Mixed examination practice 7

Short questions

1. The fourth term of an arithmetic sequence is 9.6 and the ninth term is 15.6.
   Find the sum of the first nine terms. *[5 marks]*

2. The sum of the first n terms of a series is given by:
   $$S_n = 2n^2 - n, \text{ where } n \in \mathbb{Z}^+.$$
   (a) Find the first three terms of the series.
   (b) Find an expression for the nth term of the series, giving your answer in terms of n. *[6 marks]*
   (© *IB Organization 2004*)

3. Which is the first term of this sequence which is less than 10^{-6}?
   $$\frac{1}{3}, \frac{1}{9} \cdots \frac{1}{3^n}$$ *[5 marks]*

4. The fifth term of an arithmetic sequence is three times larger than the second term. Find the ratio: $\dfrac{\text{common difference}}{\text{first term}}$ *[6 marks]*

5. A geometric sequence and an arithmetic sequence both start with a first term of 1. The third term of the arithmetic sequence is the same as the second term of the geometric sequence. The fourth term of the arithmetic sequence is the same as the third term of the geometric sequence. Find the possible values of the common difference of the arithmetic sequence. *[7 marks]*

6. Evaluate $\displaystyle\sum_{i=0}^{i=\infty} \frac{\left(2^i + 4^i\right)}{6^i}$. *[6 marks]*

7. Find the sum of all the integers between 300 and 600 which are divisible by 7. *[7 marks]*

8. Find an expression for the sum of the first 23 terms of the series
   $$\ln\frac{a^3}{\sqrt{b}} + \ln\frac{a^3}{b} + \ln\frac{a^3}{b\sqrt{b}} + \ln\frac{a^3}{b^2} + \dots$$
   giving your answer in the form $\ln\dfrac{a^m}{b^n}$, where $m, n \in \mathbb{Z}$. *[7 marks]*

Long questions

1. Kenny is offered two investment plans, each requiring an initial investment of $10 000:

Plan A offers a fixed return of $800 per year.

Plan B offers a return of 5% each year, reinvested in the plan.

(a) Find an expression for the amount in plan A after n years.

(b) Find an expression for the amount in plan B after n years.

(c) Over what period of time is plan A better than plan B?   *[10 marks]*

2. Ben builds a pyramid out of toy bricks. The top row contains one brick, the second row contains three bricks and each row after that contains two more bricks than the previous row.

(a) How many bricks are in the nth row?

(b) If a total of 36 bricks are used how many rows are there?

(c) In Ben's largest ever pyramid he noticed that the total number of bricks was four more than four times the number of bricks in the bottom row. What is the total number of bricks?   *[10 marks]*

3. A pupil writes '1' on the first line of a page, then the next two integers '2, 3' on the second line of the page then the next three integers '4, 5, 6' on the third line. He continues this pattern.

(a) How many integers are on the nth line?

(b) What is the last integer on the nth line?

(c) What is the first integer on the nth line?

(d) Show that the sum of all the integers on the nth line is $\frac{n}{2}(n^2 + 1)$.

(e) The sum of all the integers on the last line of the page is 16 400. How many lines are on the page?   *[10 marks]*

4. Selma has a mortgage of £150 000. At the end of each year 6% interest is added before Selma pays £10 000.

(a) Show that at the end of the third year the amount owing is
$$£150\,000 \times (1.06)^3 - 10\,000 \times (1.06)^2 - 10\,000 \times 1.06 - 10\,000$$

(b) Find an expression for how much is owed at the end of the nth year.

(c) After how many years will the mortgage be paid off?   *[10 marks]*

6. (a) 12 days

(b) Day 102

7. (a) 0.8192 m

(b) 15.85 m

8. (b) $25000(1.04^n - 1)$

(c) Year 29

Mixed examination practice 7

Short questions

1. 97.2

2. (a) 1, 5, 9

(b) $4n - 3$

3. 13th

4. 2

5. $d = 0, -\dfrac{1}{4}$

6. 4.5

7. 19 264

8. $\ln\left(\dfrac{a^{69}}{b^{138}}\right)$

Long questions

1. (a) $10000 + 800n$

(b) 10000×1.05^n

(c) $n < 19$ years

2. (a) $2n - 1$ (b) 6

(c) 64

3. (a) n (b) $\dfrac{n(n+1)}{2}$

(c) $\dfrac{n(n-1)}{2} + 1$

(e) 32

4. (b) $150000 \times 1.06^n - \dfrac{500000(1.06^n - 1)}{3}$

(c) 40 years

◾ Chapter 8

Exercise 8A

1. (a) 4

(b) 35

(c) 7

(d) 56

2. (a) $792x^5 y^7$ (b) $11440 a^7 b^9$

(c) $10 c^3 d^2$ (d) $36 a^2 b^7$

(e) $15 x^2 y^4$

Exercise 8B

1. (a) (i) 216 (ii) 20

(b) (i) $560 x^3 y^4$ (ii) $-280 x^3 y^4$

(c) (i) -5 (ii) 78 030

2. (a) (i) 56 (ii) 80

(b) (i) -672 (ii) -32

3. (a) (i) $32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

(ii) $729 + 1458x + 1215x^2 + 540x^3 + 135x^4 + 18x^5 + x^6$

(b) (i) $243 x^5 + 405 x^4 y + 270 x^3 y^2$

(ii) $16 c^4 - 32 c^3 d + 24 c^2 d^2$

(c) (i) $8x^6 - 36x^5 + 54x^4 - 27x^3$

(ii) $8x^{-3} + 60x^{-2}y + 150x^{-1}y^2 + 125y^3$

(d) (i) $16z^8 + 96z^5 + 216z^2 + 216z^{-1} + 81z^{-4}$

(ii) $27x^3 y^3 + 135x^3 y + 225x^3 y^{-1} + 125x^3 y^{-3}$

4. (a) n (b) $\dfrac{1}{2}n^2 - \dfrac{1}{2}n$

(c) $\dfrac{1}{6}n^3 - 6n^2 + \dfrac{1}{3}n$

5. (a) $80x\,y^4$ (b) $-80x^2 y^3$

6. 720

7. $-945 x^5$

8. 79 200 000

9. 14

10. 12

11. 9

12. 7