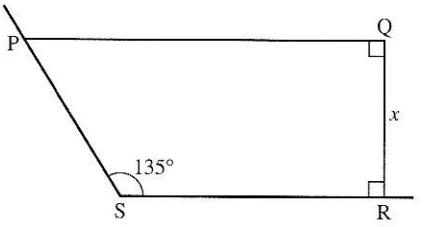
Exercise 1K: Examination questions

1 i) Write $x^2 + 6x + 16$ in the form $(x + a)^2 + b$, where a and b are integers to be found.
ii) Find the minimum value of $x^2 + 6x + 16$ and state the value of x for which this minimum occurs.
iii) Write down the maximum value of the function $\dfrac{1}{x^2 + 6x + 16}$. (MEI)

2 Show that $2x^2 - 36x + 175$ may be written in the form $a(x - b)^2 + c$, where the values of a, b and c are to be found.

State, with a reason, the least value of $2x^2 - 36x + 175$. (WJEC)

3 An enclosure PQRS is to be made as shown in the figure. PQ and QR are fences of total length 300 m. The other two sides are hedges. The angles at Q and R are right angles and the angle at S is 135°. The length of QR is x metres.



a) Show that the area, A m^2, of the enclosure is given by

$$A = 300x - \frac{3x^2}{2}$$

b) Show that A can be written as $-\dfrac{3}{2}[(x - a)^2 - b]$, where a and b are constants whose values you should determine. Hence show that A cannot exceed 15 000. (AEB Spec)

4 Given that, for all values of x,

$$3x^2 + 12x + 5 \equiv p(x + q)^2 + r$$

a) Find the values of p, q and r.
b) Hence, or otherwise, find the minimum value of $3x^2 + 12x + 5$.
c) Solve the equation $3x^2 + 12x + 5 = 0$, giving your answers to one decimal place. (EDEXCEL)

5 Find the set of values of x for which $2(x^2 - 5) < x^2 + 6$. (EDEXCEL)

6 Find the set of values of x for which $x^2 - x - 12 > 0$. (NEAB)

7 Find the set of values of x for which $2x(x + 3) > (x + 2)(x - 3)$. (EDEXCEL)

8 Find the set of values of x for which $\dfrac{x}{x + 4} > 2$. (EDEXCEL)

9 Find all the values of x for which $\dfrac{3x^2 - 1}{x^2 + 1} > 1$. (UODLE)

10 A rectangular tile has length x cm and breadth $(6 - x)$ cm. Given that the area of the tile must be at least 5 cm^2, form a quadratic inequality in x and hence find the set of possible values of x. (EDEXCEL)

11 A landscape gardener is given the following instructions about laying a rectangular lawn. The length x m is to be 2 m longer than the width. The width must be greater than 6.4 m and the area is to be less than 63 m^2.

By forming an inequality in x, find the set of possible values of x. (EDEXCEL)

12 Find the range of values of k for which the quadratic equation

$$(3+k)x^2+4x+k=0$$

has real distinct roots. (WJEC)

13 Find the range of values of k for which the quadratic equation

$$x^2+(k-4)x+(k-1)=0$$

has real distinct roots. (WJEC)

14 The quadratic equation $x^2+6x+1=k(x^2+1)$ has equal roots. Find the possible values of the constant k. (AEB 94)

15 Solve the simultaneous equations $y=x-2$, $y^2=x$. (UCLES)

16 Show that the elimination of x from the simultaneous equations

$$x-2y=1$$
$$3xy-y^2=8$$

produces the equation

$$5y^2+3y-8=0$$

Solve this quadratic equation and hence find the pairs (x, y) for which the simultaneous equations are satisfied. (EDEXCEL)

17 Solve the simultaneous equations $x+y=2$, $x^2+2y^2=11$. (UCLES)