can find the ratio between these two angles. I will find the efficiency in terms of the total amount of time when sunlight enters the vehicle with and without a sun visor.



B: Axes are not labelled.

Figure 4.

B: Should d be a suffix? Would this be achieved at noon?

The minimum sun elevation angle will be 0° at sunrise and θd at its peak angle during daytime. Hence, I will set the time it takes for the elevation angle to reach from 0° to θd to be D hours. I will define *d* as the value dividing D by θd, which represents the time (hours) it takes for the elevation angle to increase or decrease by 1° during daytime. Hence it will take 2*d* hours in a day from sunrise to sunset. Therefore, I will multiply 2*d* to each angle.

Hence, to find the efficiency in terms of the total amount of time when the sunlight enters the vehicle, we can find:

$$\text{total amount of time when sunlight enters the vehicle without a sunvisor} - \text{total amount of time when sunlight enters the vehicle with sunvisor}$$

$$= 2d\theta_c - 2d\theta_v$$

The percentage of efficiency can then be expressed as

E: This is fine, but needs more explanation.

$$\frac{2d\theta_c - 2d\vartheta_v}{2d\theta_c} \times 100$$

As 2*d* can be cancelled out from all terms, the percentage of efficiency can be expressed as

$$= \frac{\theta_c - \theta_v}{\theta_c}$$

Recalling that $\overrightarrow{OC} = \vec{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, θc will equal to $arc\tan\left(\frac{c_2}{c_1}\right)$

$$\%_{efficiency} = \frac{arc\tan\left(\frac{c_2}{c_1}\right) - arc\tan\left(-\left(\frac{b_1 - c_1}{b_2 - c_2}\right)\right)}{arc\tan\left(\frac{c_2}{c_1}\right)} \times 100$$

To solve my personal problem of finding the practical efficiency of sun visors, I will substitute real measurements to the previously found equations to determine the real efficiency of the sun visors and express them as percentages.

From my mom's car Volkswagen Polo, I have measured the angle between the ceiling and the eye level and the angle between the end of the sun visor and eye level.

C: Authentic personal engagement is seen here. Candidate considers a real-life situation.

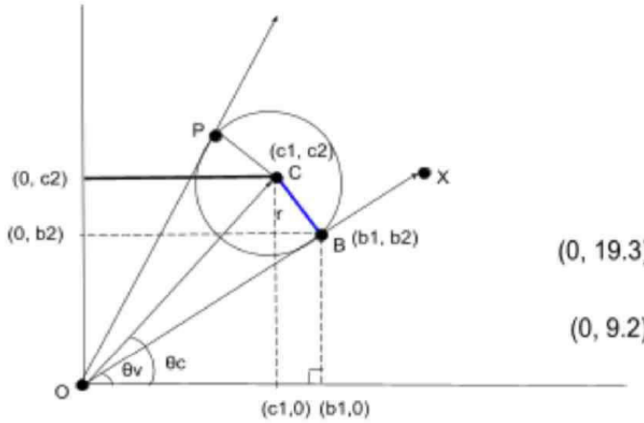


Figure 5. Self drawn hypothetical diagram

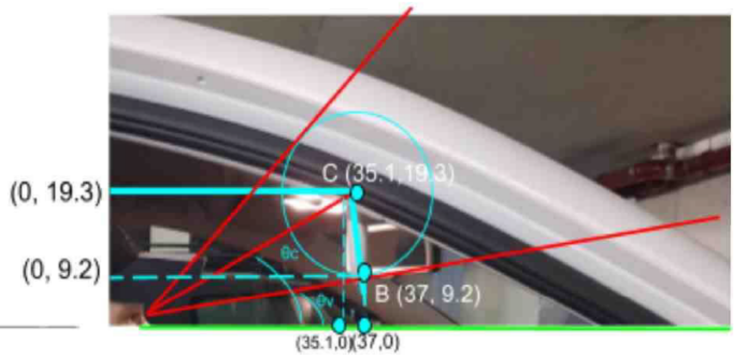


Figure 6. Real world measurements from car

By using a measurement tape with an uncertainty of ±0.05 cm, I have found the coordinates for point C , where the ceiling meets the top of the sun visor, and point B, at the end of the sun visor. Referring back to the sketched diagram of the situation, the coordinates were as the following: B (37.0,9.2) and C(35.1, 19.3). Hence, by substituting these two values to the equation above, I could find

$$\% \; Efficiency = \frac{arc\tan\left(\frac{19.3}{35.1}\right) - arc\tan\left(-\left(\frac{37.0-35.1}{9.2-19.3}\right)\right)}{arc\tan\left(\frac{19.3}{35.1}\right)} \times 100$$

$$= 63.0\;\%\;(3SF)$$

D: There is a missed opportunity to reflect on the fact that the visor can actually rotate around C.

