Chapter 7

Right angled triangle trigonometry

Contents:

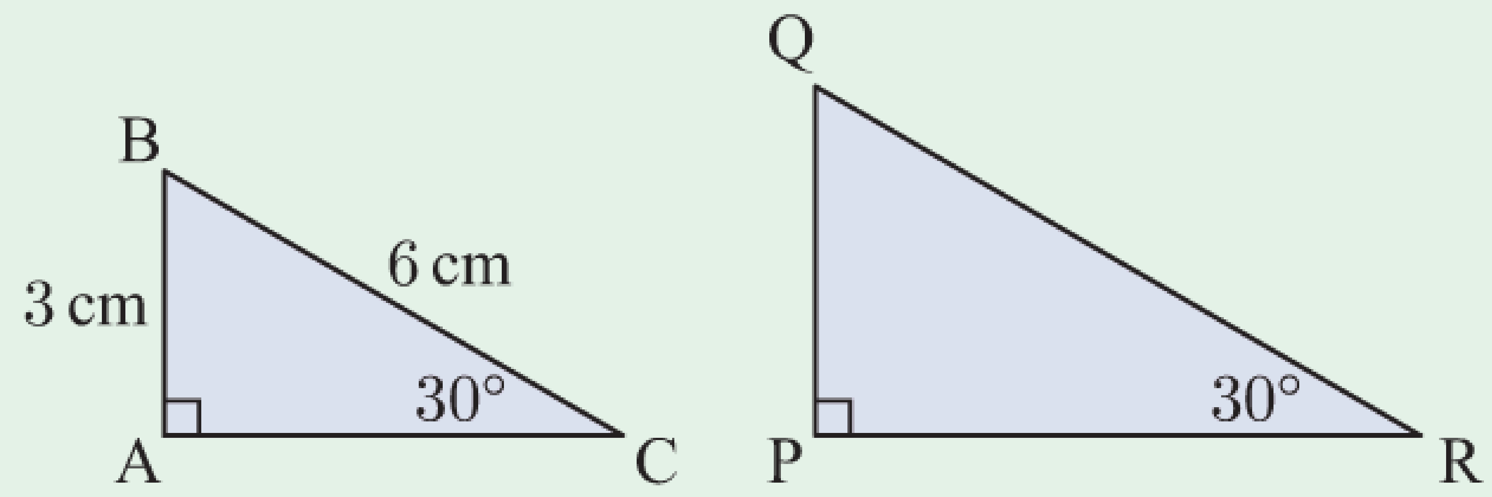
OPENING PROBLEM

Things to think about:

a Both of the right angled triangles alongside contain a 30° angle. From the information we are given, can we determine:

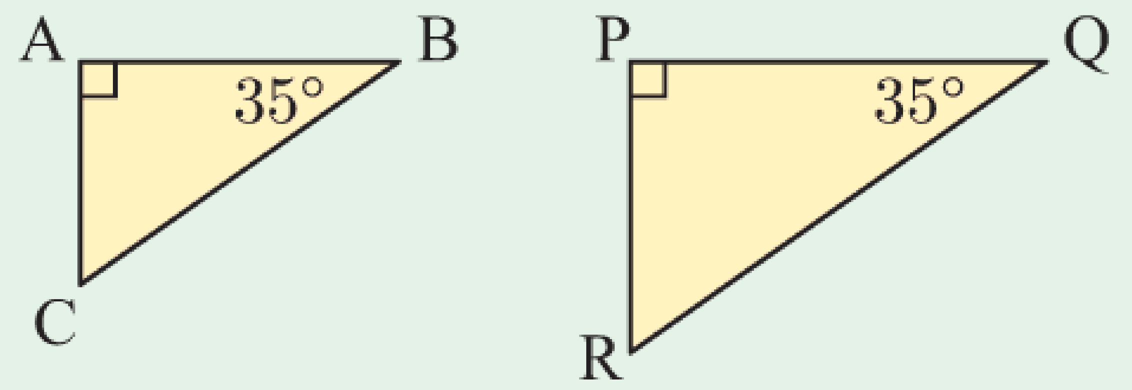
i the lengths PQ and QR

ii the *ratio* of lengths $\frac{PQ}{QR}$?



b Both of the right angled triangles alongside contain a 35° angle. Can you explain why:

i $\frac{AB}{BC} = \frac{PQ}{QR}$



ii any other right angled triangle containing a 35° angle will have corresponding sides in the same ratio?

Trigonometry is the study of the relationships between the side lengths and angles of triangles.

Trigonometry is extensively used in the real world, being essential for engineering, architecture, building, physics, astronomy, navigation, and many other industries.

THEORY OF KNOWLEDGE

The study of celestial objects such as the sun, moon, stars, and planets is called **astronomy**. It has been important to civilisations all over the world for thousands of years, not only because it allowed them to navigate at night, but because the celestial objects feature in so many of their myths and beliefs.

To create an accurate star map, astronomers measure the angles between objects in the sky. The oldest known star map was found in the Silk Road town of Dunhuang in 1907. It was made in the 7th century AD, presumably from the Imperial Observatory in either Chang'an (present day Xi'an) or Luoyang. A possible author of the map was the mathematician and astronomer **Li Chunfeng** (602 - 670). The map shows 1339 stars in 257 star groups recorded with great precision on 12 charts, each covering approximately 30 degree sections of the night sky.[1]

1 How much of what we *believe* comes from what we *observe*? Is it necessary to *understand* something, in order to *believe* it? How much of what we *study* is a quest to *understand* what we *observe*, and *prove* what we *believe*?

2 How much of what we want to know is a common desire of people and cultures all over the world?

3 How did ancient people calculate with such accuracy before computer technology?

[1] "The Dunhuang Chinese Sky: A comprehensive study of the oldest known star atlas", J-M Bonnet-Bidaud, F. Praderie, S. Whitfield, *J. Astronomical History and Heritage*, 12(1), 39-59 (2009).

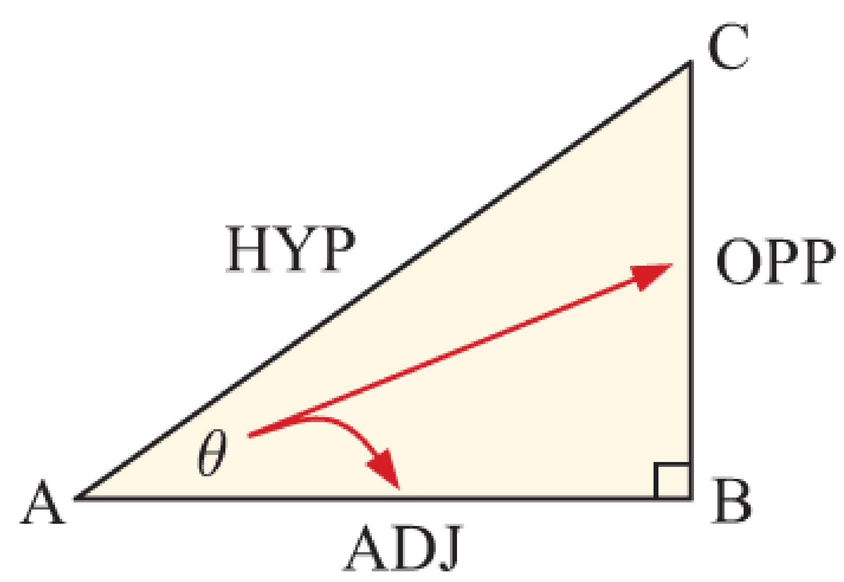
In this Chapter we will study the trigonometry of right angled triangles.

A TRIGONOMETRIC RATIOS

In previous years you should have seen that for any right angled triangle with a fixed angle θ, the side lengths are in the same ratio.

In the triangle alongside:

- The **hypotenuse (HYP)** is the side which is opposite the right angle. It is the longest side of the triangle.
- [BC] is the side **opposite (OPP)** angle θ.
- [AB] is the side **adjacent (ADJ)** to angle θ.



The **trigonometric ratios** for the angle θ are:

$$\sin\theta = \frac{\text{OPP}}{\text{HYP}} \qquad \cos\theta = \frac{\text{ADJ}}{\text{HYP}} \qquad \tan\theta = \frac{\text{OPP}}{\text{ADJ}}$$

They stand for **sine**, **cosine**, and **tangent**.

Notice that $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\text{OPP}}{\text{HYP}}}{\frac{\text{ADJ}}{\text{HYP}}} = \dfrac{\text{OPP}}{\text{ADJ}} = \tan\theta$, so $\quad \boldsymbol{\tan\theta = \dfrac{\sin\theta}{\cos\theta}}$.

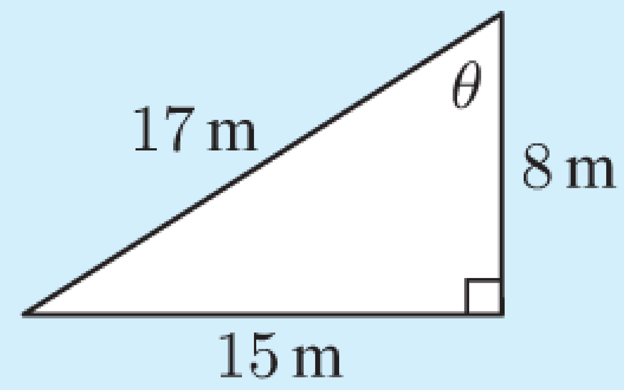
The trigonometric ratios for any angle can be found using a calculator.

GRAPHICS CALCULATOR INSTRUCTIONS

Example 1 — Self Tutor

For the following triangle, find:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



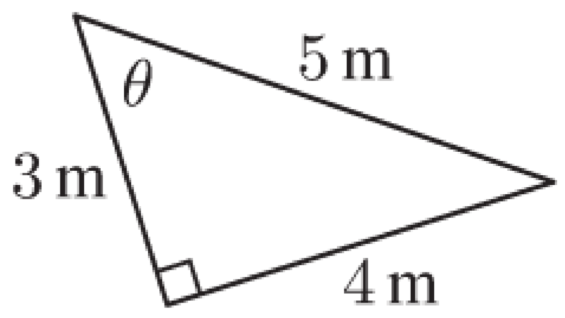
a $\sin\theta = \dfrac{\text{OPP}}{\text{HYP}} = \dfrac{15}{17}$ **b** $\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}} = \dfrac{8}{17}$ **c** $\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}} = \dfrac{15}{8}$

EXERCISE 7A

1 For each of the following triangles, find:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

a



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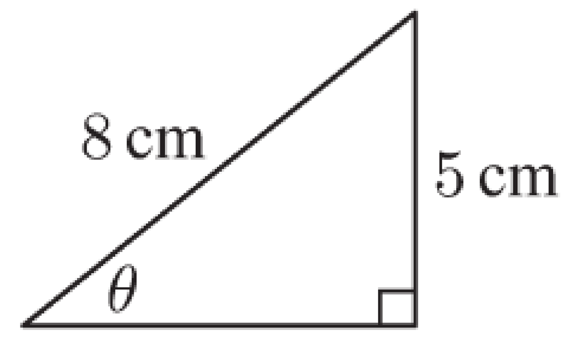
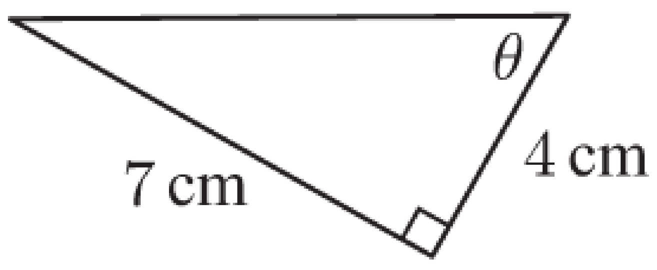


c



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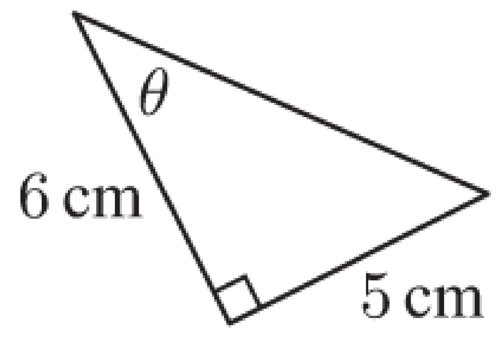
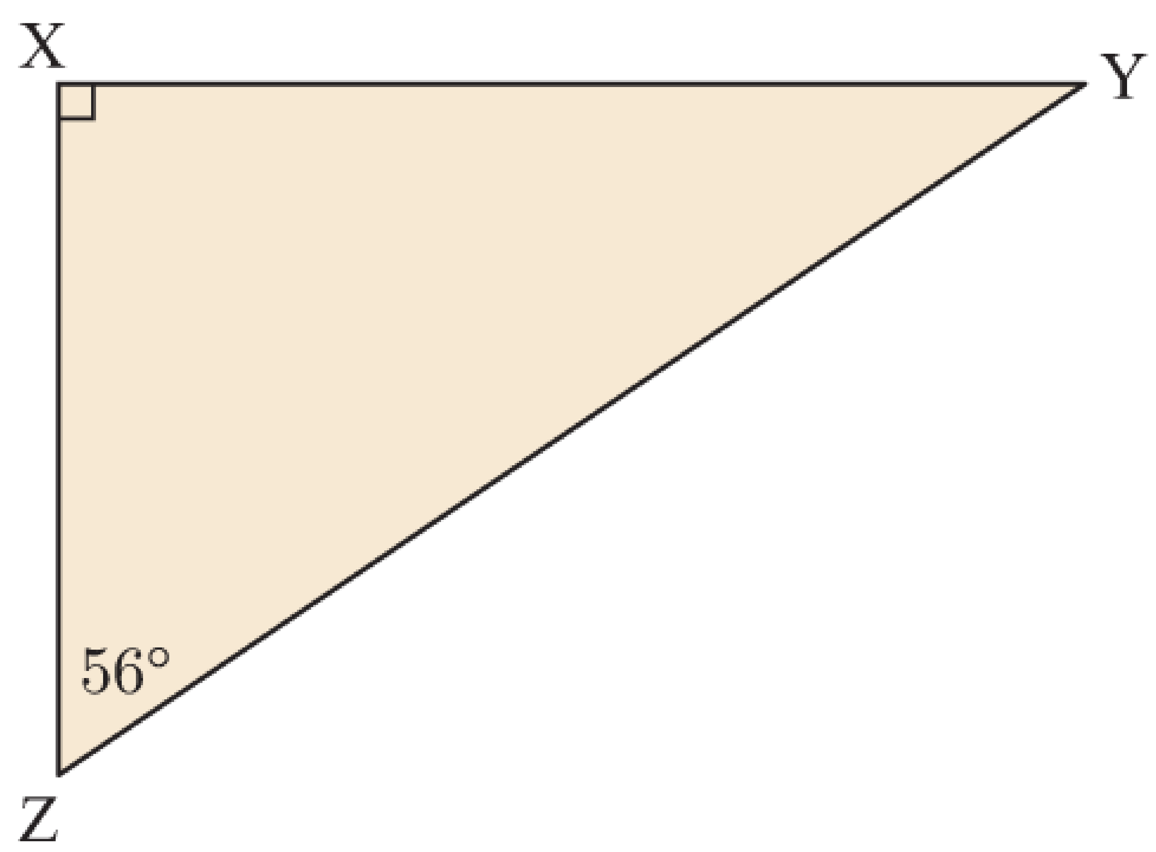


2 The right angled triangle alongside contains an angle of $56°$.



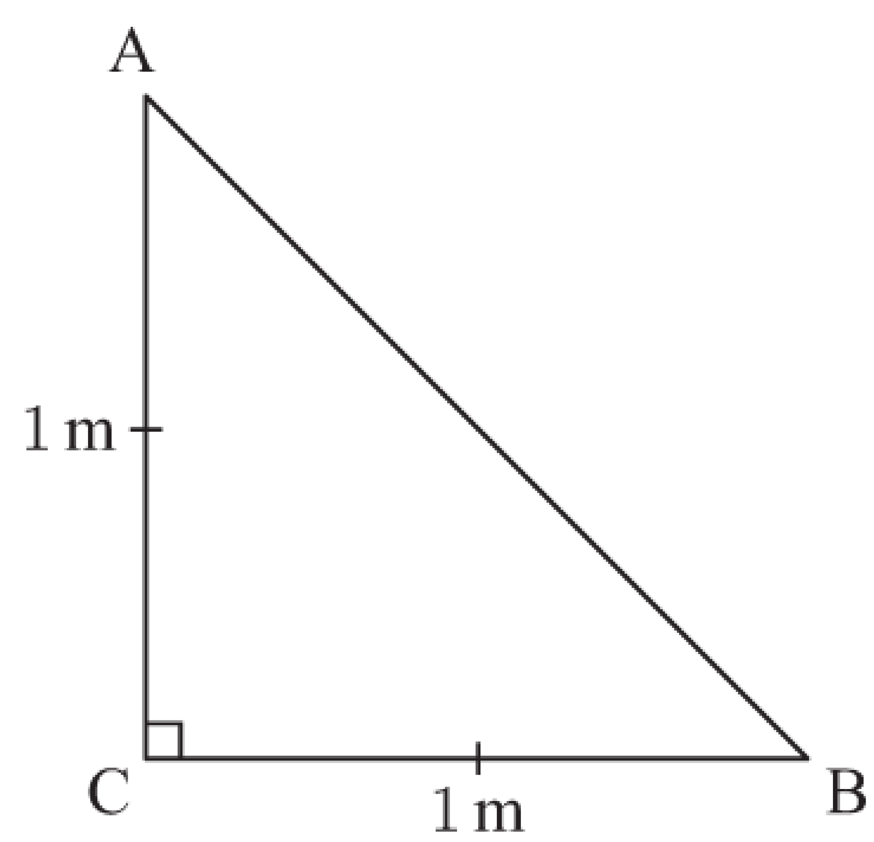
a Use a ruler to measure the length of each side, to the nearest millimetre.

b Hence estimate, to 2 decimal places:

i $\sin 56°$ **ii** $\cos 56°$ **iii** $\tan 56°$

c Check your answers using a calculator.

3 Consider the right angled isosceles triangle ABC alongside.



a Explain why $A\widehat{B}C = 45°$.

b Use Pythagoras' theorem to find AB.

