1. Find the area enclosed by the curve $y = \arctan x$, the x-axis and the line $x = \sqrt{3}$.

(Total 6 marks)

2. The function f is defined on the domain $x \geq 0$ by $f(x) = \dfrac{x^2}{e^x}$.

(a) Find the maximum value of $f(x)$, and justify that it is a maximum.

(10)

(b) Find the x coordinates of the points of inflexion on the graph of f.

(3)

(c) Evaluate $\int_0^1 f(x)\mathrm{d}x$.

(8)

(Total 21 marks)

3. Calculate the exact value of $\int_1^e x^2 \ln x \, \mathrm{d}x$.

(Total 5 marks)

4. It is given that

$$f(x) = \frac{18(x-1)}{x^2},\ f'(x) = \frac{18(2-x)}{x^3}, \text{ and } f''(x) = \frac{36(x-3)}{x^4},\ x \in \mathbb{R},\ x \neq 0.$$

(a) Find

(i) the zero(s) of $f(x)$;

(ii) the equations of the asymptotes;

(iii) the coordinates of the local maximum and justify it is a maximum;

(iv) the interval(s) where $f(x)$ is concave up.

(9)

(b) Hence sketch the graph of $y = f(x)$.

(5)

(Total 14 marks)

5. Find the gradient of the normal to the curve $3x^2y + 2xy^2 = 2$ at the point $(1, -2)$.

(Total 6 marks)

6. A normal to the graph of $y = \arctan(x - 1)$, for $x > 0$, has equation $y = -2x + c$, where $c \in \mathbb{R}$.

Find the value of c.

(Total 6 marks)

7. The function f is defined by $f(x) = x\,e^{2x}$.

It can be shown that $f^{(n)}(x) = (2^n x + n\,2^{n-1})\,e^{2x}$ for all $n \in \mathbb{Z}^+$, where $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$.

(a) By considering $f^{(n)}(x)$ for $n = 1$ and $n = 2$, show that there is one minimum point P on the graph of f, and find the coordinates of P.

(7)

(b) Show that f has a point of inflexion Q at $x = -1$.

(5)

(c) Determine the intervals on the domain of f where f is

(i) concave up;

(ii) concave down.

(2)

(d) Sketch f, clearly showing any intercepts, asymptotes and the points P and Q.

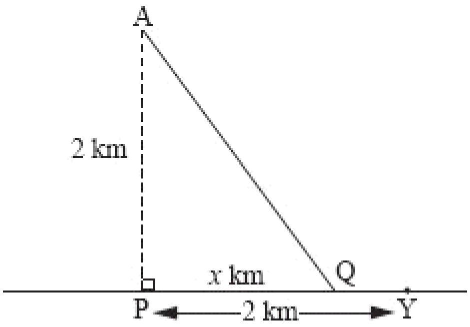
(4)

(e) Use mathematical induction to prove that $f^{(n)}(x) = (2^n x + n2^{n-1})\,e^{2x}$ for all $n \in \mathbb{Z}^+$, where $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$.

(9)

(Total 27 marks)

8. André wants to get from point A located in the sea to point Y located on a straight stretch of beach. P is the point on the beach nearest to A such that AP = 2 km and PY = 2 km. He does this by swimming in a straight line to a point Q located on the beach and then running to Y.



When André swims he covers 1 km in $5\sqrt{5}$ minutes. When he runs he covers 1 km in 5 minutes.

(a) If PQ = x km, $0 \le x \le 2$, find an expression for the time T minutes taken by André to reach point Y.

(4)

(b) Show that $\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{5\sqrt{5}\,x}{\sqrt{x^2+4}} - 5$.

(3)

(c) (i) Solve $\frac{\mathrm{d}T}{\mathrm{d}x} = 0$.

(ii) Use the value of x found in **part (c) (i)** to determine the time, T minutes, taken for André to reach point Y.

(iii) Show that $\frac{\mathrm{d}^2T}{\mathrm{d}x^2} = \frac{20\sqrt{5}}{\left(x^2+4\right)^{\frac{3}{2}}}$ and **hence** show that the time found in **part (c) (ii)** is a minimum.

(11)
(Total 18 marks)