Therefore, the efficiency of a sun visor in my car would be 63.0% from a two dimensional perspective.

II. Efficiency of sun visors in three dimension

However, in reality, vehicles are placed in our three dimensional world. Therefore, in part II, I will find the efficiency of sun visors in three dimensions. To find the efficiency of sun visors in three dimensions, I will first find the total amount of sunlight received by the front window without sun visors and then the total amount received with sun visors.

Figure 7 below has been modelled using *Autodesk Fusion 360*, which illustrates the situation without a sun visor. Point O represents the driver's eye, hence sector OBC is the field of view. The length from point O to one point of the front window, represented by line OC or OB is *r.* The vertical angle between the driver's eye level and the ceiling, between vector OA and OB, is θc[1] and the horizontal angle between the endpoints of the driver's field of view, between OB and OC, is ∅p. The curved surface ABCD represents the proportion of surface area of the vehicle's front window inside the driver's field of view. Hence, the surface area will imply the total area, in this case, amount of sunlight the driver receives.

A: It would have been better if the diagram accompanied the explanation (e.g. at the side). That way the reader would not have to scroll up and down.

[1] Angles in part I and part II should be considered as different values. They do not represent the same values.

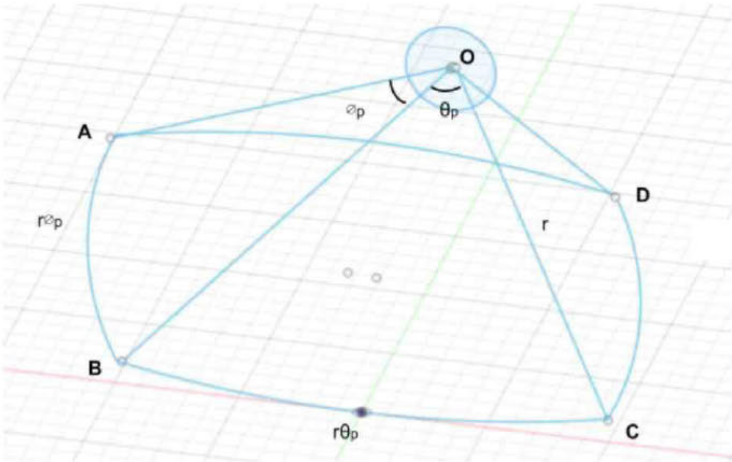


Figure 7. Illustration of driver's field of view without sun visor

As represented in figure 8 below, it is assumed that surface ABCD is an area element on a spherical surface, hence it is drawn on the spherical coordinate. The sphere has a center at O and radius of r. As mentioned above, the horizontal angle between OB and OC is θp. The vertical angle between the z-axis and line OD is Øp.

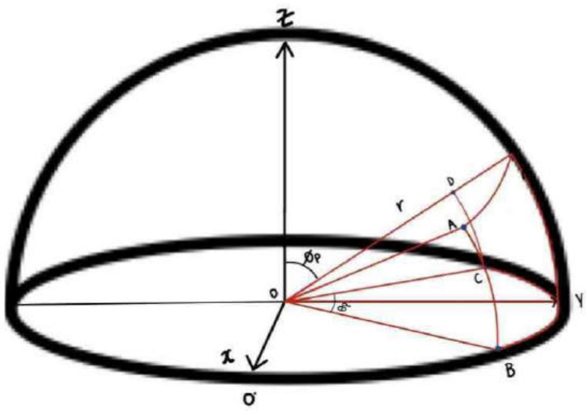
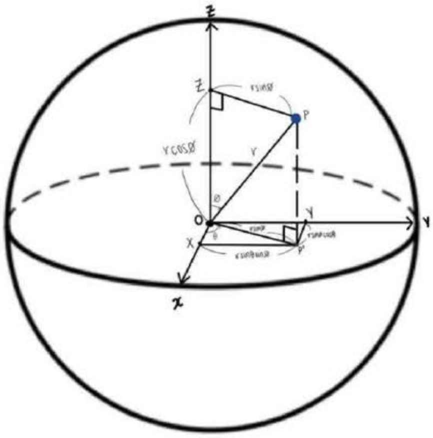