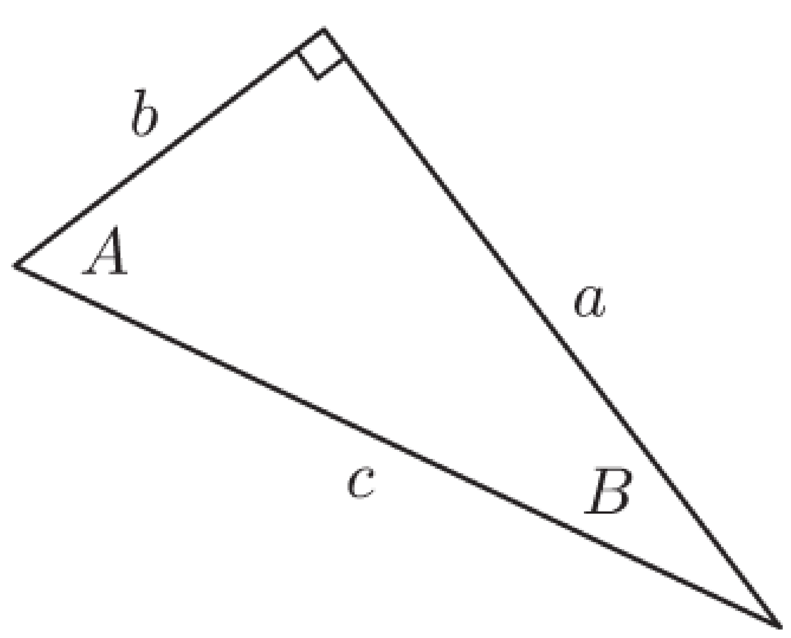
c Hence find:

i $\sin 45°$ **ii** $\cos 45°$ **iii** $\tan 45°$.

d Check your answers using a calculator.

4 Explain why it is impossible for the sine or cosine of an angle to be greater than 1.

5 Consider the right angled triangle shown.



a Write expressions for:

i $\sin A$ **ii** $\cos A$ **iii** $\tan A$

iv $\sin B$ **v** $\cos B$ **vi** $\tan B$

b State the relationship between A and B.

c Hence state the relationship between:

i $\sin\theta$ and $\cos(90° - \theta)$

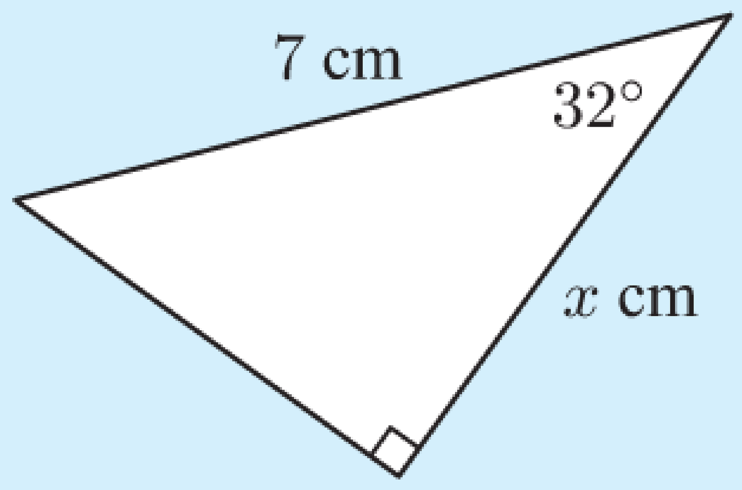
ii $\cos\theta$ and $\sin(90° - \theta)$

iii $\tan\theta$ and $\tan(90° - \theta)$

Example 2 — Self Tutor

Find, correct to 3 significant figures, the unknown length in the following triangles:

a



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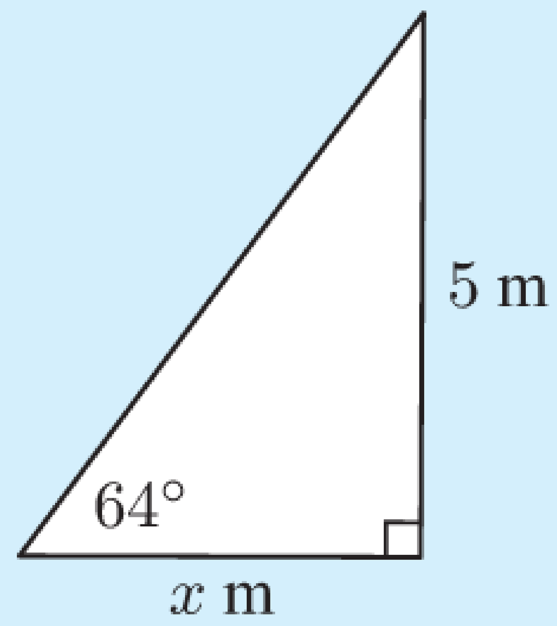
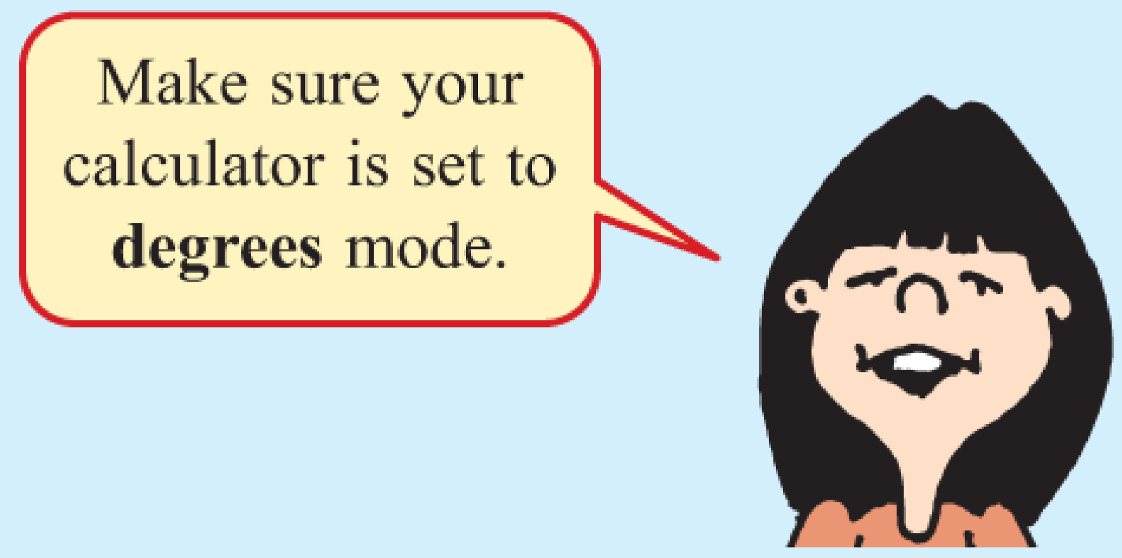
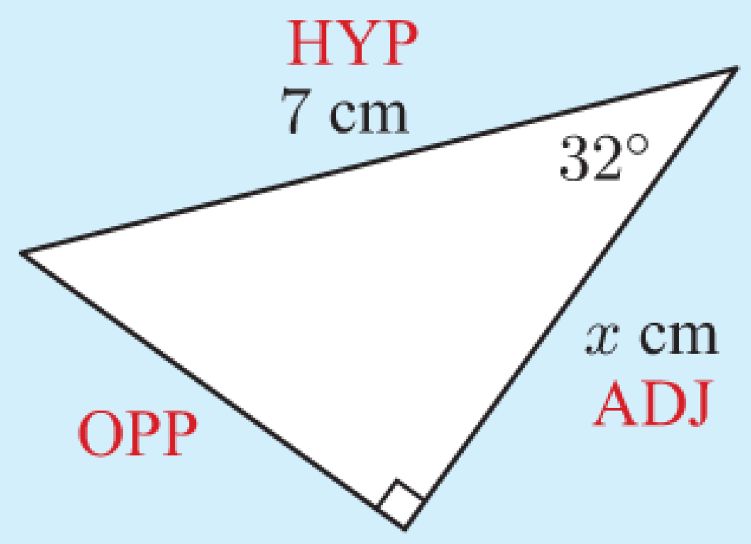




a



The relevant sides are ADJ and HYP, so we use the *cosine* ratio.

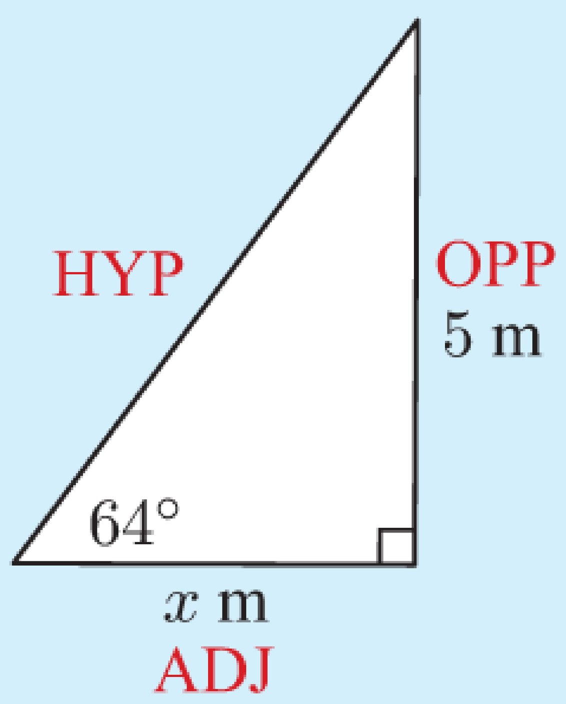
$\cos 32° = \frac{x}{7}$ $\quad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$

$\therefore\ 7 \times \cos 32° = x$ $\quad$ {multiplying both sides by 7}

$\therefore\ x \approx 5.94$ $\quad$ {using technology}

So, the side is about 5.94 cm long.

b



The relevant sides are ADJ and OPP, so we use the *tangent* ratio.

$\tan 64° = \frac{5}{x}$ $\quad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ x \times \tan 64° = 5$ $\quad$ {multiplying both sides by x}

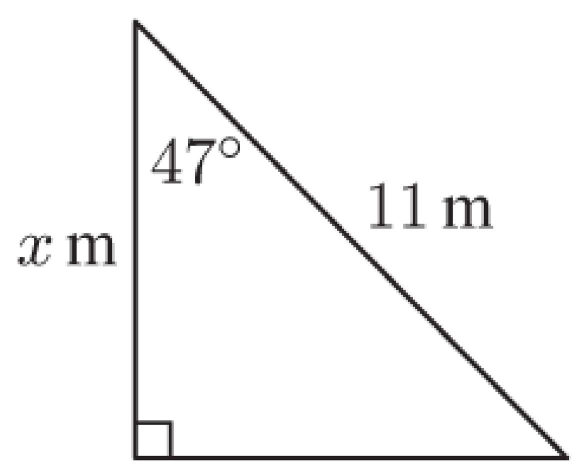
$\therefore\ x = \frac{5}{\tan 64°}$ $\quad$ {dividing both sides by $\tan 64°$}

$\therefore\ x \approx 2.44$ $\quad$ {using technology}

So, the side is about 2.44 m long.

6 Find, correct to 3 significant figures, the unknown length:

a



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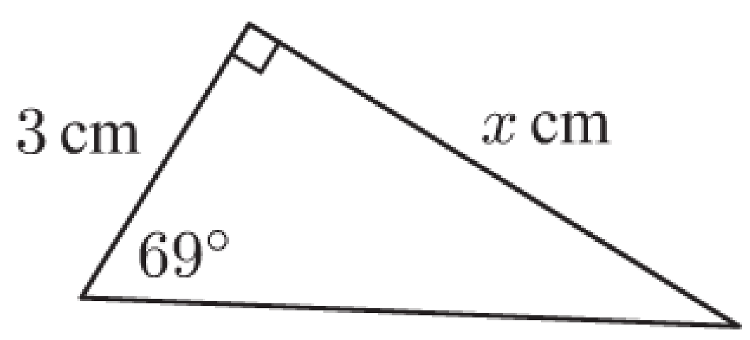

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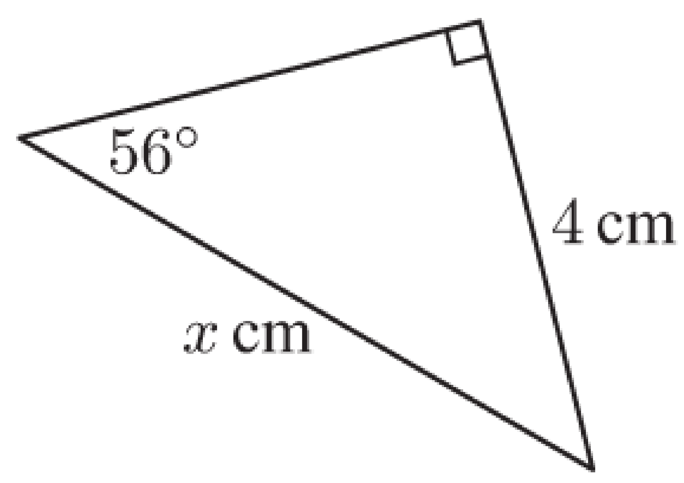

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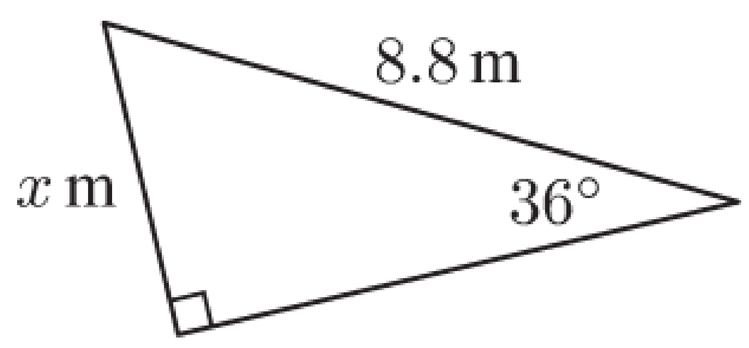

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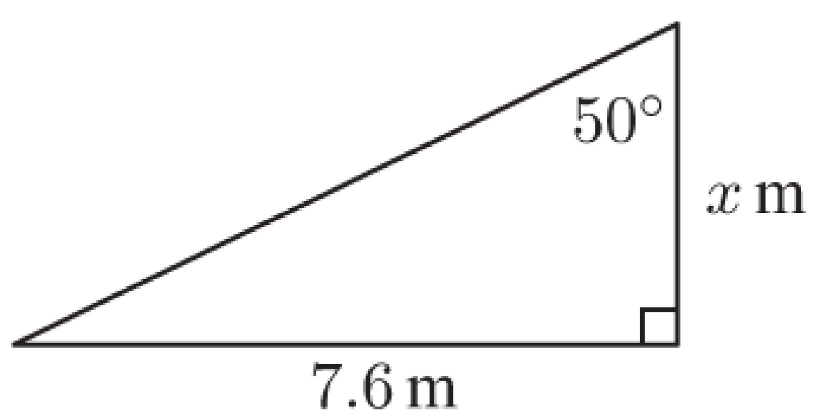

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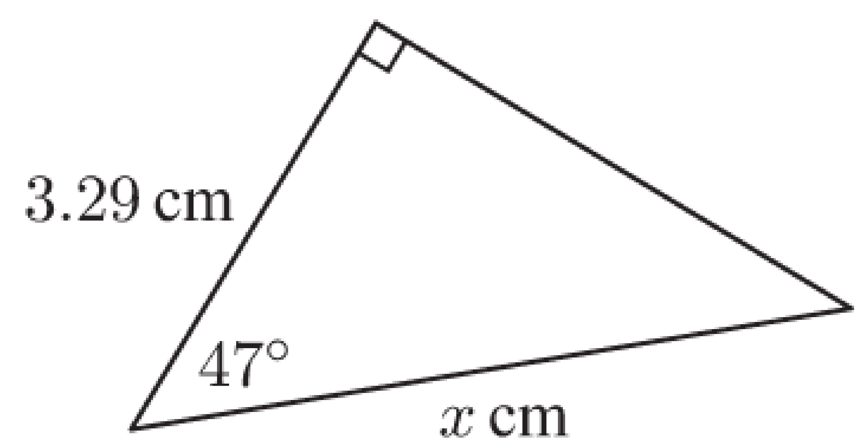


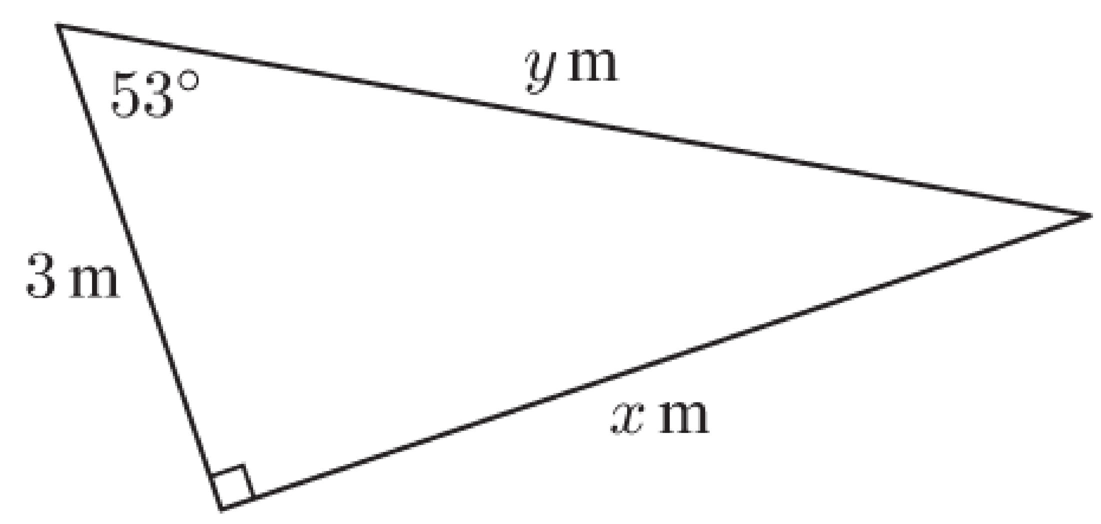
7 Consider the triangle alongside.

a Find x.

b Find y using:

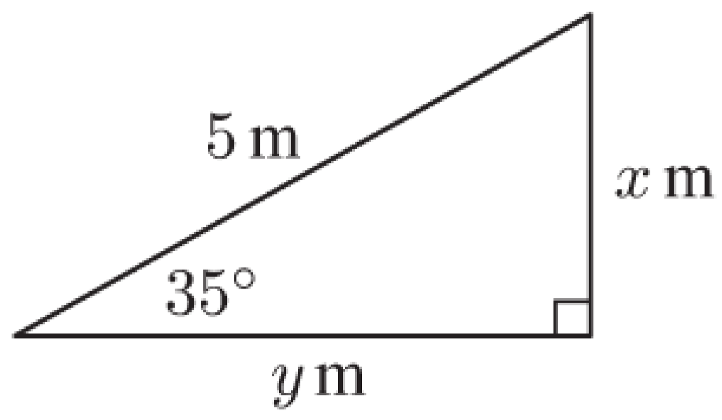
i Pythagoras' theorem

ii trigonometry.



