7 When a stone is projected vertically into the air with an initial speed of $30\,\text{m s}^{-1}$ its height, h metres, above the point of projection, at a time t seconds after the instant of projection, can be approximated by the formula $h = 30t - 5t^2$.

Find the maximum height reached by the stone, and the time at which this occurs.

8 A strip of wire of length 28 cm is cut into two pieces. One piece is bent to form a square of side x cm, and the other piece is bent to form a rectangle of width 3 cm.

a) Show that the lengths of the other two sides of the rectangle are given by $(11 - 2x)$ cm.

b) Deduce that the total combined area of the square and the rectangle is $(x^2 - 6x + 33)\,\text{cm}^2$.

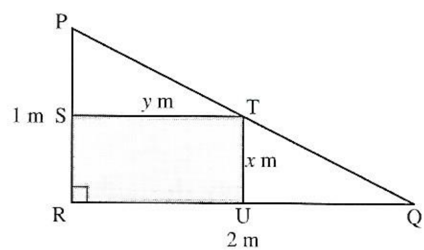
c) Prove that the minimum total area which can be enclosed in this way is $24\,\text{cm}^2$.

9 A string of length 60 cm is cut into two pieces and each piece is formed into a rectangle. The first rectangle has width 6 cm, and the second rectangle is three times as long as it is wide. Given the width of the second rectangle is x cm,

a) deduce that the total combined area enclosed by the two rectangles may be expressed as $[3(x - 4)^2 + 96]\,\text{cm}^2$

b) show that the minimum area which can be enclosed in this way is $96\,\text{cm}^2$.

10 It is required to fit a rectangle of maximum area inside a triangle, PQR, in which PR = 1 metre, RQ = 2 metres, and $\angle PRQ = 90°$. The diagram shows an arbitrary rectangle, RSTU, in which TU = x metres and ST = y metres.



a) Show that $y = 2 - 2x$.

b) Find an expression, in terms of x, for the area of the rectangle, and deduce that the rectangle of maximum area which fits inside triangle PQR has area $\frac{1}{2}\,\text{m}^2$.

*__11__ Show that, in general, for any rectangle drawn inside any right-angled triangle, the area of the rectangle cannot exceed half the area of the triangle.