Figure 8. Self-drawn diagram of surface with no sun visor on a spherical coordinates

To find surface area ABCD, I will first find the general expression for a curved surface area, and integrate this specifically for the angles in my scenario. First, I will find a general expression that represents any curved surface area. As shown in figure 9 below, assume that a point P is located on the surface of a sphere with center O and radius r. Point P' is a projection of point P on the XY axis. The angle from the z-axis to line OP is Ø and the angle from the x-axis to line OP' is θ. Using trigonometry, I could find that line ZP is rsinØ. Since OP' is a projection of line ZP, it would also be rsinØ. Hence, using trigonometry, line XP', is rsinθsinØ. Using trigonometry, line OZ is rcosØ and line YP' is rsinØcosθ. (Refer to fig 9)



A: Diagram is correct, but it is too small and it should have been accompanied by some explanation.

Figure 9. Self-drawn diagram of coordinate for general point P on spherical coordinates.

Hence expressing the general point, point P with vector coordinates,

$$x = r\sin\varnothing\ \cos\theta$$
$$y = r\sin\varnothing\ \sin\theta$$
$$z = r\cos\varnothing$$

It will be assumed that point P moves along the surface of the sphere. Referring to the previous figure 9, the radius of the sphere, length of line OP, has a fixed value of *r* but the angle Ø and θ is variable, as Ø will extend vertically downwards on the YZ plane and θ will extend horizontally on the XY plane to create a curved surface on the spherical coordinate. A curved surface can be treated like a rectangle and its surface area can be found using the vector formula of finding areas (Sharetechnote), by ✓

$$A = \left| \overrightarrow{r'}_{\varnothing} \times \overrightarrow{r'}_{\theta} \right|$$

in which $\overrightarrow{r'}_{\emptyset}$ and $\overrightarrow{r'}_{\theta}$ are tangent vectors of line AB and BC from figure 7 previously shown.

The cross product of specifically tangent vectors are found because the tangent vector is the normal at the direction θ and Ø are extending towards, hence finding the cross vector would yield the area of a curved surface.

Tangent vectors can be found by taking the derivative of the coordinates of point B each with respect to Ø and θ, since the surface area is created by extending point P in the both the vertical and horizontal direction of Ø and θ.

Hence, taking the derivative of point P with respect to Ø and treating θ as a constant, the coordinates of $\overrightarrow{r'}_{\emptyset}$ will be

$$(rcos\emptyset cos\theta, rcos\emptyset sin\theta, -rsin\emptyset)$$

Taking the derivative of point P with respect to θ and treating Ø as a constant, the coordinates of $\overrightarrow{r'}_{\theta}$ will be

$$(-rsin\emptyset sin\theta, rsin\emptyset cos\theta, 0)$$

Hence, I have found the coordinates for each $\overrightarrow{r'}_{\emptyset}$ and $\overrightarrow{r'}_{\theta}$, the two tangent vectors of a curved surface.

The surface area of the curved surface can be found by the magnitude of the cross vectors of these two vectors.

To find the area of surface element ABCD, I will now find the magnitude of the cross product of the tangent vectors $\overrightarrow{r'}_{\emptyset}$ and $\overrightarrow{r'}_{\theta}$ since,

$$A = \left| \overrightarrow{r'}_{\emptyset} \times \overrightarrow{r'}_{\theta} \right|$$

First, I will find the cross product of the two tangent vectors:

$$\overrightarrow{r'}_{\varnothing} \times \overrightarrow{r'}_{\theta} = \begin{vmatrix} i & j & k \\ r\cos\varnothing\cos\theta & r\cos\varnothing\sin\theta & -r\sin\varnothing \\ -r\sin\varnothing\sin\theta & r\sin\varnothing\cos\theta & 0 \end{vmatrix}$$

$$= \begin{pmatrix} 0-\left(-r^2\sin^2\varnothing\cos\theta\right) \\ -\left(0-\left(r^2\sin^2\varnothing\sin\theta\right)\right) \\ r^2\sin\varnothing\cos\varnothing\cos^2\theta-\left(-r^2\sin\varnothing\cos\varnothing\sin^2\theta\right) \end{pmatrix}$$

$$= \begin{pmatrix} r^2\sin^2\varnothing\cos\theta \\ r^2\sin^2\varnothing\sin\theta \\ r^2\sin\varnothing\cos\varnothing\cos^2\theta+r^2\sin\varnothing\cos\varnothing\sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} r^2\sin^2\varnothing\cos\theta \\ r^2\sin^2\varnothing\sin\theta \\ r^2\cos^2\theta\sin\varnothing\cos\varnothing + r^2\sin^2\theta\sin\varnothing\cos\varnothing \end{pmatrix}$$