8 Find, correct to 2 decimal places, *all* unknown sides:

a



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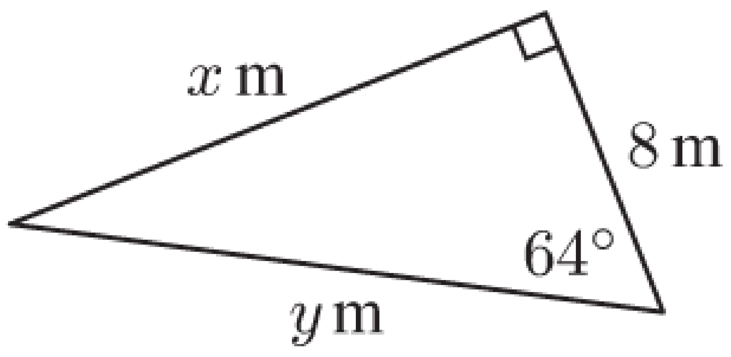

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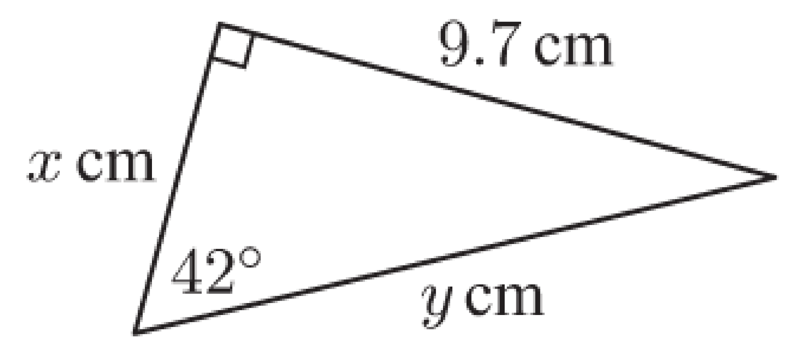
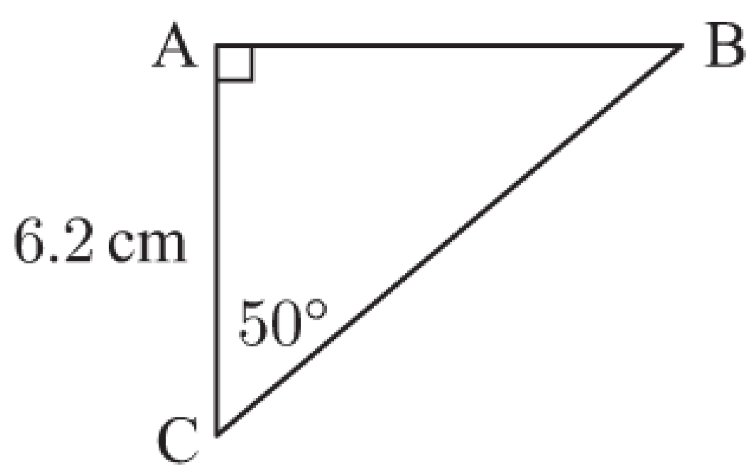


9 Find the perimeter and area of triangle ABC.

a



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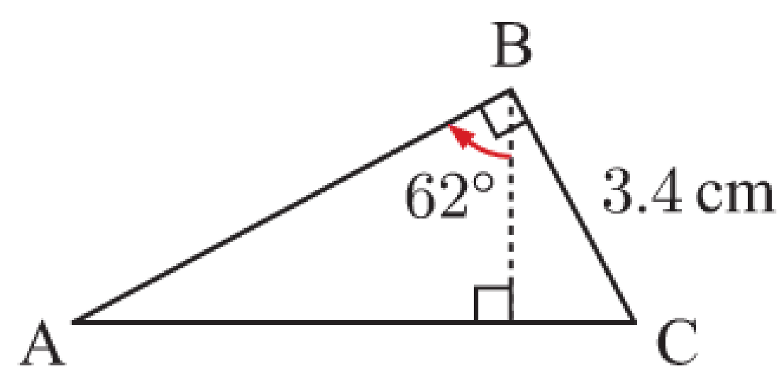
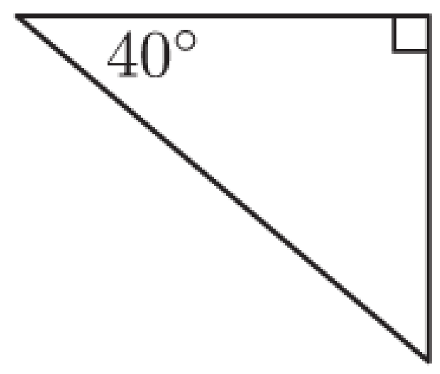


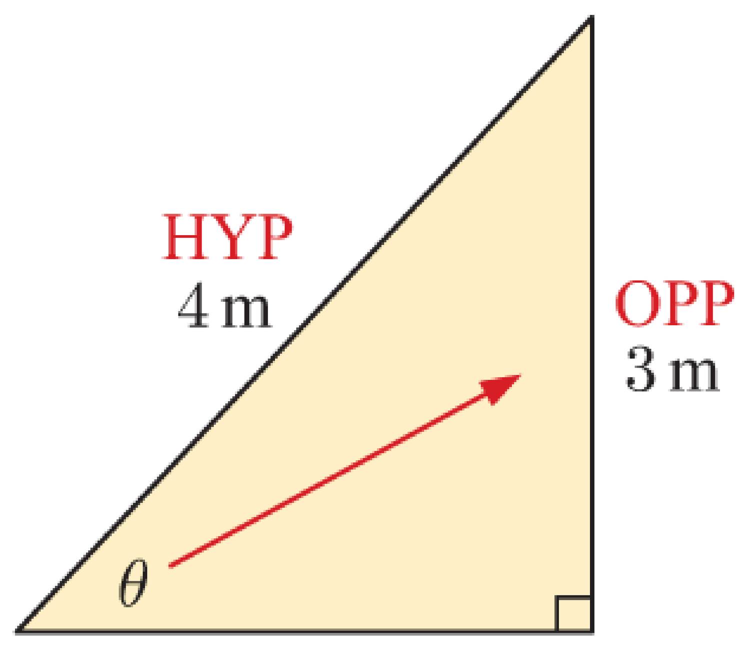
10 This triangle has area 20 cm^2. Find its perimeter.



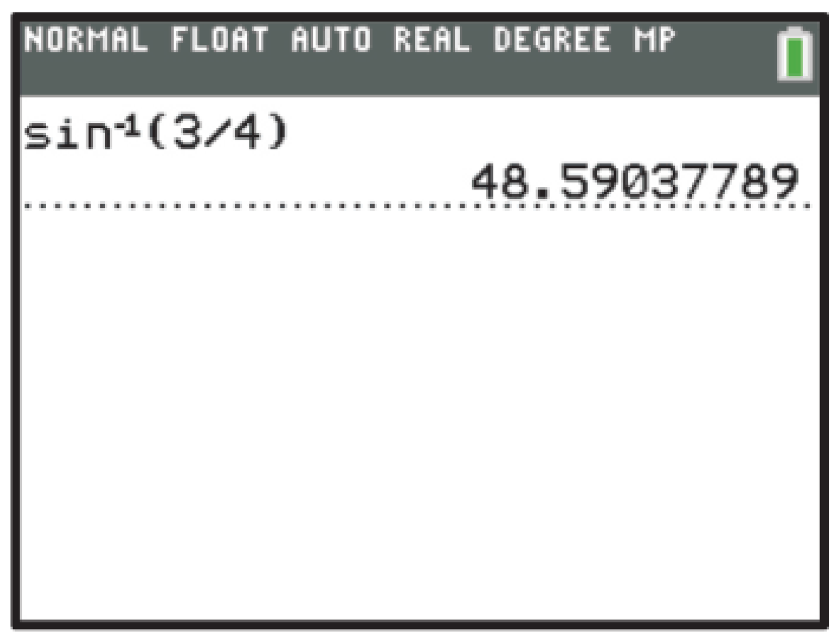
B INVERSE TRIGONOMETRIC RATIOS

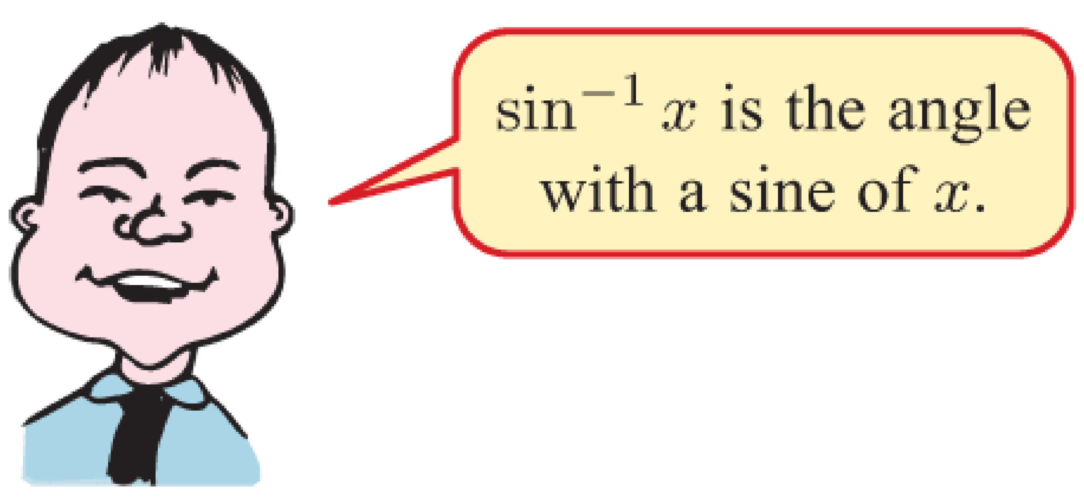
In the triangle alongside, $\sin\theta = \dfrac{\text{OPP}}{\text{HYP}} = \dfrac{3}{4}$.

To find θ, we need to find the angle whose sine is $\frac{3}{4}$.



We say that θ is the **inverse sine** of $\frac{3}{4}$, and write $\theta = \sin^{-1}\left(\frac{3}{4}\right) \approx 48.6°$.





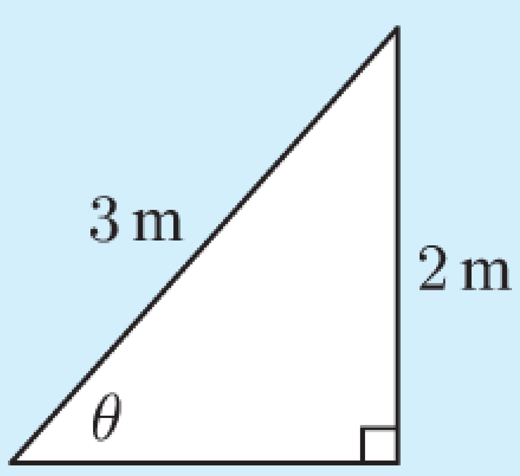
GRAPHICS CALCULATOR INSTRUCTIONS

We define **inverse cosine** and **inverse tangent** in a similar way.

Example 3 Self Tutor

Find, correct to 3 significant figures, the measure of the angle marked θ:

a



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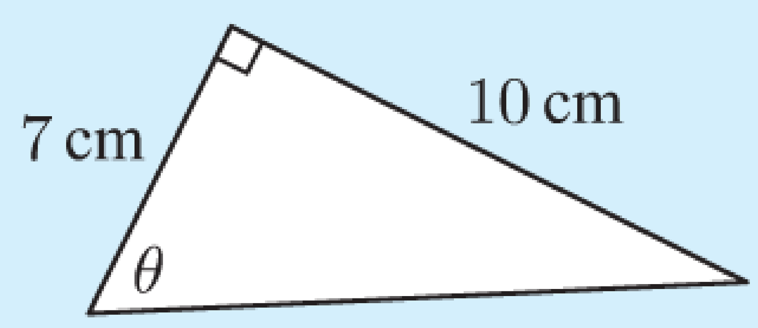
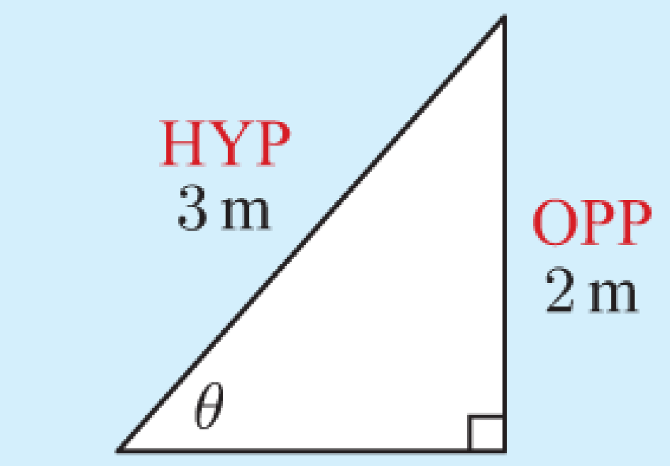


a

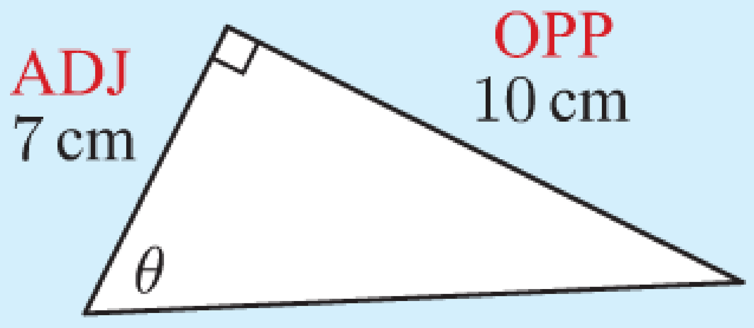


$\sin\theta = \frac{2}{3}$ $\quad \{\sin\theta = \frac{\text{OPP}}{\text{HYP}}\}$

$\therefore \quad \theta = \sin^{-1}\left(\frac{2}{3}\right)$

$\therefore \quad \theta \approx 41.8^\circ$ $\quad$ {using technology}

b



$\tan\theta = \frac{10}{7}$ $\quad \{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

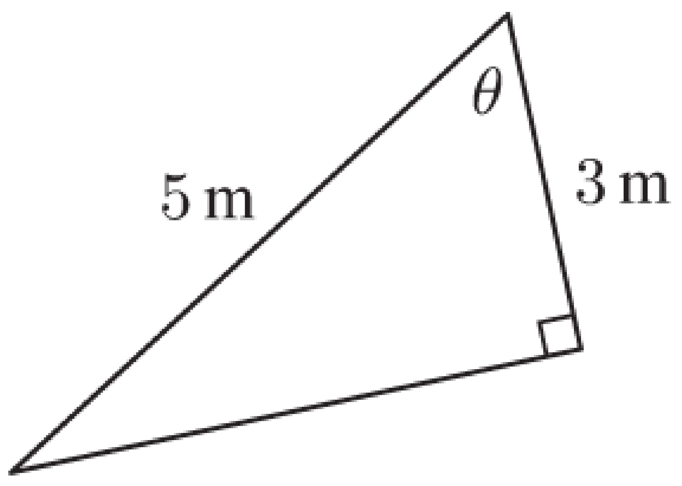
$\therefore \quad \theta = \tan^{-1}\left(\frac{10}{7}\right)$

$\therefore \quad \theta \approx 55.0^\circ$ $\quad$ {using technology}

EXERCISE 7B

1 Find, correct to 3 significant figures, the measure of the angle marked θ:

a



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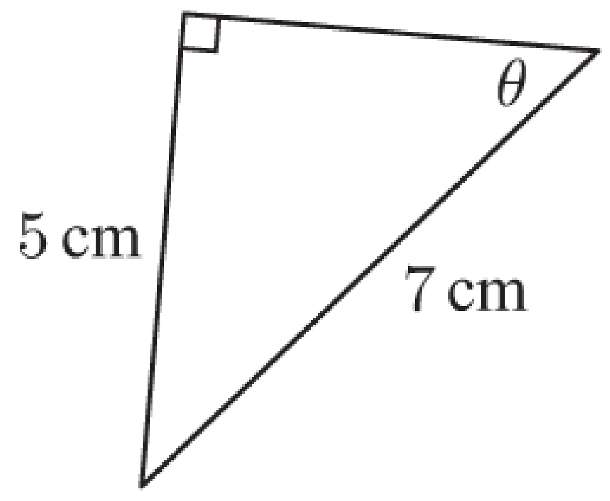

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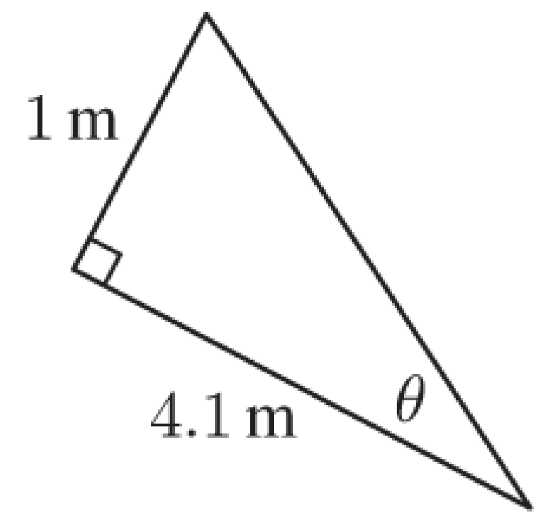

