SL 1.3 Geometric sequences

In this presentation we will review and practice problems involving geometric sequences.

Definition

Geogemtric sequence

A sequence u_n is geometric if the **ratio** between consecutive terms of this sequence is constant:

$$\frac{u_{n+1}}{u_n} = costant$$

We use r to denote the common ratio in a geometric sequence.

Definitions - exercise

Decide if the following sequences are geometric:

- 10, 20, 30, 40, ... not geometric (it is in fact arithmetic),
- 9, 18, 36, 72, ... geometric with common ratio $r = 2$,
- 3, 6, 7, 10, ... not geometric $\frac{6}{3} \neq \frac{7}{6}$,
- 1, 4, 9, 16, ... not geometric $\frac{4}{1} \neq \frac{9}{4}$,
- 20, -10, 5, -2.5, ... geometric with common ratio $r = -\frac{1}{2}$,
- 108, 72, 48, 32, ... geometric with common ratio $r = \frac{2}{3}$,

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Formula

Geometric sequence

The formula for the n-th term of a geometric sequence:

$$u_n = u_1 \cdot r^{n-1}$$

For example we have:

$$u_{21} = u_1 \cdot r^{20}$$

To calculate the 21-st term we need to multiply the first term by r 20 times.

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Exercise 1

The first term of a geometric sequence is 5, the fourth term is 40. Find the seventh term.

We set up an equation to calculate r:

$$u_4 = u_1 \cdot r^3$$
$$40 = 5r^3$$

We solve to get $r = 2$. We can now calculate the seventh term:

$$u_7 = u_1 \cdot r^6$$
$$u_7 = 5 \cdot 2^6 = 320$$

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Exercise 2

The first term of a geometric sequence is 256, the last term is 1, the common ratio is $\frac{1}{2}$. How many terms does this sequence have?

We have $u_1 = 256$ and $u_n = 1$. We set up an equation:

$$u_n = u_1 \cdot r^{n-1}$$

$$1 = 256 \cdot \left(\frac{1}{2}\right)^{n-1}$$

We solve (remember that you can use the GDC) to get $n = 9$. So the sequence has 9 terms.

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Geometric series

Recall that a **series** is a sum of a sequence.

$8+4+2+...+\frac{1}{16}$ is an example of a **geometric series**, because $8,\ 4,\ 2,\ ...,\ \frac{1}{16}$ is a geometric sequence.

Remember that instead of the word "series" you can just use "sum of a sequence".

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Formula

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$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

Exercise 3

Calculate:

$$8+4+2+...+\frac{1}{16}$$

Exercise 3

This is a geometric series. We want to apply the formula:

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

We know that $u_1 = 8$ and $r = \frac{1}{2}$, but again we're missing n.

We can write:

$$u_n = u_1 \cdot r^{n-1}$$

$$\frac{1}{32} = 8 \cdot \left(\frac{1}{2}\right)^{n-1}$$

This can be solved algebraically, but you can also just use the SOLVER on your GDC and get $n = 9$, so:

$$S_9 = \frac{8((0.5)^9 - 1)}{0.5 - 1} = 15.96875 \approx 16.0$$

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In case of any questions you can email me at t.j.lechowski@gmail.com or message me via Librus.