SL 1.2, SL 1.3 Sequences

In this presentation we will look at an exam-style question that involves both arithmetic and geometric sequences.

Exam-style question

Maria is offered two pay schemes at her company.

Scheme A: she earns 60 000 PLN in the first year and the salary is increased by 4000 PLN each year.

Scheme B: she earns 55 000 PLN in the first year and the salary is increased by 6% in each subsequent year.

a) Find how much she would earn in the 5th year of work according to each scheme.

b) Find her total earnings in the first 5 years according to each scheme.

c) Find the first year in which she would earn more in scheme B.

d) Find the first year in which her total earnings are greater in scheme B.

Exam-style question (a)

We will do the first two parts algebraically.

Scheme A is an arithmetic sequence with $u_1 = 60000$ and $d = 4000$.
Scheme B is a geometric sequence with $v_1 = 55000$ and $r = 1.06$.

We have:

$$u_5 = 60000 + 4 \cdot 4000 = 76000$$

and:

$$v_5 = 55000 \cdot 1.06^4 \approx 69400$$

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Now we want to sum these sequences.

For Scheme A we have:

$$S_{5A} = \frac{(60000 + 76000) \cdot 5}{2} = 340000$$

For Scheme B we have:

$$S_{5B} = \frac{55000(1.06^5 - 1)}{1.06 - 1} \approx 310000$$

Exam-style question (b)

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Exam-style question (c)

We will use tables now. Press $Y =$ and then put $Y1 = 60000 + (n-1) \cdot 4000$, which is the formula for yearly salary according to Scheme A and let's put $Y2 = 55000 \cdot 1.06^{n-1}$, which is the yearly salary according to Scheme B. Then press TABLE (shift + GRAPH).

You can check your answers to part (a) for $n = 5$.

We can see that when $n = 12$, we have $Y1 < Y2$ for the first time, so in year 12 Maria would earn more with Scheme B for the first time.

As your solution you should write down the inequality you're trying to solve, namely:

$$60000 + (n-1) \cdot 4000 < 55000 \cdot 1.06^{n-1}$$

and then the answer $n = 12$.

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Exam-style question (d)

We will use tables again, but this time we need to use formulae for the sums.

$$Y1 = \frac{(120000 + (n-1)\cdot 4000)\cdot n}{2}$$

$$Y2 = \frac{55000\cdot(1.06^n - 1)}{1.06 - 1}$$

Again we can check our answers to part (b) for $n = 5$.

We can see that the sum is greater for Scheme B for the first time when $n = 18$, so in year 18 her total earning will be greater for the first time with Scheme B. Again as your solution you should write down the inequality:

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