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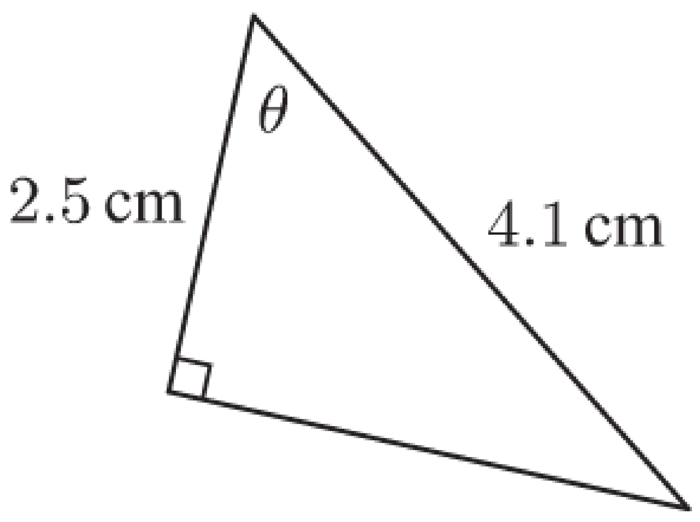

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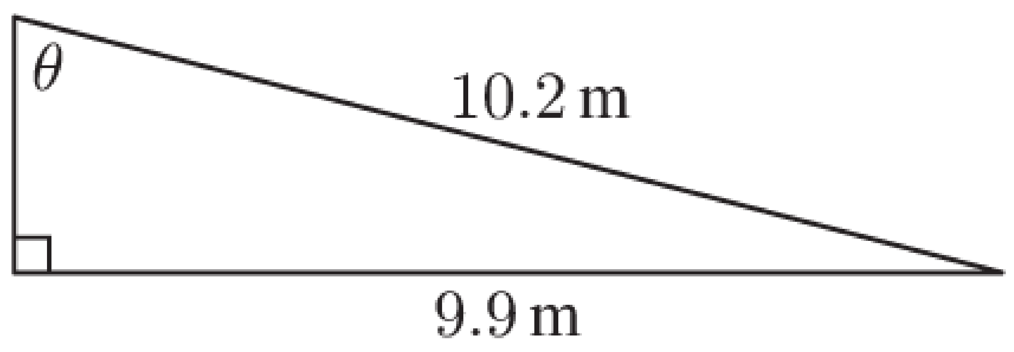

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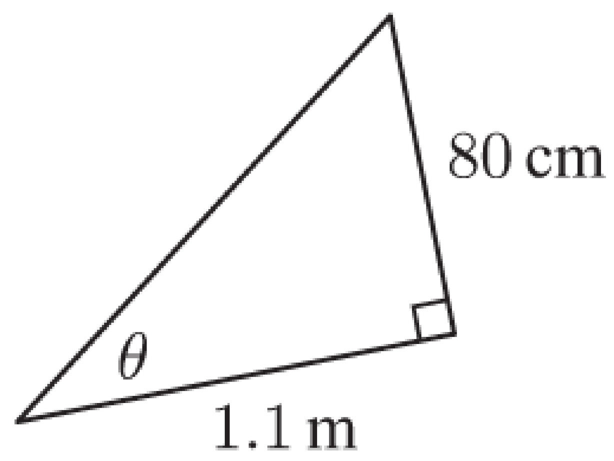


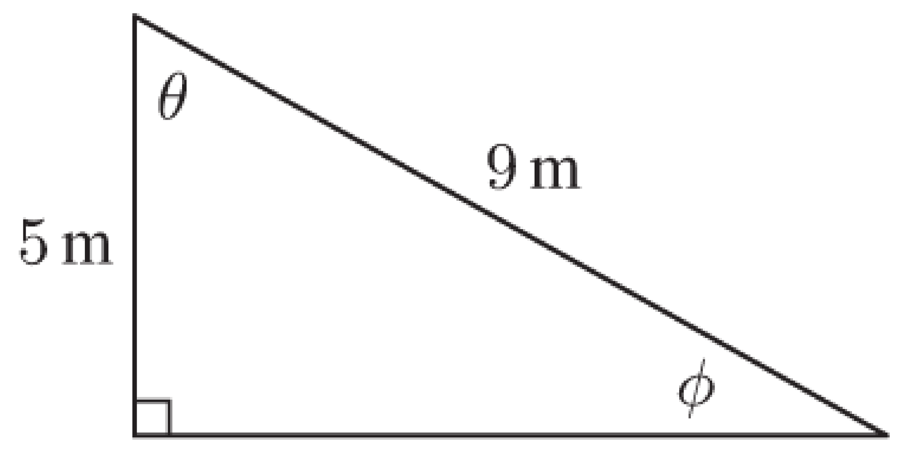
2 Consider the triangle alongside.

a Find θ, correct to 1 decimal place.

b Find ϕ using:

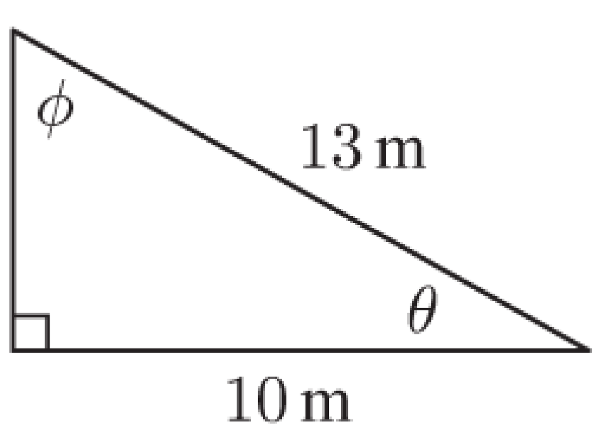
i the angles in a triangle theorem

ii trigonometry.



3 Find, correct to 1 decimal place, all unknown angles:

a



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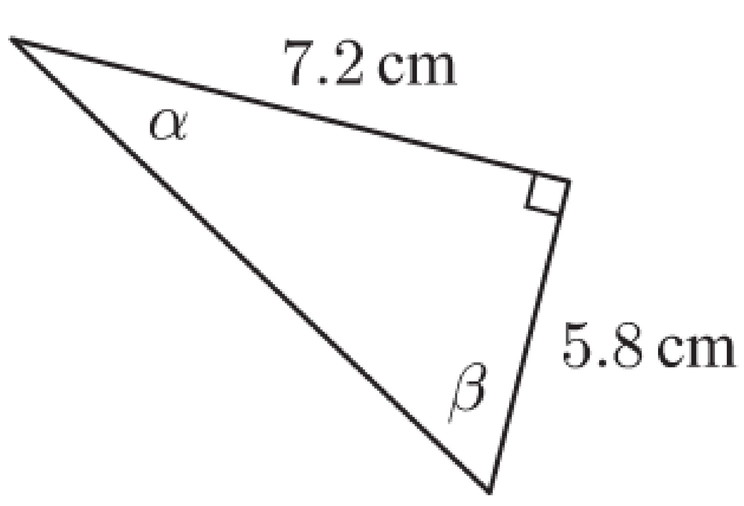

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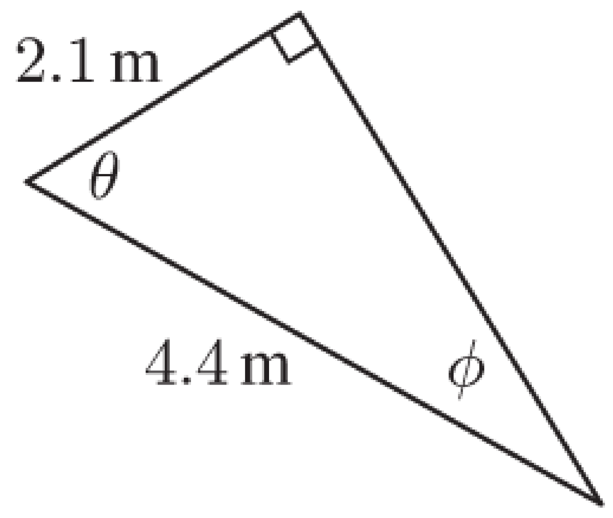
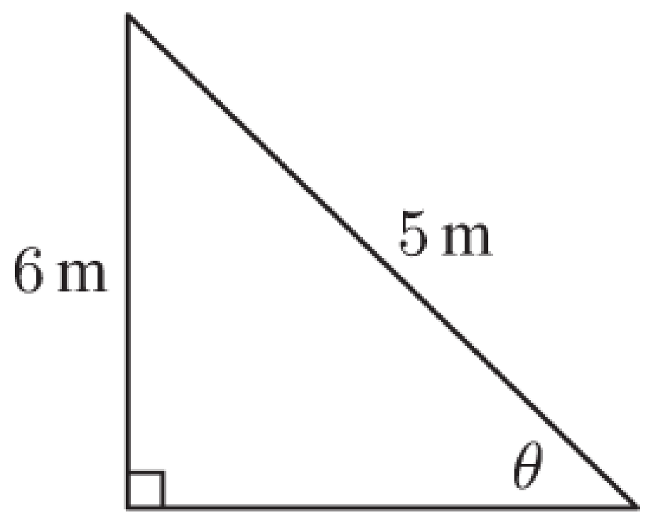


4 Try to find θ in the following diagrams. What conclusions can you draw?

a



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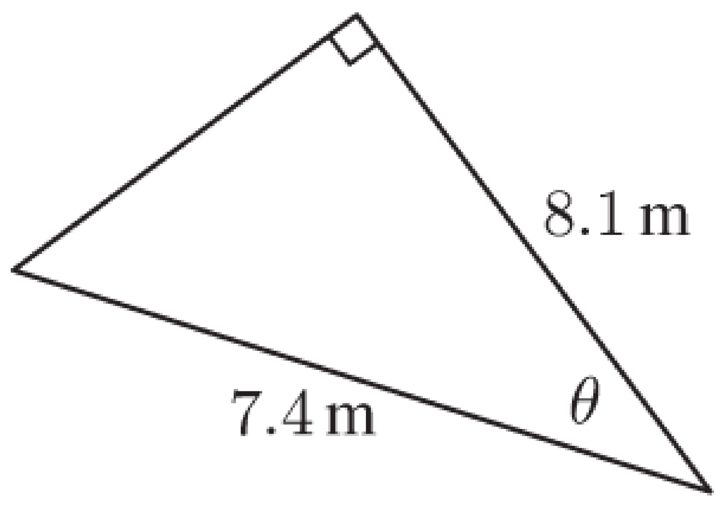

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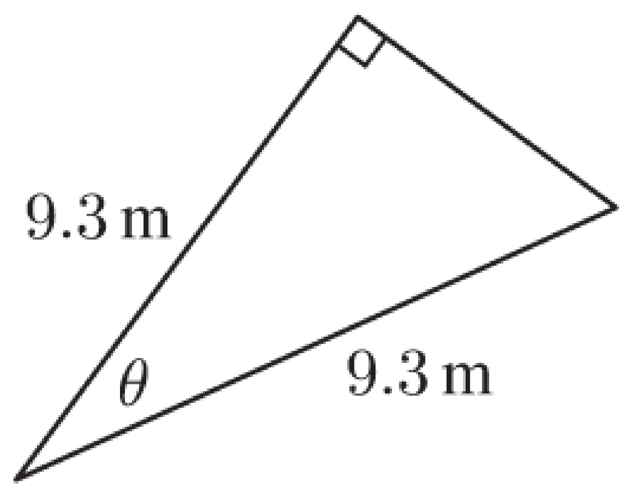
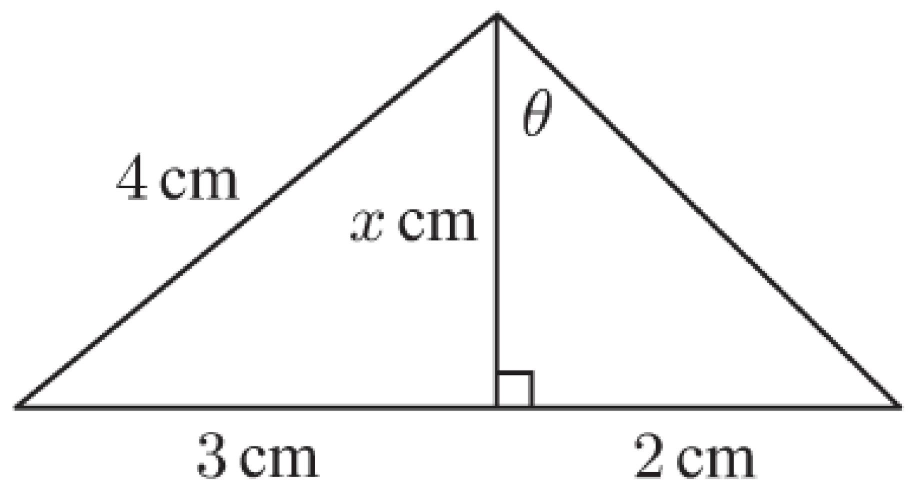


5 Find all unknown sides and angles in these figures:

a



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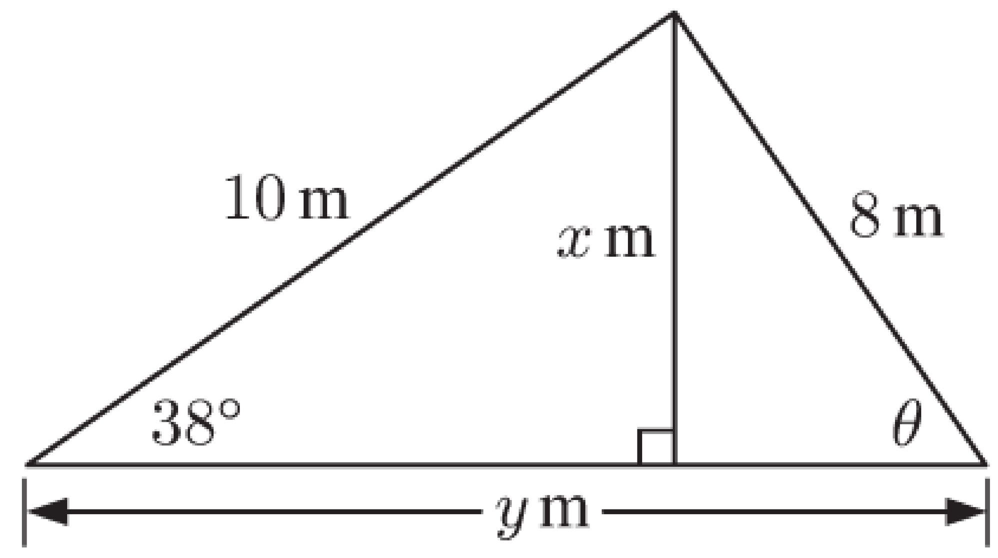
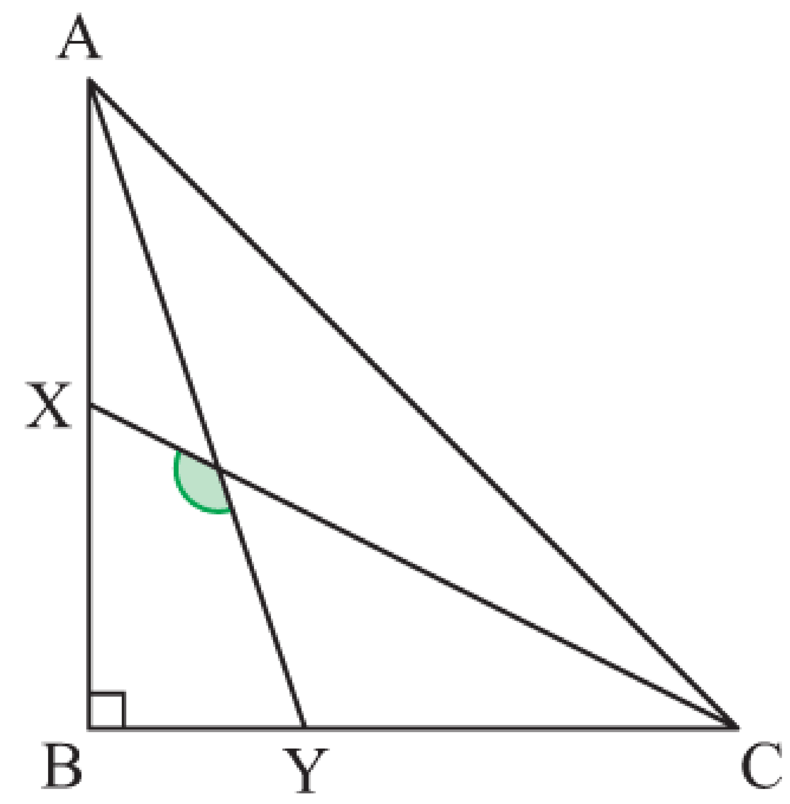
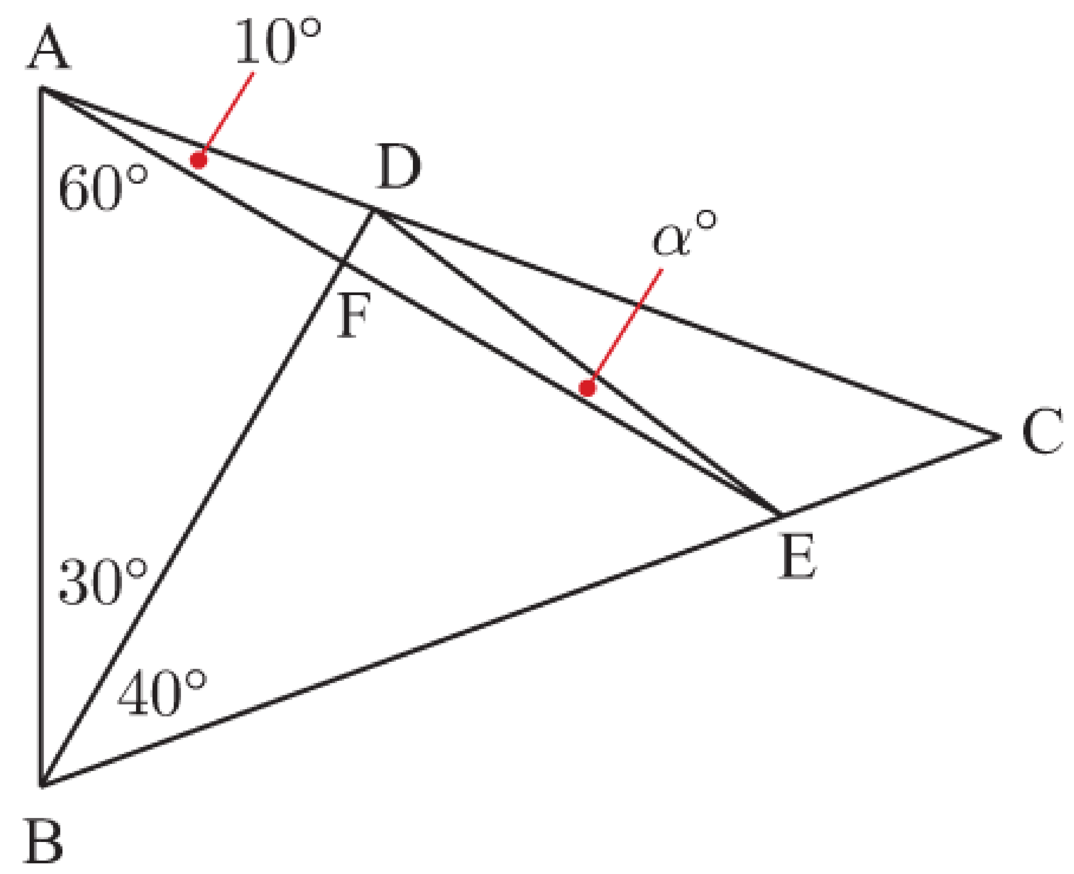