Since for the z-coordinate $r^2 sin\emptyset\ cos\emptyset$ is a common factor in both two terms,

$$= \begin{pmatrix} r^2\sin^2\varnothing\cos\theta \\ r^2\sin^2\varnothing\sin\theta \\ r^2\sin\varnothing\cos\varnothing\left(\cos^2\theta+\sin^2\theta\right) \end{pmatrix}$$

Since $cos^2\theta + sin^2\theta \ = \ 1$,

$$= \begin{pmatrix} r^2\sin^2\varnothing\cos\theta \\ r^2\sin^2\varnothing\sin\theta \\ r^2\sin\varnothing\cos\varnothing \end{pmatrix}$$

Hence, finding the magnitude of the cross product of the two tangent vectors found above,

$$\left|\overrightarrow{r'_{\varnothing}} \times \overrightarrow{r'_{\theta}}\right| = \sqrt{\left(r^2\sin^2\varnothing\cos\theta\right)^2 + \left(r^2\sin^2\varnothing\sin\theta\right)^2 + \left(r^2\sin\varnothing\cos\varnothing\right)^2}$$

$$= \sqrt{r^2\sin^4\varnothing\cos^2\theta + r^4\sin^4\varnothing\sin^2\theta + r^4\sin^2\varnothing\cos^2\varnothing}$$

$$= \sqrt{r^2\sin^4\varnothing\left(\cos^2\theta + \sin^2\theta\right) + r^4\sin^2\varnothing\cos^2\varnothing}$$

Since $cos^2\theta + sin^2\theta = 1,$

$$= \sqrt{r^4\sin^4\varnothing + r^4\sin^2\varnothing\cos^2\varnothing}$$

$$= \sqrt{r^4\sin^2\varnothing\left(\sin^2\varnothing + \cos^2\varnothing\right)}$$

Since $sin^2\emptyset + cos^2\emptyset = 1,$

$$= \sqrt{r^4\sin^2\varnothing}$$

$$= r^2\sin\varnothing$$

Hence, I have found the general expression, $r^2 sin\emptyset$,for the area of a curved surface.

A: Some concluding explanatory notes would have helped the reader.

E: Double integrals are not covered in the syllabus so some explanation is required.

By taking the double integral of this expression, we can find the surface area of ABCD. Double integrals are commonly used when finding the surface area of a sphere, when an expression for an area element is found and this is integrated over the whole sphere for the surface area of the whole sphere (Socratic Q&A). However, in my scenario, double integrals will be used not to integrate over the whole sphere but over specific values of vertical and horizontal angles. In order for a curved surface to be created on a spherical coordinates, a point must move both in respect to two different variables - the horizontal direction, in terms of θ and vertical in terms of ∅, as it is on a three dimensional spherical coordinate. Hence, I will use double integrals.

Referring back to figure 8, I will find the interval of the integrals. First, I will integrate with respect to ∅, the angle extending vertically on the YZ plane. Figure 10 below shows the YZ plane of the spherical coordinate from figure 8. On a spherical coordinate, the z-axis is considered as 0 and y axis as $\frac{\pi}{2}$. Hence from the yz plane, the angle measured down from the z-axis to line OA will be Øp. Thus, the curved surface ABCD is created by vertically expanding Øp to $\frac{\pi}{2}$, so the first integral will be integration of the general curved surface area expression from the interval Øp to $\frac{\pi}{2}$, with respect to Ø.

