SL 1.2, Arithmetic Sequences

In this presentation we will review arithmetic sequences.

We will start by reviewing the definitions and formulae. Then we will go through some more complicated exam-style question focussing on the use of GDC.

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$$u_{n+1} - u_n = costant$$

We use d to denote the common difference in an arithmetic sequence.

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Definitions - exercise

Decide if the following sequences are arithmetic:

- 5, 8, 11, 14, ..., arithmetic with common difference $d = 3$,
- 9, 18, 36, 72, ..., not arithmetic $18 - 9 \neq 36 - 18$,
- 7, 1, -5, -11, ..., arithmetic with common difference $d = -6$,
- 1, 4, 9, 16, ..., not arithmetic $4 - 1 \neq 9 - 4$,
- 3, 7, 11, 16, ..., not arithmetic $7 - 3 \neq 16 - 11$.

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Formulae

Arithmetic sequence

The formula for the *n*-th term of an arithmetic sequence:

$$u_n = u_1 + (n-1)d$$

For example we have:

$$u_{17} = u_1 + 16d$$

To calculate the 17-th term we need to add the common difference to the first term 16 times.

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Exercise 1

The first term of an arithmetic sequence is 7, the fifth term is 23. Find the tenth term.

We set up an equation using the information given in order to calculate d:

$$u_5 = u_1 + 4d$$
$$23 = 7 + 4d$$

We solve to get $d = 4$. Now we calculate the tenth term:

$$u_{10} = u_1 + 9d$$
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Exercise 2

The eight term of an arithmetic sequence is 123, the seventeenth term is 15. Find the the first term and the common difference.

We set up a sytem of equations

$$\begin{cases} u_8 = u_1 + 7d \\ u_{17} = u_1 + 16d \end{cases}$$

In our case we have:

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Exercise 2 - GDC

On **CASIO** we go:

EQUATION $\rightarrow$ F1 SIMULTANEOUS $\rightarrow$ F1 2 UNKNOWNS

and then you enter the coefficients 1 (coefficient of u_1), 7 (coefficient of d), 123 (the result) and 1, 16, 15 in the second line.

On **Ti-84** you press APPS and look for PlySmlt2 and choose 2: Simultaneous Eqn Solver.

We should get $u_1 = 207$ and $d = -12$.

Exercise 3

The first two terms of an arithmetic sequence are 1 and 7 respectively. The last term is equal to 145. How many terms does this sequence has?

We have $u_1 = 1$ and u_7. Let n be the number of terms in our sequence. Then we also have $u_n = 145$ (the last term is 145).

We clearly have $d = 6 (= u_2 - u_1)$. So we can set up an equation involving u_1 and u_n:

$$u_n = u_1 + (n-1) \cdot d$$

We substitute the known values:

$$145 = 1 + (n-1) \cdot 6$$

Solving this gives $n = 25$, so the sequence has 25 terms.

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Series

A **series** is a sum of a sequence.

$2 + 5 + 8 + 11 + ... + 95$ is an example of an **arithmetic series**, because $2, 5, 8, 11, ..., 95$ is an arithmetic sequence.

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Formulae

Arithmetic series

$$S_n = \frac{(u_1 + u_n) \cdot n}{2}$$

where S_n is the sum of the fist n-terms of an arithmetic sequence. u_1 is the first term, u_n is the last, n-th term and n is the number of terms.

Note that we already have a formula for $u_n = u_1 + (n-1)d$. We can substitute this formula to the equation above to get another formula for an arithmetic series:

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Exercise 4

Calculate:

$$2 + 5 + 8 + 11 + \ldots + 95$$

Exercise 4

This is an arithmetic series, so we can just apply the formula:

$$S_n = \frac{(u_1 + u_n) \cdot n}{2}$$

We know the first term $u_1 = 2$, we know the last term $u_n = 95$. However we don't know how many terms there are.

We can write:

$$u_n = u_1 + (n-1)d$$
$$95 = 2 + (n-1) \cdot 3$$

Solving this gives us $n = 32$, there are 32 terms in the series, so we can calculate the sum:

$$S_{32} = \frac{(2+95) \cdot 32}{2} = 1552$$

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These were the basics. Now let's take a look at an exam-style questions.

Exam-style question

Tomasz trains pull-ups. In the first week of training he does 70 pull-ups and in every subsequent week he increases the number of pull-ups per week by 7.

a) Calculate the number of pull-ups Tomasz does in his eight week of training.

b) In which week of training will Tomasz do more than 200 pull-ups for the first time?

c) Calculate the total number of pull-ups Tomasz does in the first six weeks of training.

d) In which week the total number of pull-ups Tomasz has done will first exceed 1000 pull-ups?

Exam-style question (a)

We have an arithmetic sequence with $u_1 = 70$ and $d = 7$. We want u_8.

$$u_8 = u_1 + 7d = 70 + 7 \cdot 7 = 119$$

Tomasz will do 119 pull ups in the eight week of training.

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Exam-style question (b)

The formula for the number of pull-ups Tomasz does in week n is:

$$u_n = u_1 + (n-1)d = 70 + (n-1)\cdot 7 = 63 + 7n$$

We want to solve:

$$u_n > 200$$

So:

$$63 + 7n > 200$$

which gives $n > 19.5714...$, so $n = 20$. Tomasz will do more than 200 pull-ups for the first time in 20th week of training.

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Exam-style question (b) - GDC

Let's see how this can be done on GDC using tables. I strongly recommend the use of tables for questions on sequences.

On **CASIO** we go MENU → TABLE and we enter our formula $Y1 = 63 + 7x$ (it doesn't need to be simplified, you can enter $u_n = 70 + (x - 1)7$). Then press F5 for SETTINGS and set START:1 (we start with the first term), END: 50 (we don't know when to stop, you can change this latter), STEP:1 (we want to see every term).

On **Ti-84** you need to press $Y =$ and enter your formula and then press TABLE (shift+GRAPH).

Then you can go down the table and see that when $n = 19$, Tomasz does 196 pull-ups, and when $n = 20$ he does 203. So the answer to our question is indeed $n = 20$.

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On the exam paper you can't just give the answer, you must write down your solution. If you use tables, then the solution may look as follows:

$$u_{19} = 196, \quad u_{20} = 203, \quad \text{so} \quad n = 20$$

or you can write down the inequality and the answer:

$$70 + (n-1) \cdot 7 > 200 \quad \text{so} \quad n = 20$$

Exam-style question (c)

We want to find the sum of an arithmetic sequence here. We will use the second formula:

$$S_n = \frac{(2u_1 + (n-1)d) \cdot n}{2}$$

We have:

$$S_6 = \frac{(2 \cdot 70 + (6-1) \cdot 7) \cdot 6}{2} = 525$$

He will do a total of 525 in the first six weeks.

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Exam-style question (d)

This part is similar to (b), but this time we want the sum to be greater than 1000. The formula for the total after n weeks is:

$$S_n = \frac{(2u_1 + (n-1)d) \cdot n}{2} = \frac{(140 + (n-1) \cdot 7)n}{2}$$

We don't need to simplify this. Let's just enter the formula into the table on GDC.

We should get: $S_9 = 882$ and $S_{10} = 1015$, so the answer is week 10.

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In case of any questions you can email me at t.j.lechowski@gmail.com or message me via Librus.