6 ABC is a right angled isosceles triangle. AX = XB and BY : YC = 1 : 2.
Find the measure of the shaded angle.



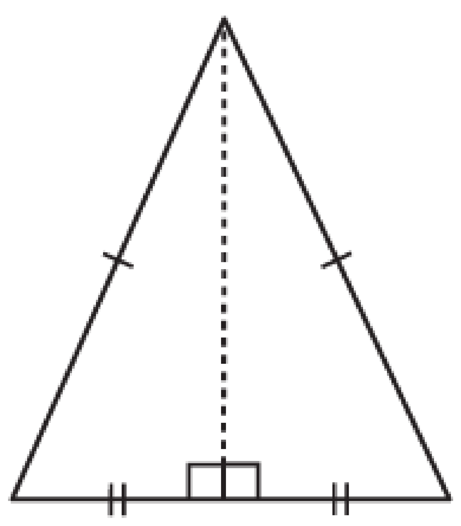
7 Find α:



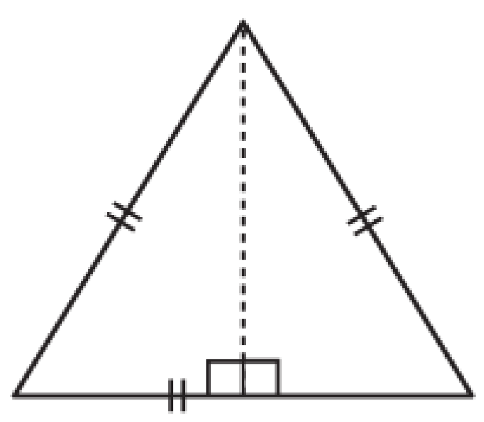
C RIGHT ANGLES IN GEOMETRIC FIGURES

Many geometric figures contain right angles which we can use to help solve problems:

- In an **isosceles triangle** and an **equilateral triangle**, the altitude bisects the base at right angles.

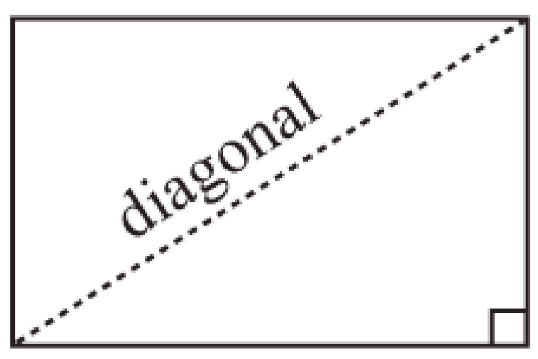


isosceles triangle

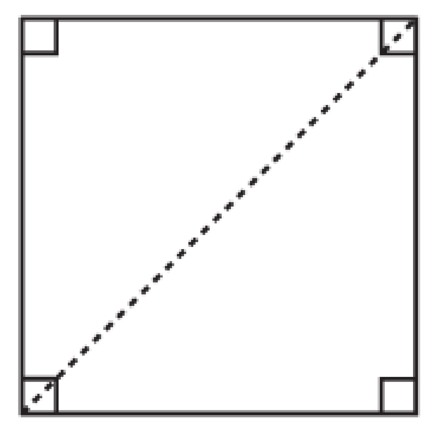


equilateral triangle

- The corners of a **rectangle** and a **square** are right angles. We can construct a diagonal to form a right angled triangle.

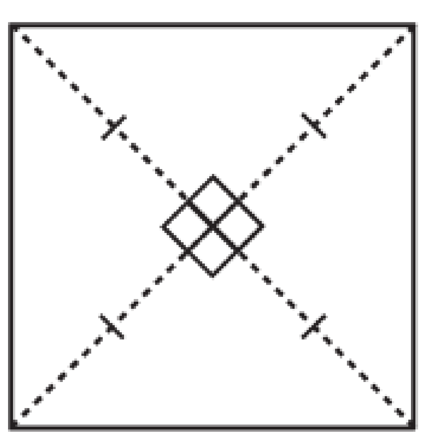


rectangle

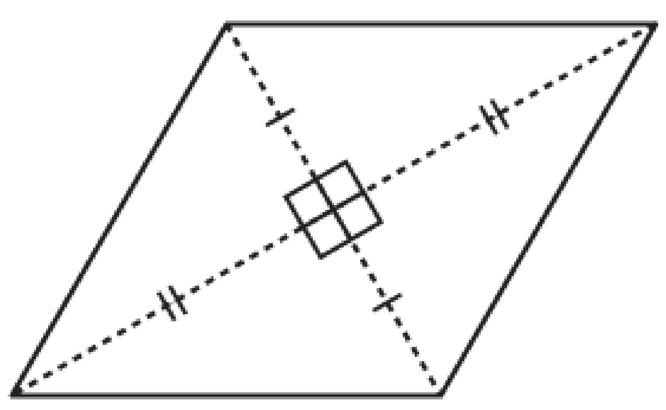


square

- In a **square** and a **rhombus**, the diagonals bisect each other at right angles.

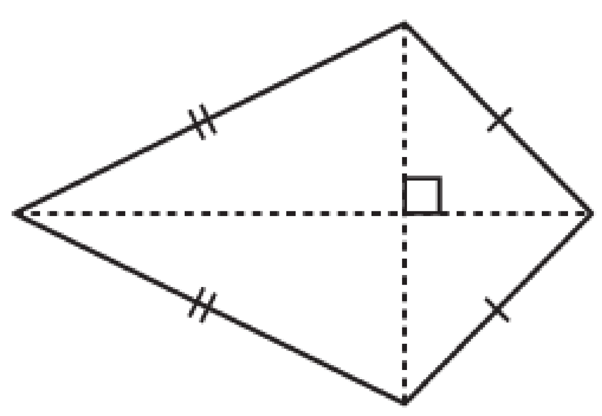


square



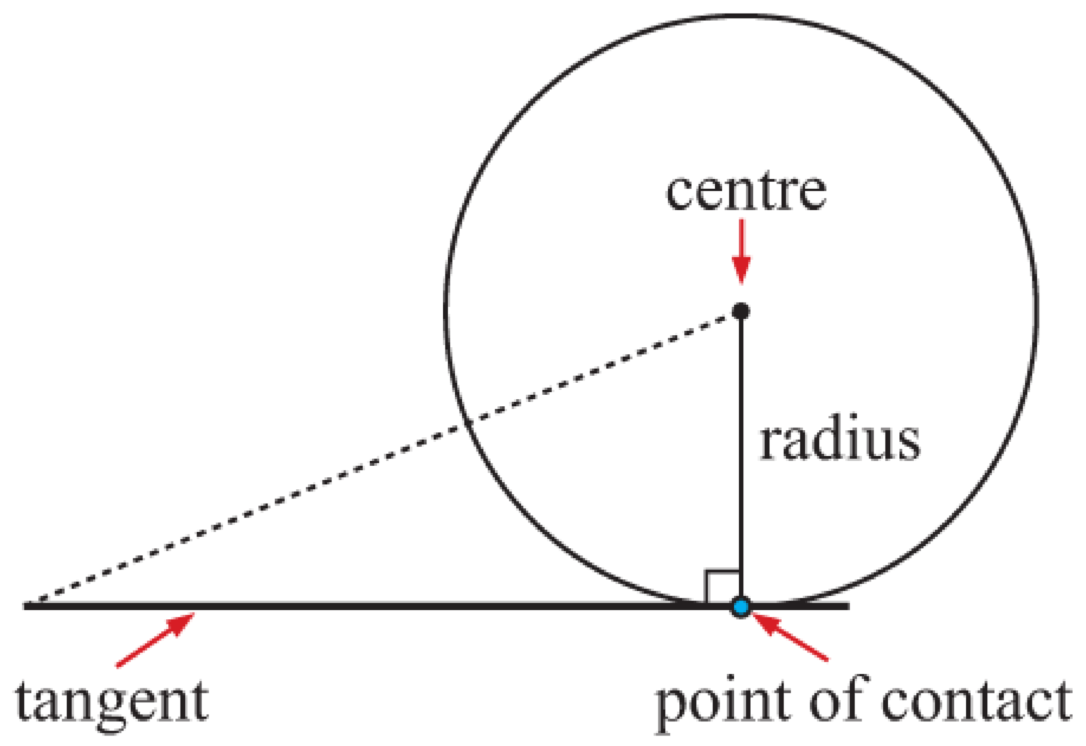
rhombus

- In a **kite**, the diagonals intersect at right angles.

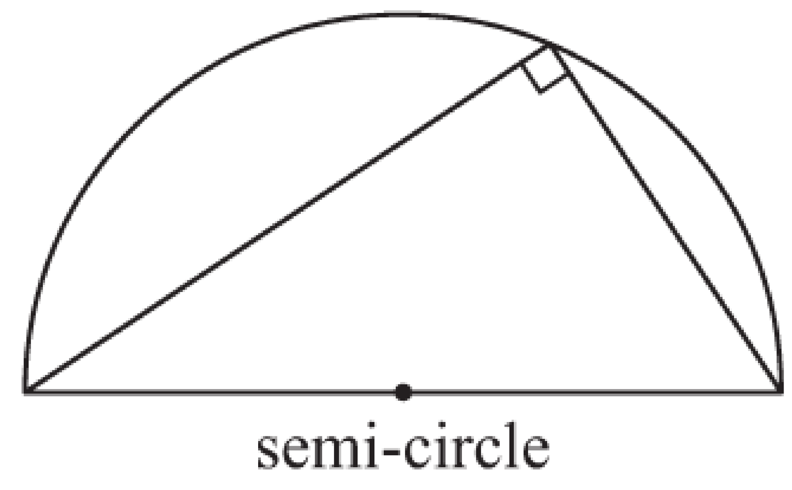


kite

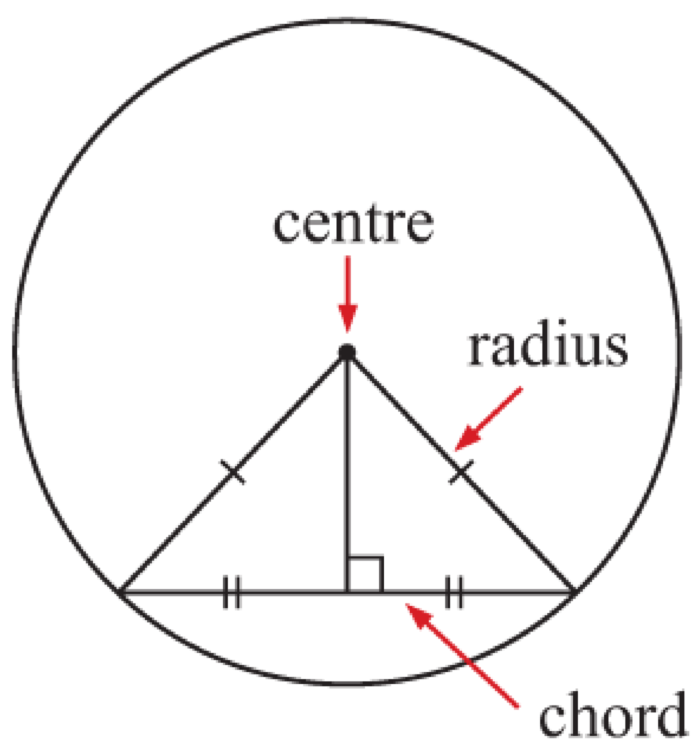
- A **tangent** to a circle and a radius at the point of contact meet at right angles.
  We can form a right angled triangle with another point on the tangent.



- The angle in a **semi-circle** is always a right angle.



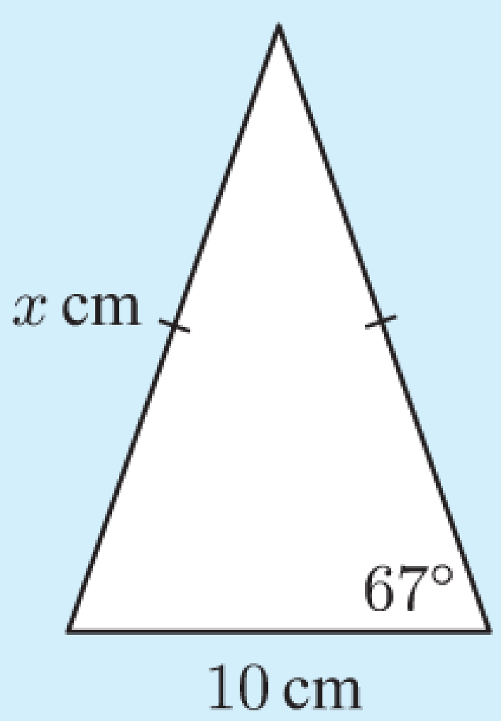
- The line drawn from the centre of a circle at right angles to a **chord**, bisects the chord.



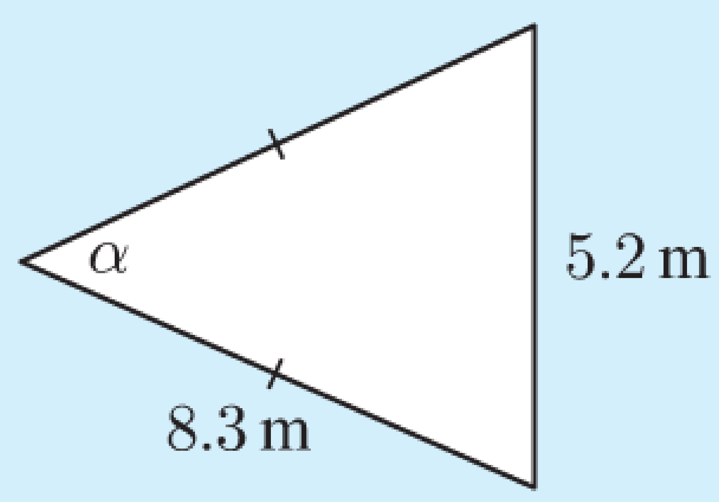
Example 4 **Self Tutor**

Find the unknowns:

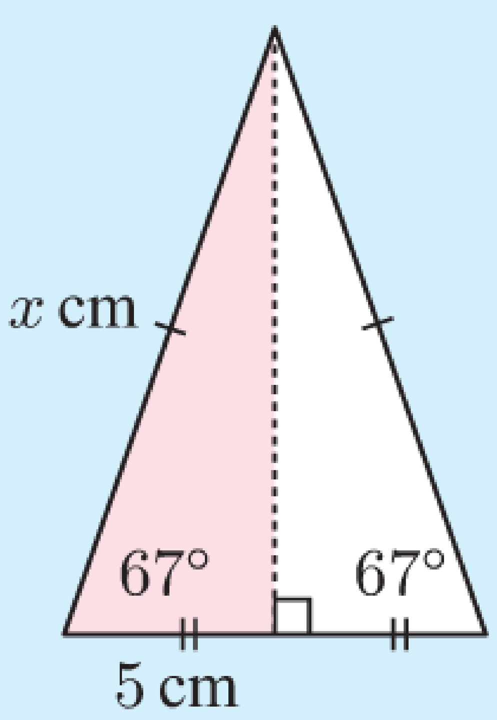
a



b



a

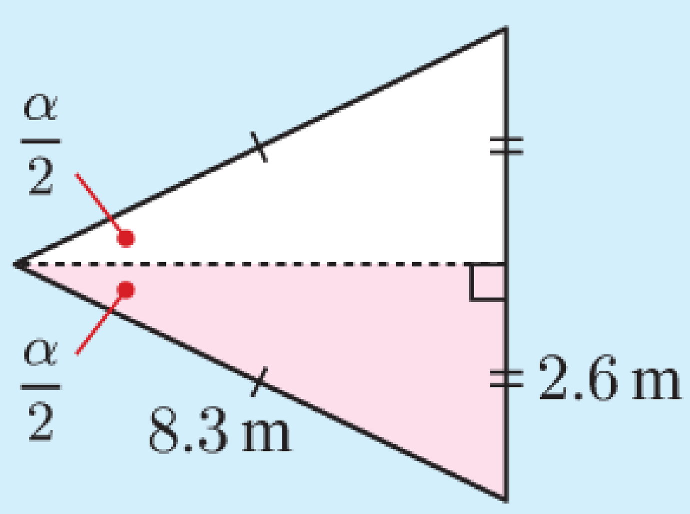


In the shaded right angled triangle,

$$\cos 67° = \frac{5}{x}$$

$$\therefore \quad x = \frac{5}{\cos 67°} \approx 12.8$$

b



In the shaded right angled triangle,

$$\sin \frac{\alpha}{2} = \frac{2.6}{8.3}$$

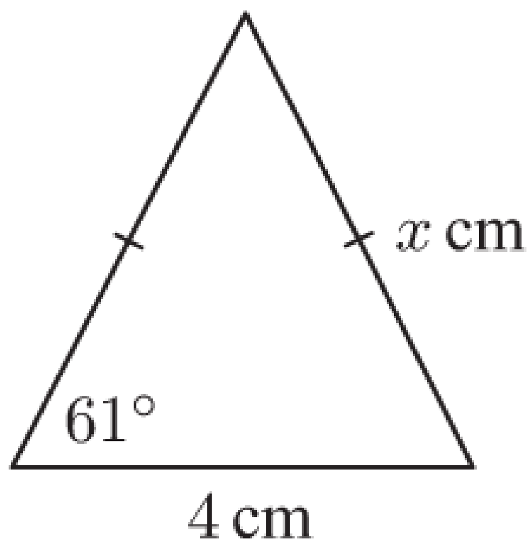
$$\therefore \quad \frac{\alpha}{2} = \sin^{-1}\left(\frac{2.6}{8.3}\right)$$

$$\therefore \quad \alpha = 2\sin^{-1}\left(\frac{2.6}{8.3}\right) \approx 36.5°$$