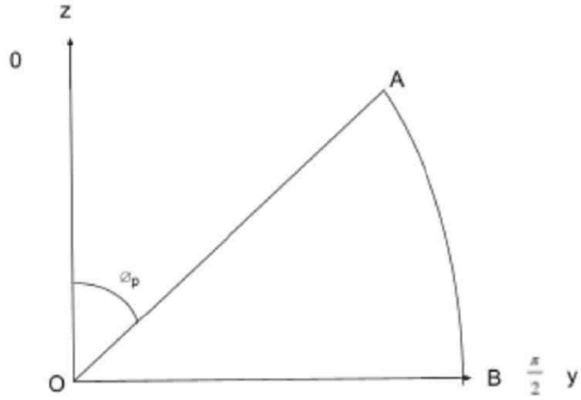


Figure 10. Vertical angles on a YZ plane

Similarly, as shown in figure 11 below, from the XY plane, θp is the angle between OB and OC. OB will be assumed to be 0 and OC is hence θp, as the curved surface ABCD is created by horizontally extending OB to OC. Therefore, the expression integrated with respect to Ø will be integrated again with respect to θ.

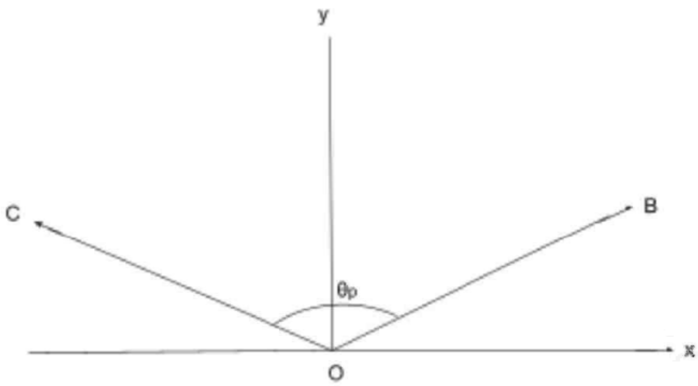


Figure 11. Horizontal angles on a XY plane

Hence setting up the equation for surface area of ABCD using double integrals,

B: This is a double integral.
$d\emptyset$ is missing.

$$SA = \int_0^{\theta_p} \int_{\emptyset_p}^{\frac{\pi}{2}} \left(r^2 \sin\emptyset \ d\emptyset \right) d\theta$$

$$= \int_0^{\theta_p} \left[- r^2 \cos\emptyset \right]_{\emptyset_p}^{\frac{\pi}{2}} d\theta$$

$$= \int_0^{\theta_p} \left[- r^2 \cos\frac{\pi}{2} - \left(- r^2 \cos\left(\emptyset_p \right) \right) \right] d\theta$$

$$= \int_0^{\theta_p} \left[0 + r^2 \cos\left(\emptyset_p \right) \right] d\theta$$

E: The evaluation of double integrals needs to be explained in more detail to demonstrate understanding.

$$= \int_0^{\theta_p} \left(r^2 \cos\left(\emptyset_p \right) \right) d\theta$$

Treating Øp as a constant and integrating with respect to θ for the second integral,

$$= \left[r^2 \theta \cos\left(\emptyset_p \right) \right]_0^{\theta_p}$$

$$= \left[r^2 \theta_p \cos\left(\emptyset_p \right) - 0 \right]$$

$$= r^2 \theta_p \cos\left(\emptyset_p \right)$$

Hence, the surface area of ABCD (see fig 7) can be expressed as $r^2\theta_p cos(\emptyset_p)$

However, when there is a sun visor present, less sunlight will shine through the surface. Figure 12 illustrates this situation. Similar to the previous situation (see fig 7) curved surface ABCD represents the proportion of surface area of the vehicle's front window inside the driver's field of view. However, different from the illustration of the previous situation in figure 7, in this scenario, a sun visor of curved surface AEFD is present. Hence sunlight will only pass through the curved surface EBCF. Therefore, I will find the surface area of EBCF to later compare with the amount of sunlight received without a sun visor.

D: But the sun visor is not extended along the whole windscreen of the car.

