8. A bacteria population, N thousand, has a growth rate modelled by the equation

$$\frac{dN}{dt} = \frac{4000}{1+0.5t}, t \geq 0$$

where t is measured in days. Initially there are 250 bacteria in the population. Find the population size after 10 days.

12. Find $f(x)$ given that $f''(x) = 12x + 4$ and that the gradient at the point (1,6) is 12.

13. Find $f(x)$ given that $f'(x) = ax^2 + b$, where the gradient at the point (1,2) is 4 and that the curve passes through the point (3,4).

10. Show that $\frac{2x+6}{x^2+6x+5} \equiv \frac{1}{x+1} + \frac{1}{x+5}$. Hence evaluate $\int_0^2 \frac{2x+6}{x^2+6x+5}dx$

1. Find the area of the region enclosed by the curve $y = \frac{x^2}{x^3+1}$ and the lines $x = 0$ and $x = 2$.

12. Given that $\int_a^b f(x)dx = m$ and $\int_a^b g(x)dx = n$, find

(a) $\int_a^b 2f(x)dx - \int_a^b g(x)dx$ (b) $\int_a^b (f(x) - 1)dx$ (c) $\int_b^a 3g(x)dx$

(d) $\int_a^b (af(x) - m)dx$ (e) $\int_a^b (b^2 g(x) - 2nx)dx$

15. The rate of flow of water, $\frac{dV}{dt}$ litres/hour , pumped into a hot water system over a 24-hour period from 6:00 am, is modelled by the relation $\frac{dV}{dt} = 12 + \frac{3}{2}\cos\frac{\pi}{3}t$, $t \geq 0$.

(a) Sketch the graph of $\frac{dV}{dt}$ against t.

(b) For what percentage of the time will the rate of flow exceed 11 litres/hour .

(c) How much water has been pumped into the hot water system by 8:00 am?

10. Find the area of the region enclosed by the curve $y = x(x+1)(x-2)$ and the x–axis.

15. Find the area of the region enclosed by the curve $f(x) = \frac{2}{(x-1)^2}$

(a) the x–axis, the lines $x = 2$ and $x = 3$,

(b) the y–axis, the lines $y = 2$ and $y = 8$.

21. (a) Sketch the graph of the function $f(x) = |e^x - 1|$.

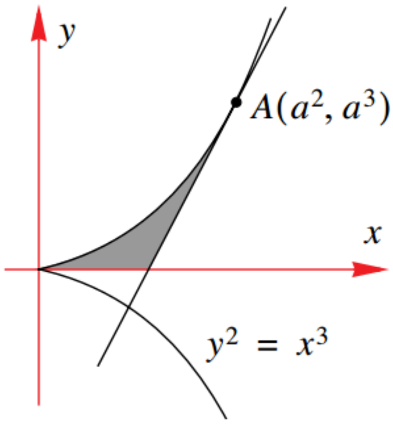
(b) Find the area of the region enclosed by the curve $y = f(x)$,

i. the x–axis and the lines $x = -1$ and $x = 1$.

ii. the y–axis and the line $y = e - 1$.

iii. and the line $y = 1$. Discuss your findings for this case.

23. Consider the curve with equation $y^2 = x^3$ as shown in the diagram.
A tangent meets the curve at the point $A(a^2, a^3)$.



(a) Find the equation of the tangent at A.

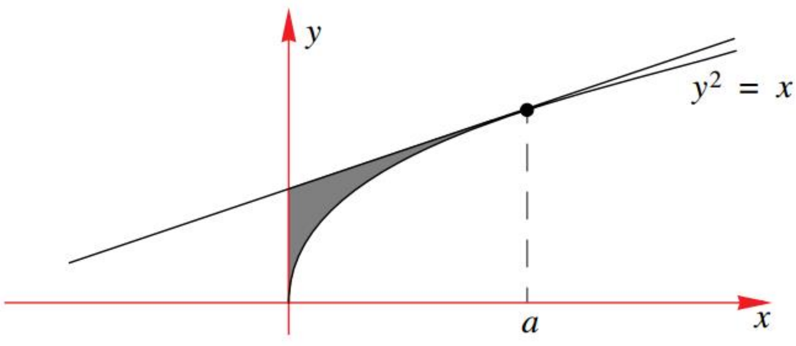
(b) Find the area of the shaded region enclosed by the curve, the line $y = 0$ and the tangent.

24. (a) On a set of axes, sketch the graph of the curve $y = e^{x-1}$ and find the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$.

(b) Hence evaluate $\int_{e^{-1}}^{1} (\ln x + 1)dx$.

(c) Find the area of the region enclosed by the curves $y = e^{x-1}$ and $y = \ln x + 1$ over the $e^{-1} \le x \le 1$.

25. The area of the shaded region enclosed by the y-axis, the tangent to the curve at $x = a$ and the curve $y^2 = x$, $y \ge 0$, as shown in the diagram below, measures $\frac{16}{3}$ sq. units.



Find the exact value of a.