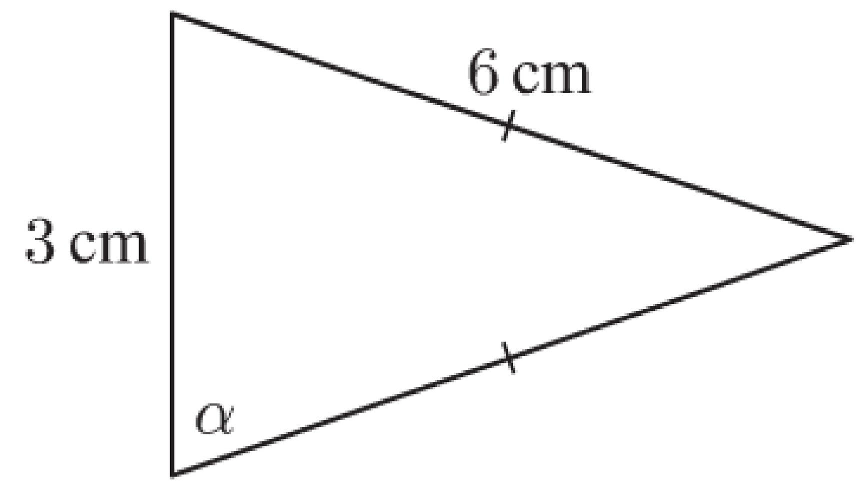
EXERCISE 7C

1 Find the unknown, correct to 3 significant figures:

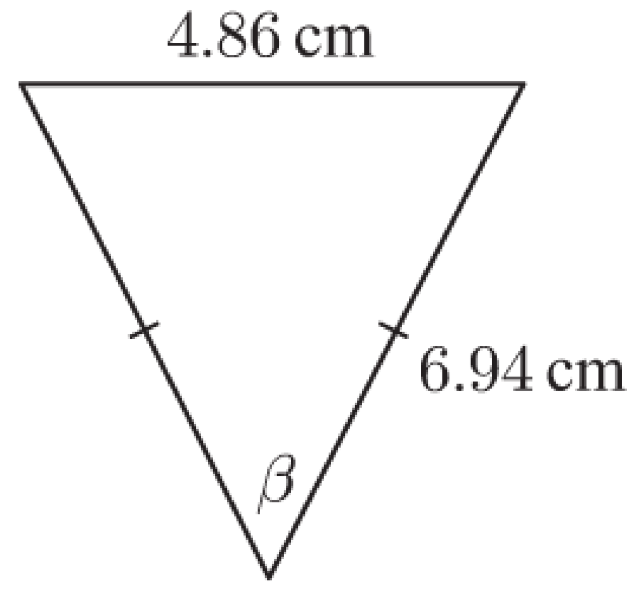
a



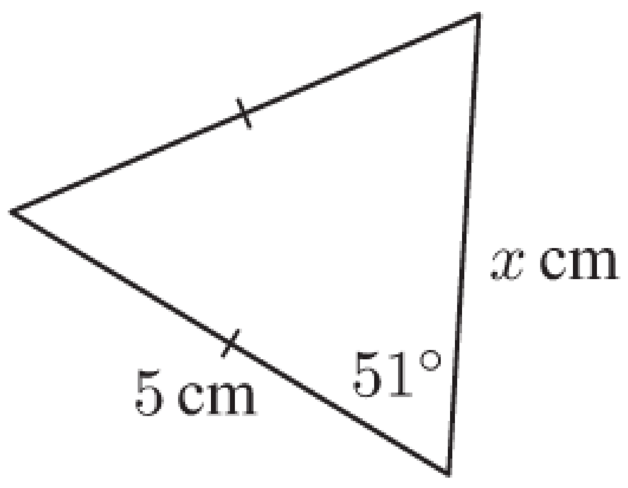
b



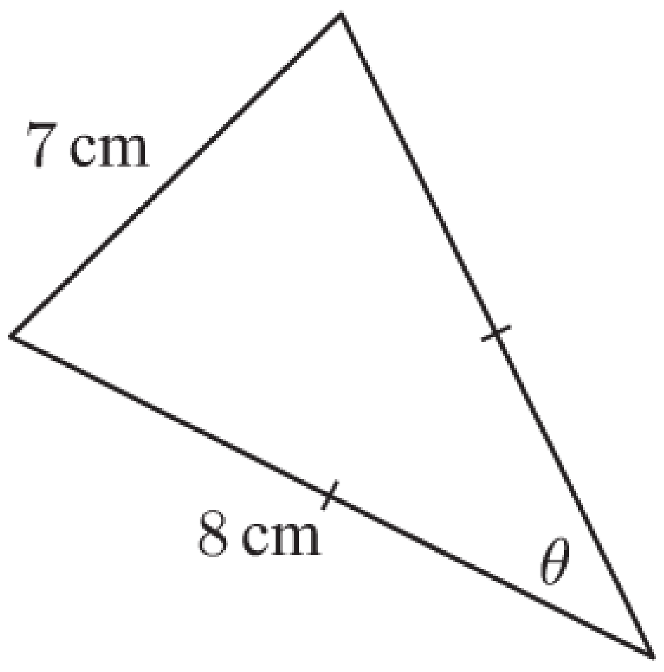
c



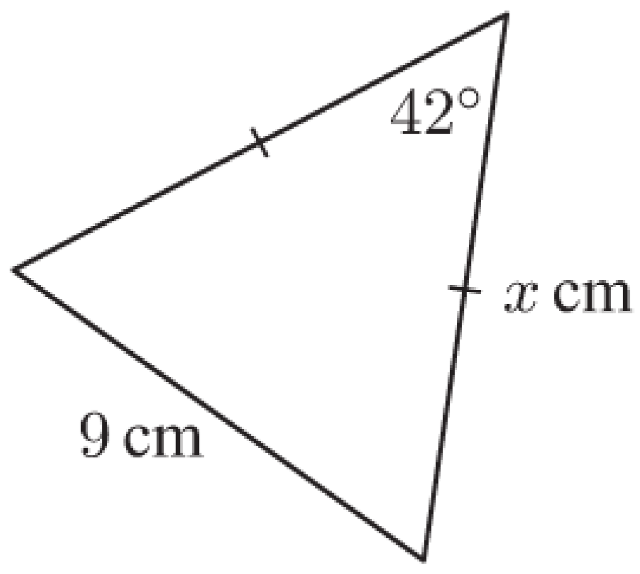
d



e



f

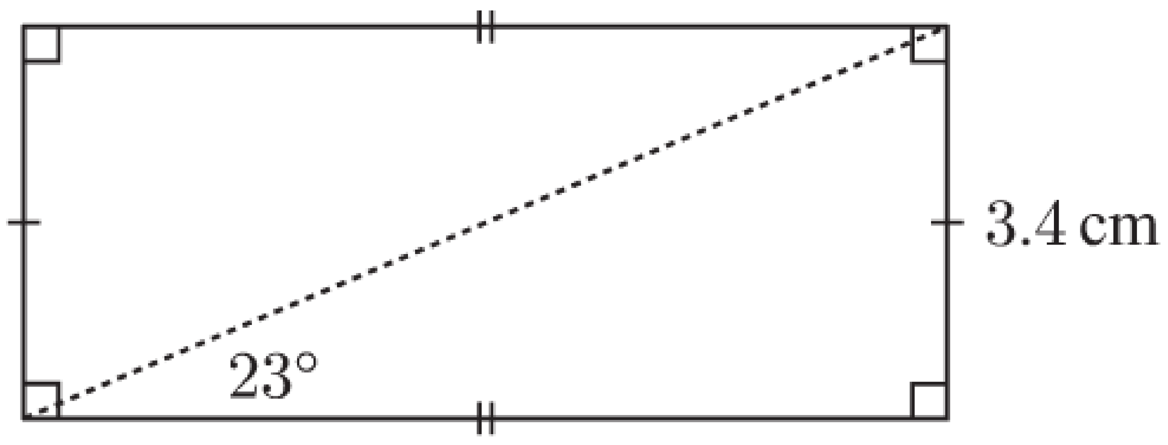


2 A rectangle is 9.2 m by 3.8 m. What angle does its diagonal make with its longer side?

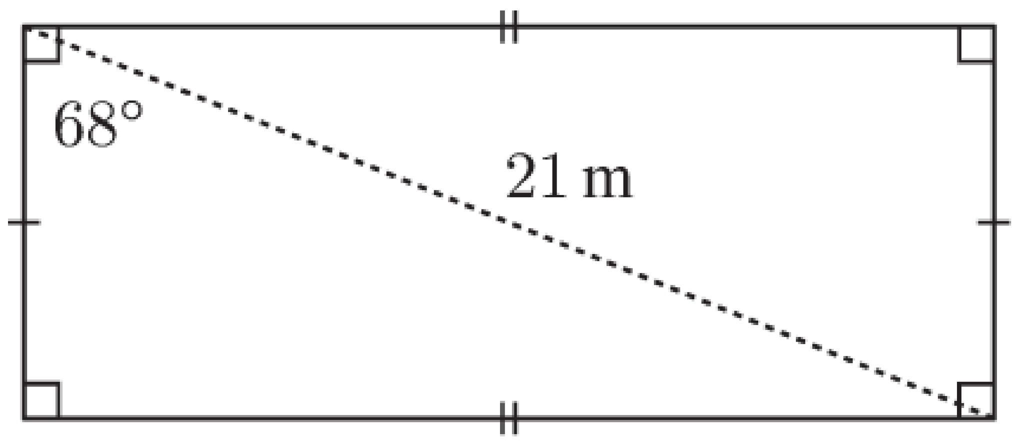
3 The diagonal and the longer side of a rectangle make an angle of $43.2°$. If the longer side is 12.6 cm, find the length of the shorter side.

4 Find the area of the rectangle:

a

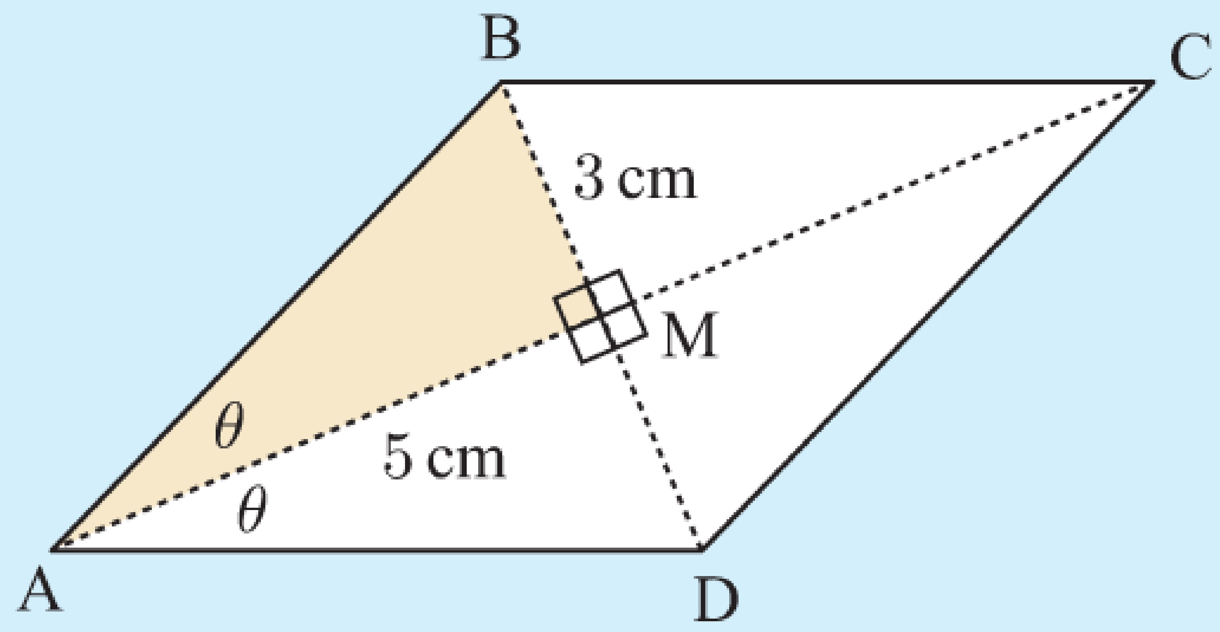


b



Example 5 **Self Tutor**

A rhombus has diagonals of length 10 cm and 6 cm. Find the smaller angle of the rhombus.



The diagonals bisect each other at right angles, so AM = 5 cm and BM = 3 cm.

In $\triangle$ABM, θ will be the smallest angle as it is opposite the shortest side.

$$\tan\theta = \frac{3}{5}$$
$$\therefore \quad \theta = \tan^{-1}\left(\frac{3}{5}\right)$$
$$\therefore \quad \theta \approx 30.964°$$

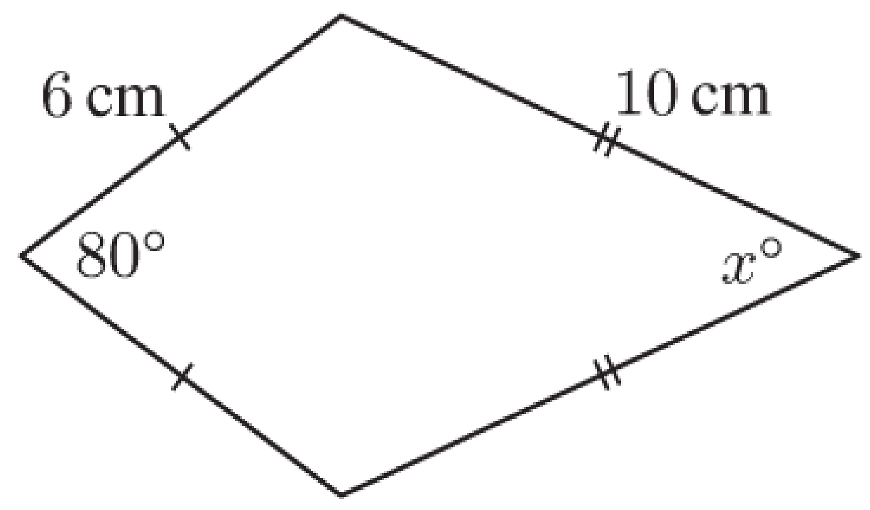
The required angle is 2θ as the diagonals bisect the angles at each vertex.
So, the angle is about $61.9°$.

5 A rhombus has diagonals of length 12 cm and 7 cm. Find the larger angle of the rhombus.

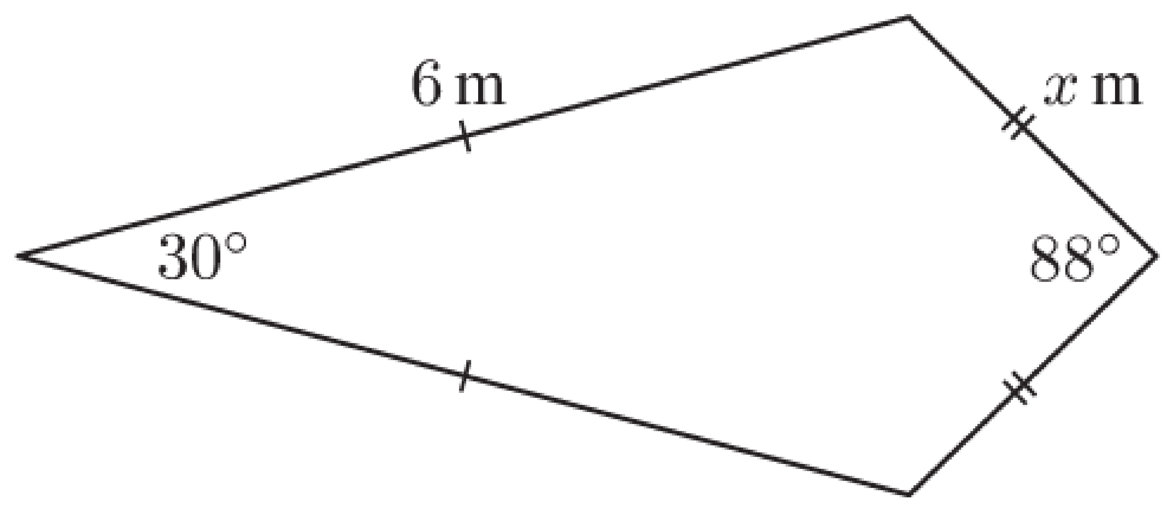
6 The smaller angle of a rhombus measures $21.8°$ and the shorter diagonal has length 13.8 cm. Find the lengths of the sides of the rhombus.