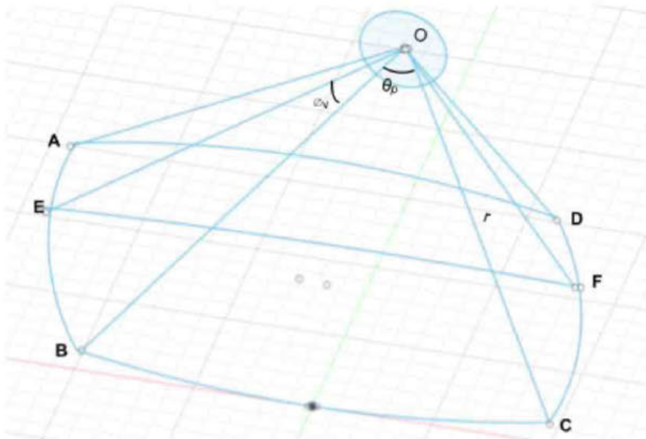


Figure 12. Illustration of driver's field of view with sun visor (AEFD)

As represented in figure 13 below, it is assumed that surface EBCF is an area element on a spherical surface, hence it is drawn on the spherical coordinate. The sphere has a center at O and radius of r. As mentioned above, the horizontal angle between OB and OC is θp. The vertical angle between the z-axis and line OF is Øv.

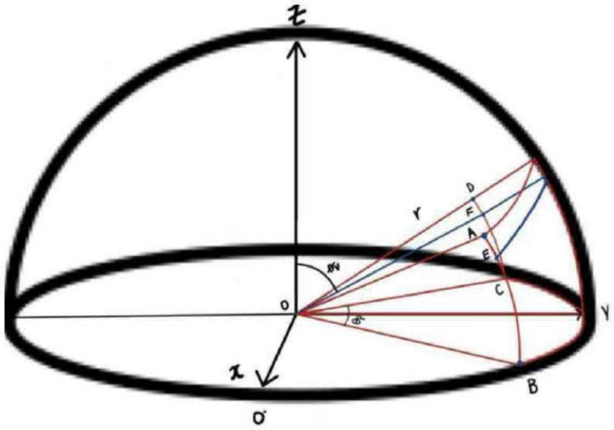


Figure 13. Self-drawn diagram of surface with no sun visor on a spherical coordinates

Utilizing the previously found general expression for the surface area of any curved surface, and the same method of double integral, I can find the surface area of EBCF. In terms of the intervals of double integration in the situation of sun visors present, as shown in figure 14 below, on the YZ plane, line OA is the line from the driver's eye to the ceiling. Line OE is the line from the driver's eye to the endpoint of the sun visor. Hence, as previously mentioned, because the z-axis is 0 and y-axis is on spherical coordinates, the angle between the z-axis and line OE will be øv. Thus, the curved surface area EBCF is created by vertically expanding from øv to $\frac{\pi}{2}$.

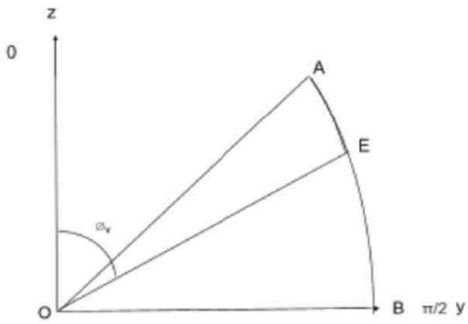


Figure 14. Vertical angles on YZ plane with sun visors

D: There are two sun visors in a car, and not one that spans across the whole windshield.

The second integral with respect to θ will be in the same interval from 0 to θp as sun visors do not affect the horizontal angles. (Refer to figure 11) Hence, double integrals will be used as previous, to find the total amount of sunlight received by the front window even with the presence of a sun visor, the surface area of EBCF will be found, which is

$$SA = \int_0^{\theta_p} \int_{\varnothing v}^{\frac{\pi}{2}} \left(r^2 \sin\varnothing \ d\varnothing \right) d\theta$$

B: $d\emptyset$ is missing.

$$SA = \int_0^{\theta_p} \int_{\varnothing v}^{\frac{\pi}{2}} \left(r^2 \sin\varnothing \ d\varnothing \right) d\theta$$

$$= \int_0^{\theta_p} \left[-r^2 \cos\varnothing \right]_{\varnothing v}^{\frac{\pi}{2}} d\theta$$

$$= \int_0^{\theta_p} \left[-r^2 \cos\frac{\pi}{2} - \left(-r^2 \cos(\varnothing_v) \right) \right] d\theta$$

$$= \int_0^{\theta_p} \left[0 + r^2\cos(\varnothing_v)\right] d\theta$$

Treating $r^2cos\ (\emptyset v)$as a constant and integrating with respect to θ,

$$= r^2\cos(\varnothing_v)\int_0^{\theta_p}[1]\, d\theta$$

$$= r^2\cos(\varnothing_v)\ [1]_0^{\theta_p}$$

$$= r^2\cos(\varnothing_v)[\theta_p]$$

E: An error in the integral, which is corrected on the next line.

$$= r^2\theta_p\cos(\varnothing_v)$$

Hence, the surface area of EBCF (see fig 12) can be expressed as $r^2\theta_p cos(\emptyset_v)$.

