Name:

Mathematics preIB Examination

June 11, 2021

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A scientific calculator is required for this paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [**80 marks**].
- Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to **show all working**.
- Write your solutions in the space provided. Underline your answer to each part of the question.

1. [Maximum mark: 4]

An IB class consists of 31 students. 17 of them chose Mathematics HL and 9 chose Physics. 10 students chose neither Mathematics HL nor Physics.

(a) Find the number of students who chose both Mathematics HL and Physics. [1]

(b) Write down the number of students who chose **exactly one** of the two subjects. [1]

(c) Two students are chosen at random. Find the probability that both of them chose both Maths HL and Physics. [2]

2. [Maximum mark: 5]

The probability that Joe cycles to school is $\frac{1}{3}$. If he doesn't cycle, he takes a bus. If he cycles, the probability that he is late is $\frac{1}{10}$. If the takes a bus, the probability that he is late is $\frac{1}{5}$.

(a) Find the probability that on a random school day Joe is late. [2]

(b) Joe was **not** late today. Find the probability that he cycled to school. [3]

3. [Maximum mark: 4]

Consider the line l given by the equation $y = 2x - 6$.

(a) Sketch l clearly indicating the points where it intersects the coordinate axes. [2]

(b) Another line m passes through $(4, 1)$ and is perpendicular to l. Find the equation of m. [2]

4. [Maximum mark: 4]

Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, where $x \in \mathbb{R}$.

(a) Find the composition $(g \circ f)(x)$. Give your answer in the form $(g \circ f)(x) = ax^2 + bx + c$. [2]

(b) Find $f^{-1}(x)$, the inverse of $f(x)$. [2]

5. [Maximum mark: 5]

Let $f(x) = (2 - x)(x + 4)$, where $x \in \mathbb{R}$.

(a) Find the coordinates of the vertex of the graph of $y = f(x)$. [2]

(b) Sketch the graph of $y = f(x)$. [2]

(c) Let $g(x) = \sqrt{f(x)}$. State the largest possible domain of $g(x)$. [1]

6. [Maximum mark: 6]

Solve the following equations:

(a)

$$8^{3x-1} = \left(\frac{1}{2}\right)^{2-x}$$

[2]

(b)

$$\log x + \log(x-3) = 1$$

[2]

(c)

$$2^{2x+1} + 4 = 9 \times 2^{x}$$

[2]

7. [Maximum mark: 5]

Consider the function $f(x) = x^2 + 4x + 7$.

(a) Show that: [1]

$$(x+2)^2 + 3 \equiv x^2 + 4x + 7$$

(b) Hence describe the transformation that turns the graph of $y = x^2$ into the graph $y = f(x)$. [1]

(c) Let $g(x) = 2x^2 + 4x + 10$. Find the transformations that turn the graph of $y = x^2$ into the graph of $y = g(x)$. [3]

8. [Maximum mark: 5]

The graph of the function $f(x) = 2^x$ has been reflected in the y-axis and then translated by the vector $\binom{2}{-1}$ to get the graph of $g(x)$.

(a) Find the equation of $g(x)$. [2]

(b) Sketch the graph of $g(x)$ clearly indicating the axes intercepts and the asymptote. [3]

9. [Maximum mark: 6]

Consider an arithmetic sequence with the third term equal to 7 and the seventh term equal to 23.

(a) Find the first term of the sequence. [2]

(b) Find how many terms of this sequence are smaller than 100. [2]

(c) Find the sum of the first fifteen terms of this sequence. [2]

10. [Maximum mark: 4]

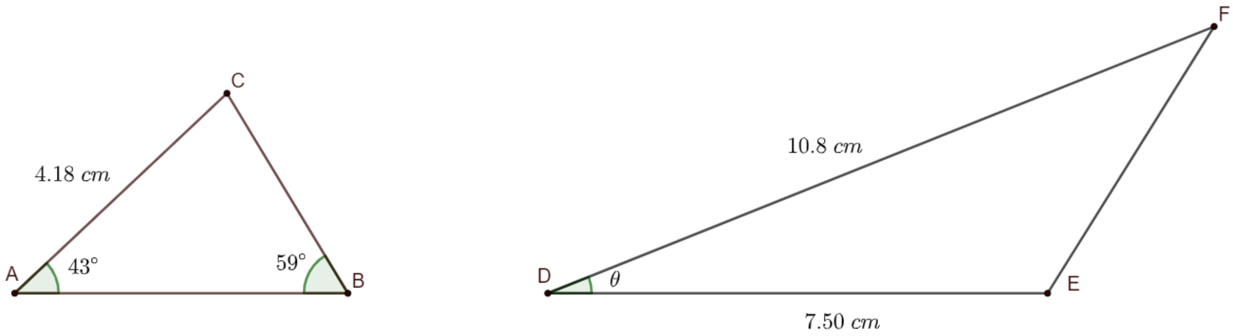
$2, x+1, 4x-2$ are the first three terms of an increasing geometric.

(a) Find the value of x. [3]

(b) Write down the fourth term of this sequence. [1]

11. [Maximum mark: 6]

Consider the following triangles.



(a) Find the length of AB [2]

(b) Show that the area is $\triangle ABC$ is 6.80 cm^2, correct to 3 significant figures. . [2]

(c) The area of $\triangle DEF$ is twice that of $\triangle ABC$. Find the size of the angle θ. [2]

12. [Maximum mark: 6]

A hiker starts at a camp C and walks 5 km at a bearing of $37°$. He then walks another 7 km at a bearing of $121°$ to reach point P. He then wants to return directly to the camp.

(a) Calculate the distance he needs to walk back. [3]

(b) Calculate the bearing of the camp C from the point P. [3]

13. [Maximum mark: 5]

Consider the following data:

$$1,\ 2,\ 2,\ 2,\ 2,\ 3,\ 3,\ 3,\ 4,\ 5$$

(a) Find: [3]

(i) mean,

(ii) median,

(iii) mode.

The standard deviation is equal to 1.1.

(b) Each element of the data set has been increased by 3. Write down the mean and the standard deviation of the new set. [2]

14. [Maximum mark: 5]

1st year students at Batory have to choose one of the four languages: French, German, Italian or Spanish. The following table shows the choices of the students.

	French	German	Italian	Spanish
Girls	16	6	13	10
Boys	4	12	5	14

(a) A student is selected at random. Find the probability that it is a girl given that the student chose French. [2]

χ^2 test was conducted to test if the choice of language is independent of gender.

(b) State the null hypothesis H_0. [1]

(c) Find the expected number of girls who chose French. [1]

(d) The p-value for this test is 6.23×10^{-3} correct to 3 significant figures. State the conclusion of the test. Assume the significance level to be 5%. [1]

15. [Maximum mark: 5]

Consider the matrices:

$$A = \begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}$$

(a) Find the inverse of A and show that B is singular (not invertible). [3]

(b) **Using your answer to part (a)** solve the system: [2]

$$\begin{cases} 2x + 3y = -1 \\ 3x - y = 15 \end{cases}$$

16. [Maximum mark: 5]

Consider vectors $\vec{v} = \binom{3n}{n-1}$ and $\vec{u} = \binom{n+1}{n-3}$.

(a) Find the angle between theses vectors when $n = 2$. [2]

(b) Show that there is no value of n for which these vectors are perpendicular. [3]