7 Find the value of x:

a

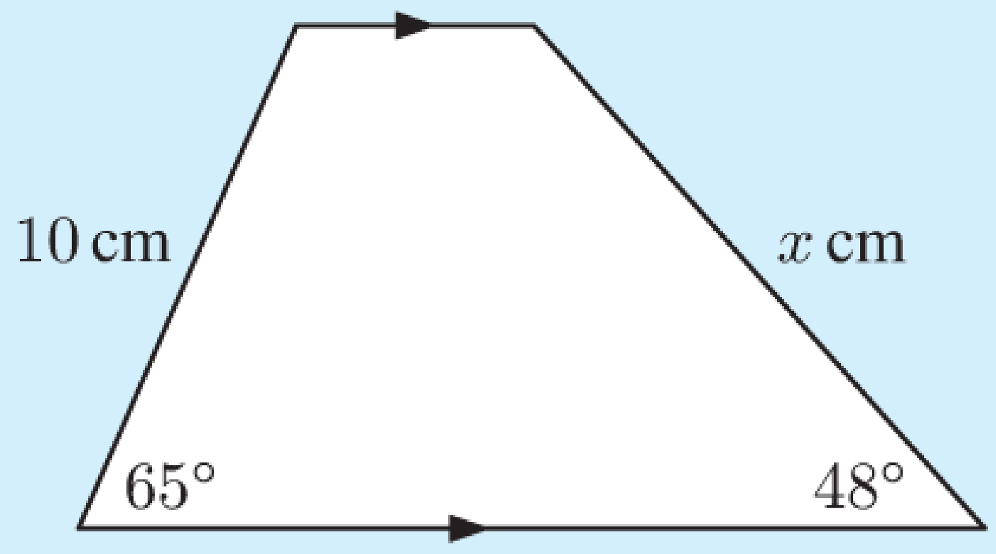


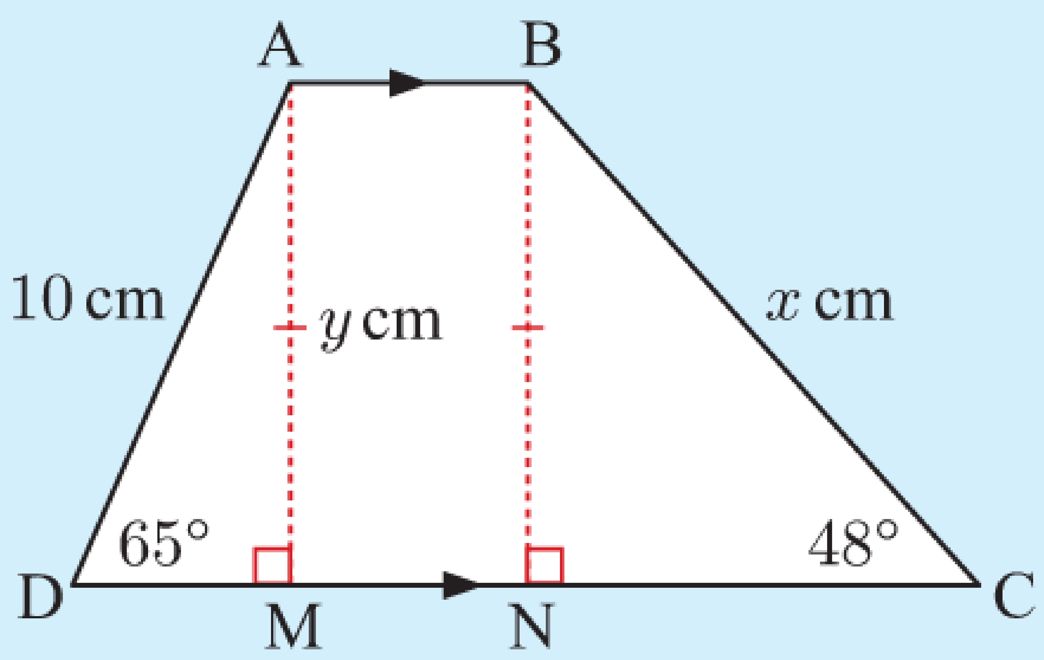
b



Example 6 — Self Tutor

Find x:





We draw perpendiculars [AM] and [BN] to [DC], creating right angled triangles and the rectangle ABNM.

In $\triangle$ADM, $\sin 65° = \dfrac{y}{10}$

$\therefore\ y = 10 \sin 65°$

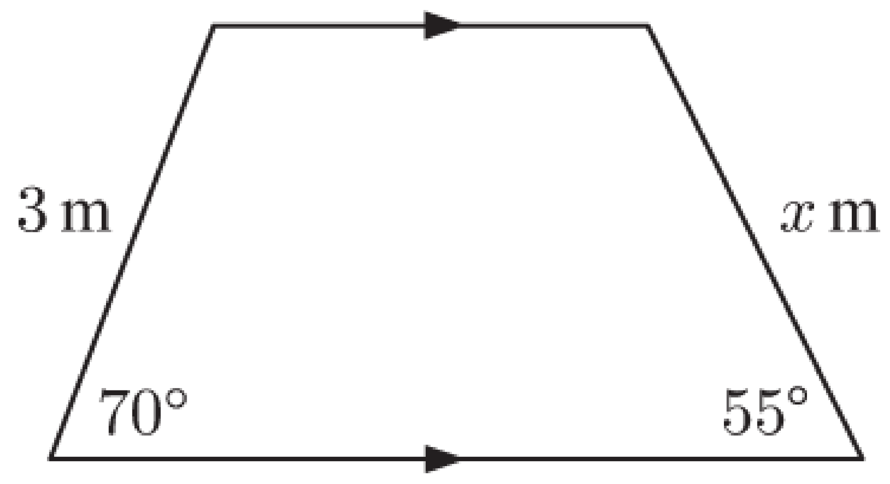
In $\triangle$BCN, $\sin 48° = \dfrac{y}{x}$

$= \dfrac{10 \sin 65°}{x}$

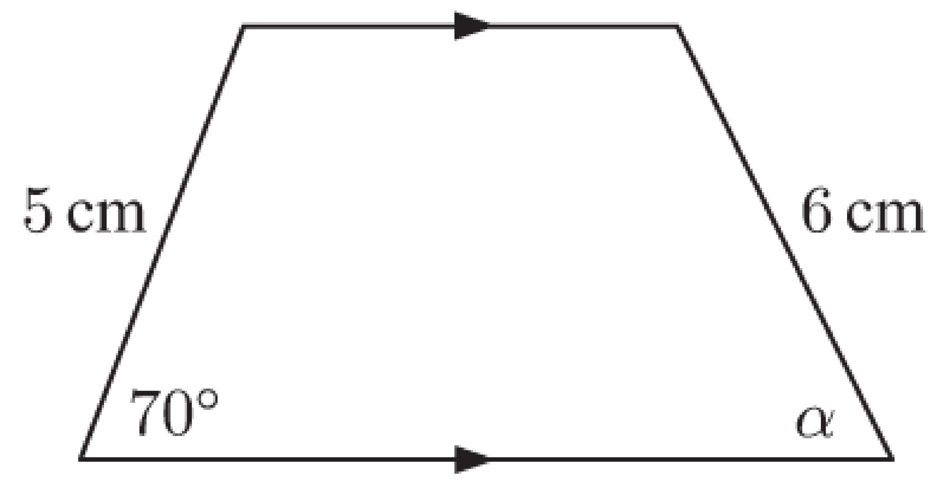
$\therefore\ x = \dfrac{10 \sin 65°}{\sin 48°} \approx 12.2$

8 Find the unknown value:

a

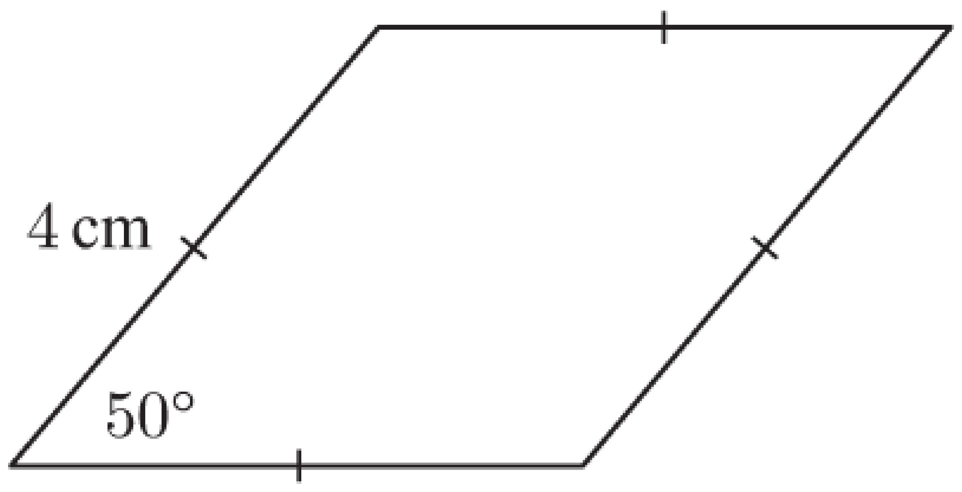


b



9 Find the area of the figure:

a



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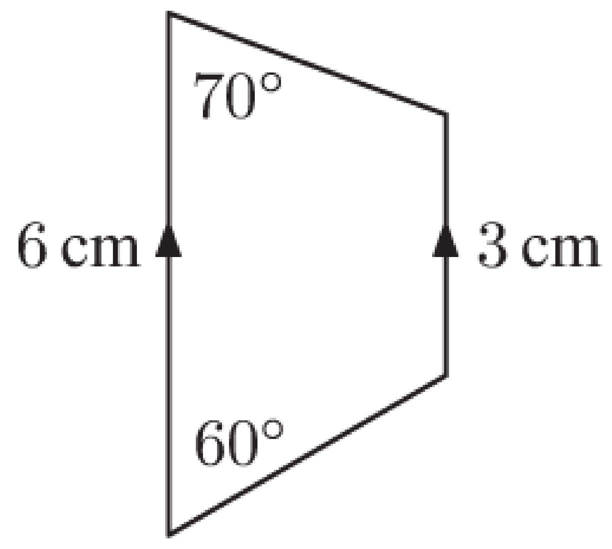


10 A rhombus has sides of length 8 metres, and the longer diagonal has length 13 metres.

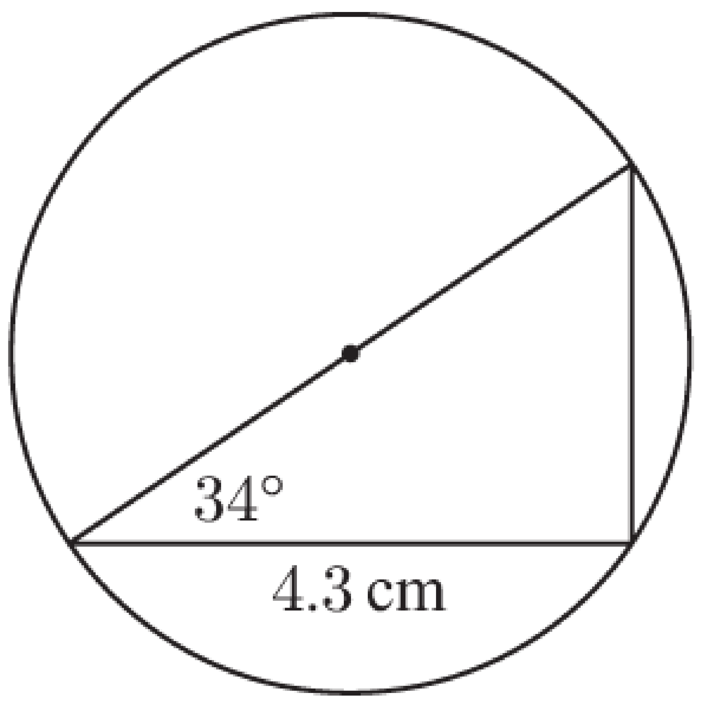
a Draw a diagram and label it with the given information.

b Find the length of the shorter diagonal of the rhombus.

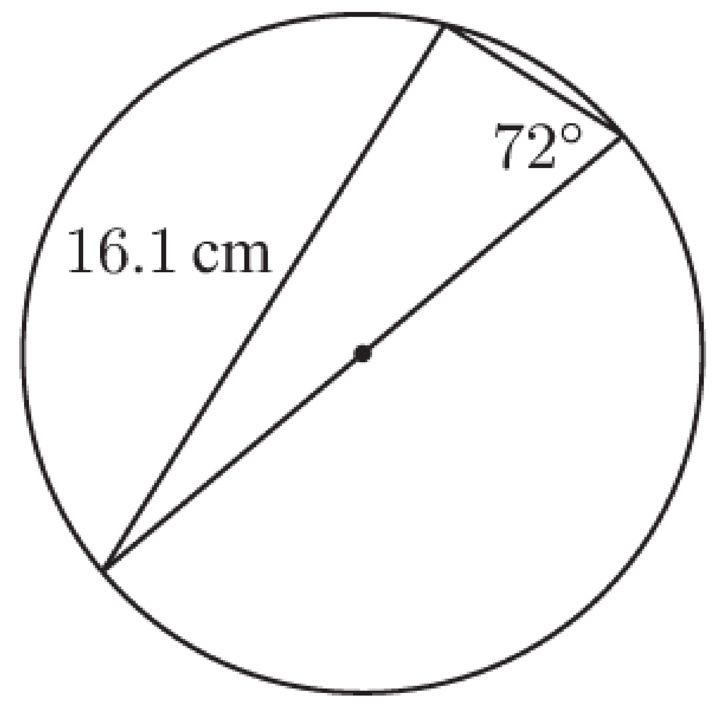
c Find the measure of the smaller angle in the rhombus.

11 Find the radius of the circle:

a

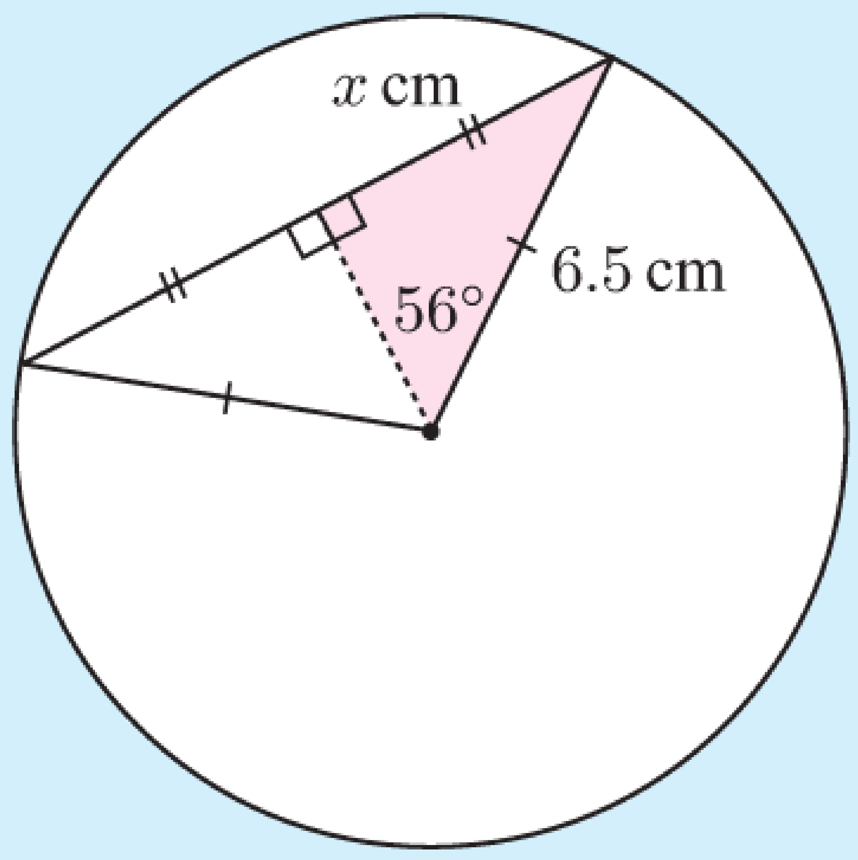


b



Example 7 **Self Tutor**

A circle has radius 6.5 cm. A chord of the circle subtends an angle of 112° at its centre. Find the length of the chord.



We complete an isosceles triangle and draw the line from the apex to the base.

For the shaded triangle, $\sin 56° = \dfrac{x}{6.5}$

$\therefore\ 6.5 \times \sin 56° = x$

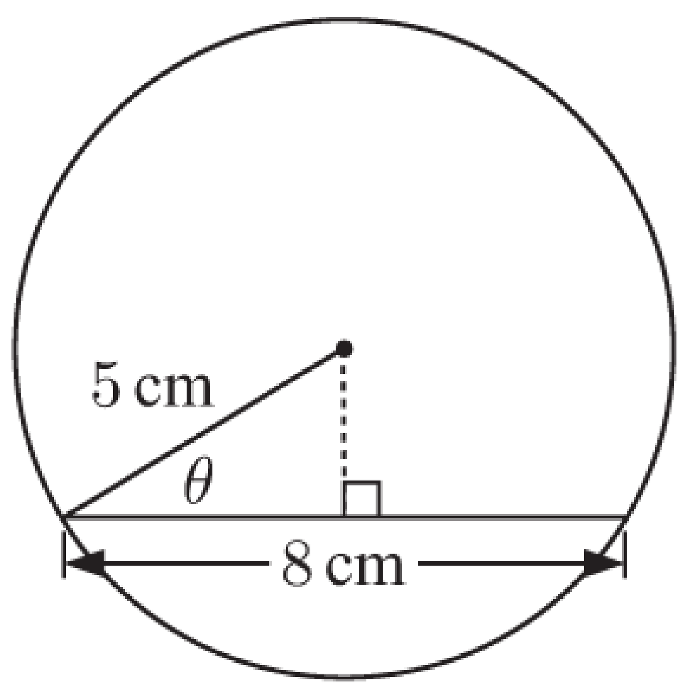
$\therefore\ x \approx 5.389$

$\therefore\ 2x \approx 10.78$

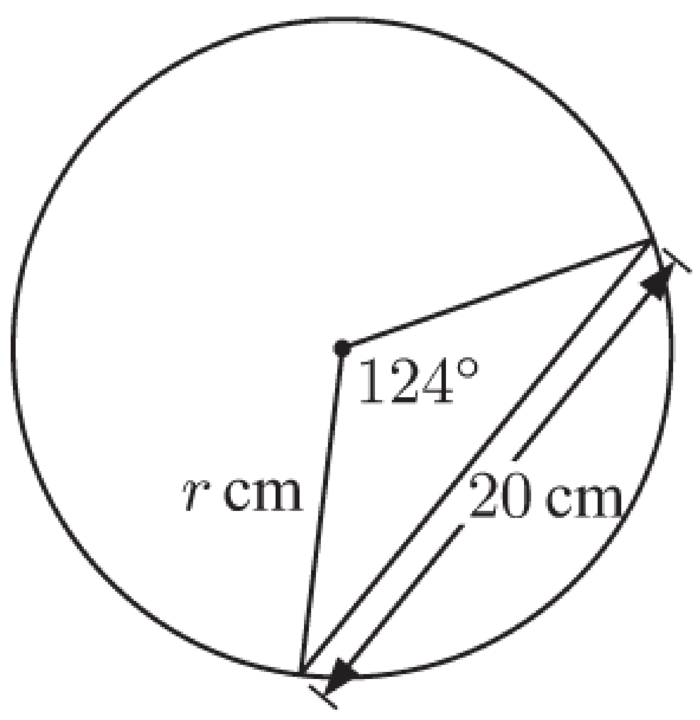
$\therefore$ the chord is about 10.8 cm long.