The percentage of efficiency of sun visors can be therefore expressed as

$$\%\ Eff\,iciency = \frac{SA\ of\ \ curved\ surf\,ace\ ABCD\ -\ SA\ of\ \ curved\ surf\,ace\ EBCF}{SA\ of\ \ curved\ surf\,ace\ ABCD} \times 100$$

$$\%\ Eff\,iciency = \frac{r^2\theta_p\cos(\varnothing_p) - r^2\theta_p\cos(\varnothing_v)}{r^2\theta_p\cos(\varnothing_p)} \times 100$$

Since $r^2\theta_p$ is a common term in all terms of the numerator and denominator, we can factor out $r^2\theta_p$ from both,

$$= \frac{r^2\theta_p\left(\cos(\varnothing_p) - \cos(\varnothing_v)\right)}{r^2\theta_p\left(\cos(\varnothing_p)\right)} \times 100$$

Hence by cancelling out $r^2\theta_p$, we obtain

$$= \frac{\left(\cos(\varnothing_p) - \cos(\varnothing_v)\right)}{\left(\cos(\varnothing_p)\right)} \times 100$$

Hence, this expression is the efficiency of sun visors found from a three dimensional perspective.

To solve my personal problem, I will substitute real measurements from my parent's car to the previously found expression to determine efficiency considered in three dimensions and express them with percentages.

Previously in part I, as shown in figure 15 below, θc was the angle from the horizontal line at the driver's eye level to the endpoint of the ceiling of the vehicle, measured as $arctan\ \frac{19.3}{35.1}$ (0.5027338… rad). θv was the angle from the horizontal line at driver's eye level to the endpoint of the sun visor, measured $arctan(\frac{1{,}9}{10.1})$(0.1859445… rad). However, this was a measurement on the XY plane (see fig 11) with vertical axis, y-axis being $\frac{\pi}{2}$ and x-axis 0, as in part I, I investigated from a two-dimensional perspective. In part II, since I investigated from a three-dimensional perspective on the spherical coordinates, the vertical angles ($\emptyset_v$ and $\emptyset_p$) are represented on the YZ plane (see fig 14), where opposite from the XY plane, the vertical axis, z-axis is 0 and y-axis is $\frac{\pi}{2}$. Hence, because the axis value are reversed, referring to the two figures below, $\emptyset_p$ should be

D: No justification given for reasonable degree of accuracy.

$$\emptyset_p = \frac{\pi}{2} - \theta_c$$

$$= \frac{\pi}{2} - arc\tan\left(\frac{19.3}{35.1}\right)$$

And $\emptyset_v$ should be

$$\emptyset_v = \frac{\pi}{2} - \theta_v$$

$$= \frac{\pi}{2} - arc\tan\left(\frac{1.9}{10.1}\right)$$

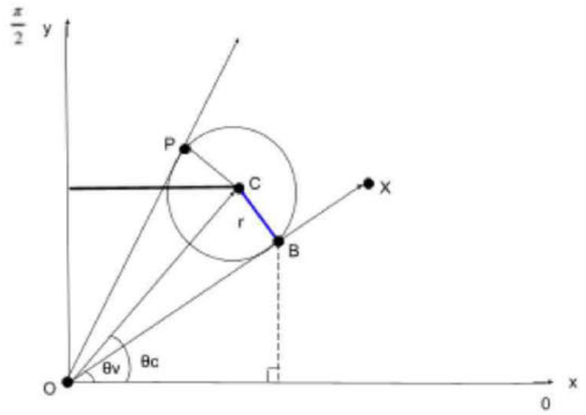


Figure 15. Angles on XY plane

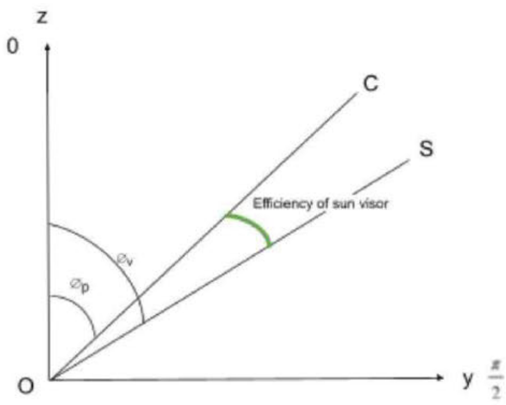


Figure 16. Angles on YZ plane

Hence using a GDC to substitute the value of $\varnothing_p$ and $\varnothing_v$ to the expression of efficiency found in part II,

$$= \frac{\left(\cos\left(\varnothing_p\right) - \cos\left(\varnothing_v\right)\right)}{\left(\cos\left(\varnothing_p\right)\right)} \times 100$$

$$\%\ Efficiency = \frac{\cos\left(\frac{\pi}{2} - arc\tan\left(\frac{19.3}{35.1}\right)\right) - \cos\left(\frac{\pi}{2} - arc\tan\left(\frac{1.9}{10.1}\right)\right)}{\cos\left(\frac{\pi}{2} - arc\tan\left(\frac{19.3}{35.1}\right)\right)} \times 100$$

= 61.6 % (3SF)

Therefore, the efficiency of a sun visor in my car would be 61.6% from a three dimensional perspective.

Conclusion

From part I and part II, I have investigated both in two and three dimensions about the efficiency of sun visors in terms of their effectiveness at blocking sunlight from directly hitting the driver's eye. In part I, I have investigated by considering the angle of sunlight, and in part II I have investigated by considering the total surface area the vehicle receives.

In conclusion, from part, it can be determined that efficiency of sun visors can be expressed as

$$\frac{\theta_c - \theta_v}{\theta_c} \times 100$$

Hence substituting the values from my own car, it could be concluded that sun visors are 63.0% efficient at blocking sunlight,therefore not much effective as its efficiency doesn't exceed 70%, which I defined to be the standard for being efficient. From part II, the efficiency of sun visors can be expressed as

$$\frac{\left(\cos\left(\varnothing_p\right) - \cos\left(\varnothing_v\right)\right)}{\left(\cos\left(\varnothing_p\right)\right)} \times 100$$

Hence substituting values from my own car, it could be concluded that sun visors are 61.6% efficient at blocking sunlight, yielding a similar percentage efficiency as found in part I from a two dimensional perspective only with an error of 2.2% between the efficiency values. Although it didn't exceed the standard 70%, considering that it still blocks more than half of the sun entering the vehicle, it is worth using it for safety and convenience while driving.

D: Some limited reflection is seen in the paragraphs here.

Evaluation

The method used to determine the efficiency of sun visors can be said to be accurate to some extent. The limitations of my result includes the facts that there are certain assumptions in the process of investigation. The assumptions are visible in part II, as when determining the efficiency in three dimensions by calculating the surface area of the front window, the shape of the front window was assumed to be a rectangular shape with arc-shaped sides. This may have affected my results as not all vehicles have the window shapes as such and it may be that the length and the width are not the same shape, which then would have changed my results. Hence, for further investigation, it would be interesting to investigate the efficiencies of sun visors in vehicles with different front window shapes. Other minor limitations could have been that when measuring the real values from a real life vehicle to substitute into the expressions, the measuring tape had an uncertainty of ± 0.5cm. Hence, this would have created systematic errors in my result. Especially since I rounded the real values to whole numbers for convenience in finding the approximate efficiency ratio, there would have been certain errors. Also, when I was measuring the values, there would have been human errors involved.

Despite the small limitations in the process of my investigation, the mathematics provided an accurate insight into the efficiency of sun visors. I have achieved to use previously abstract mathematical concepts such as vectors and integrals to solve a real-world problem that I faced. This topic was worth investigating as it proves that sun visors are efficient only to some extent, implying we could efficiently use it. However, because it is not fully efficient, this could be considered to develop methods to make it more efficient, such as extending the sun visor length. An additional topic worth investigating could be therefore to what extent a 1cm extension in sun visors would be efficient in blocking sunlight, considering it would not block the driver's vision. This investigation also raised the question of the relationship between efficiency and angle of sun visors, if they are adjustable. My results are significant as it could be used for further developments such as fully automatic sun visors for vehicles using optical sensors in the future for greater safety and convenience.

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