12 Find the value of the unknown:

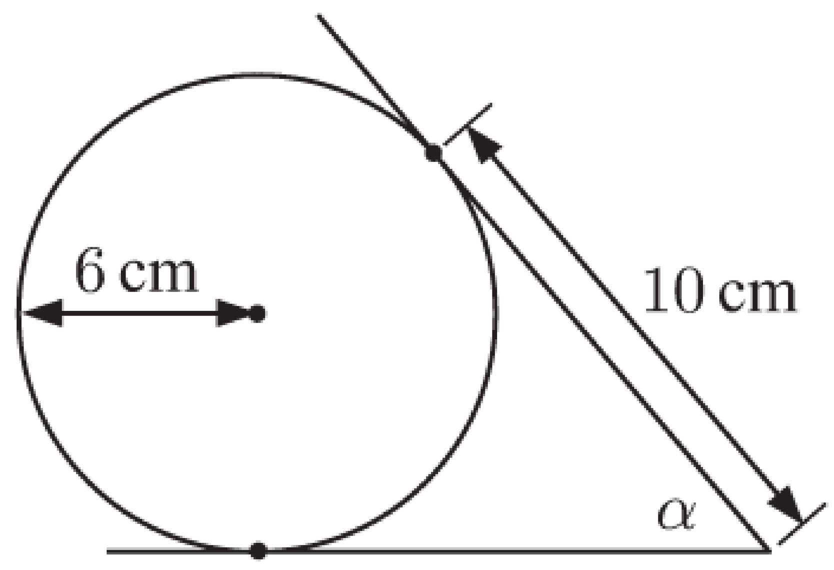
a



b



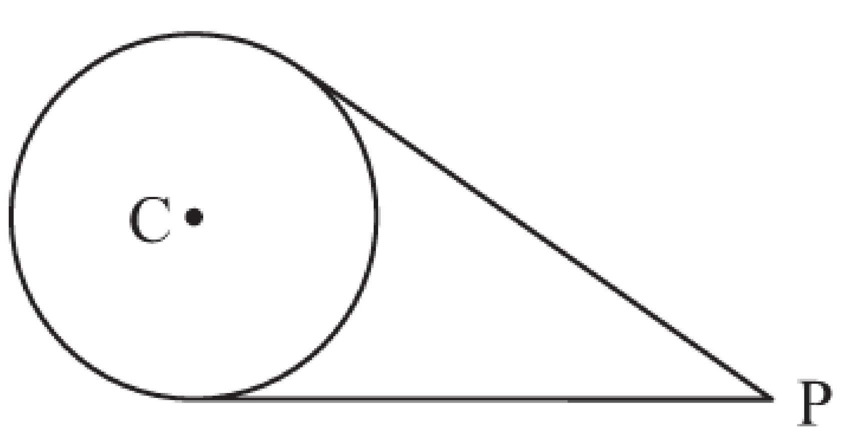
c



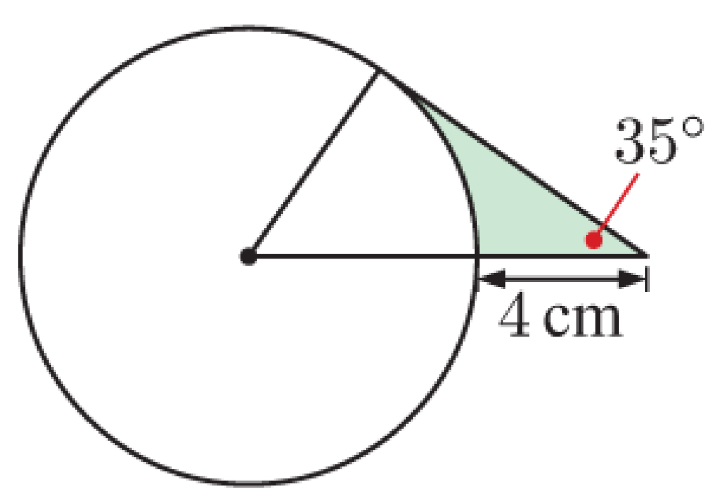
13 A circle has diameter 11.4 cm. A chord of the circle subtends an angle of 89° at its centre. Find the length of the chord.

14 A chord of a circle is 13.2 cm long and the circle's radius is 9.4 cm. Find the angle subtended by the chord at the centre of the circle.

15 PC = 10 cm, and the circle has radius 4 cm. Find the angle between the tangents.



16 Find the shaded area.



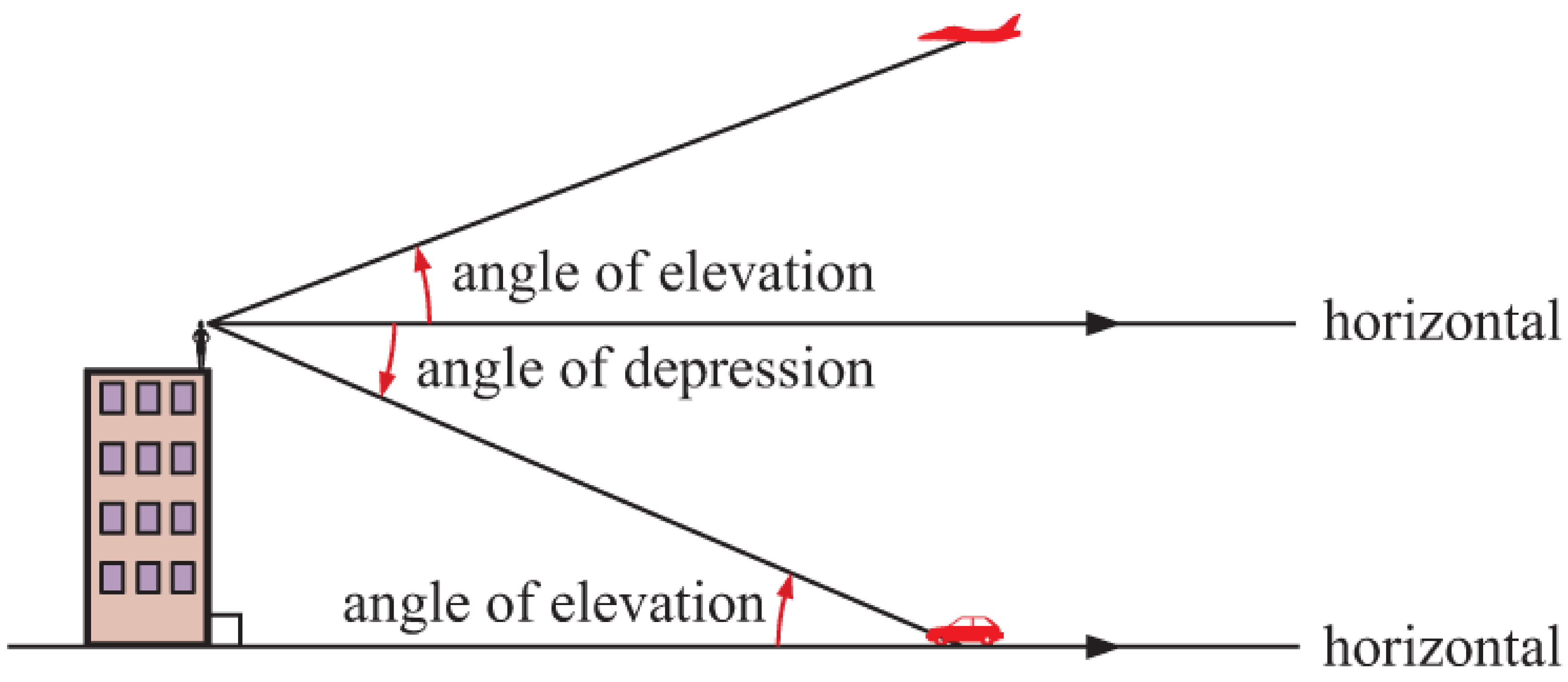
D PROBLEM SOLVING WITH TRIGONOMETRY

In this Section we consider practical applications of trigonometry. It allows us to find heights and distances which are very difficult or even impossible to measure directly.

ANGLES OF ELEVATION AND DEPRESSION

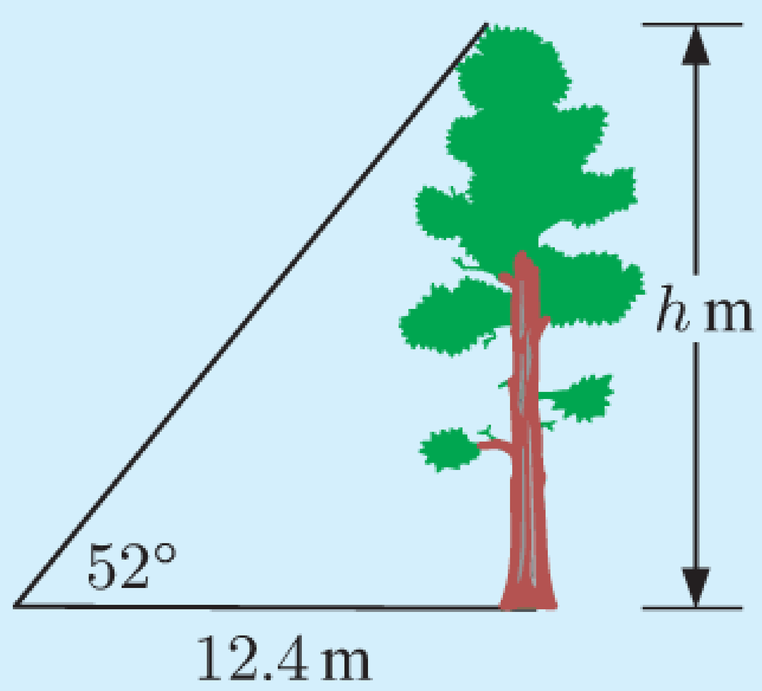
The angle between the horizontal and your line of sight to an object is called:

- the **angle of elevation** if you are looking upwards
- the **angle of depression** if you are looking downwards.



Example 8 Self Tutor

A tree casts a shadow of 12.4 m when the angle of elevation to the sun is 52°. Find the height of the tree.



Let the tree's height be h m.

For the 52° angle, OPP $= h$ m, ADJ $= 12.4$ m

$$\therefore \tan 52^\circ = \frac{h}{12.4}$$

$$\therefore 12.4 \times \tan 52^\circ = h$$

$$\therefore h \approx 15.9$$

$\therefore$ the tree is about 15.9 m high.

EXERCISE 7D

1 A flagpole casts a shadow of 9.32 m when the angle of elevation to the sun is 63°. Find the height of the flagpole.

2 A steep hill is inclined at 18° to the horizontal. It runs down to the beach so its base is at sea level.

  a If I walk 150 m up the hill, what is my height above sea level?

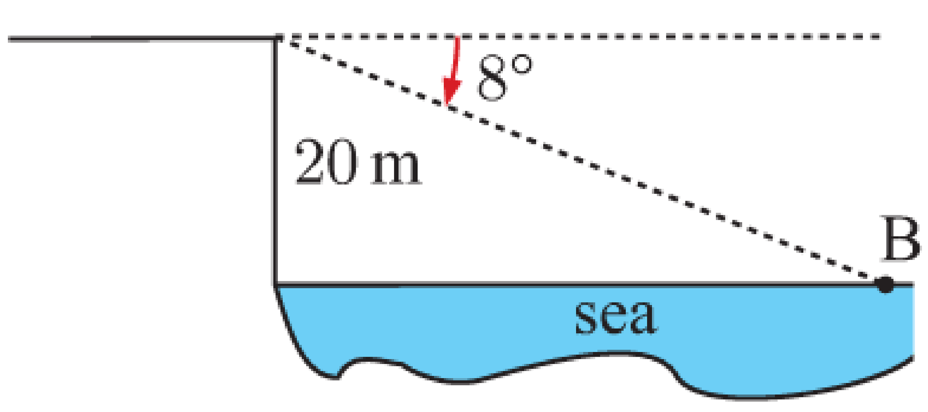
  b If I climb to a point 80 m above sea level, how far have I walked?

3 A train must climb a constant gradient of 5.5 m for every 200 m of track. Find the angle of incline.

4 **a** Find the angle of elevation to the top of a 56 m high building from point A which is 113 m from its base.

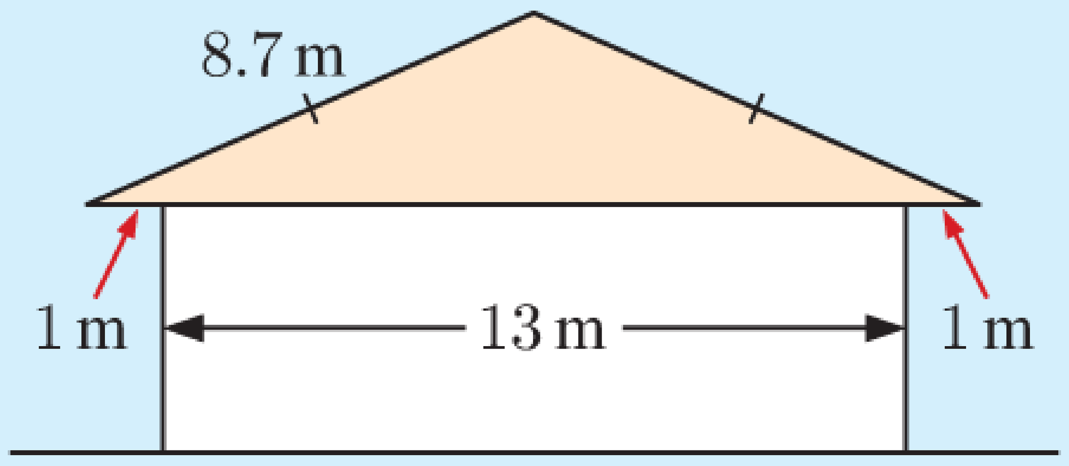
  b What is the angle of depression from the top of the building to A?

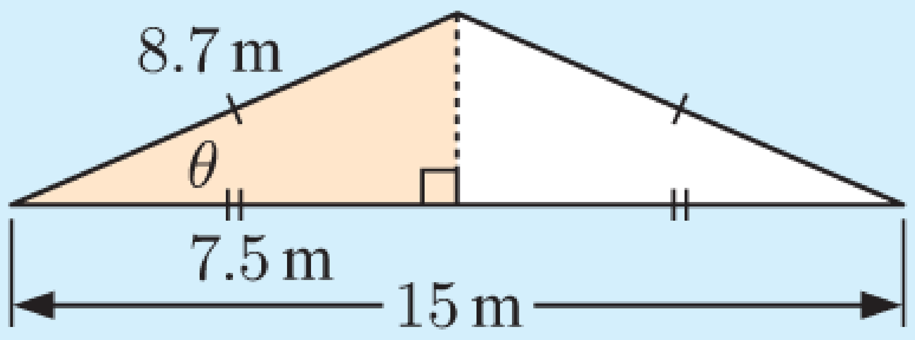
5 The angle of depression from the top of a 20 m high vertical cliff to a boat B is 8°.
How far is the boat from the base of the cliff?



Example 9 **Self Tutor**

A builder has designed the roof structure illustrated. Find the pitch of this roof.





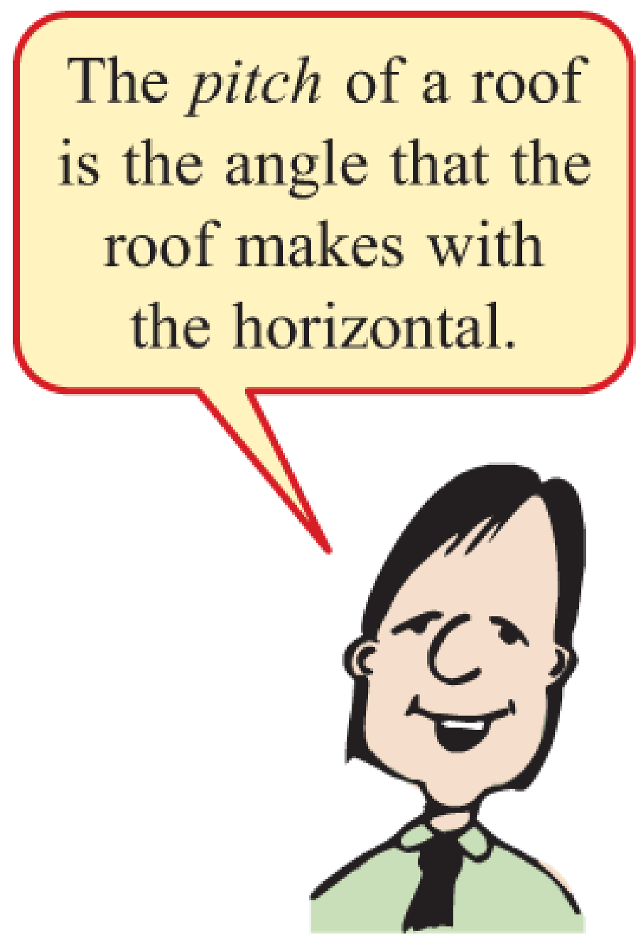
By constructing an altitude of the isosceles triangle, we form two right angled triangles.

$$\cos\theta = \frac{7.5}{8.7} \qquad \{\cos\theta = \frac{\text{ADJ}}{\text{HYP}}\}$$

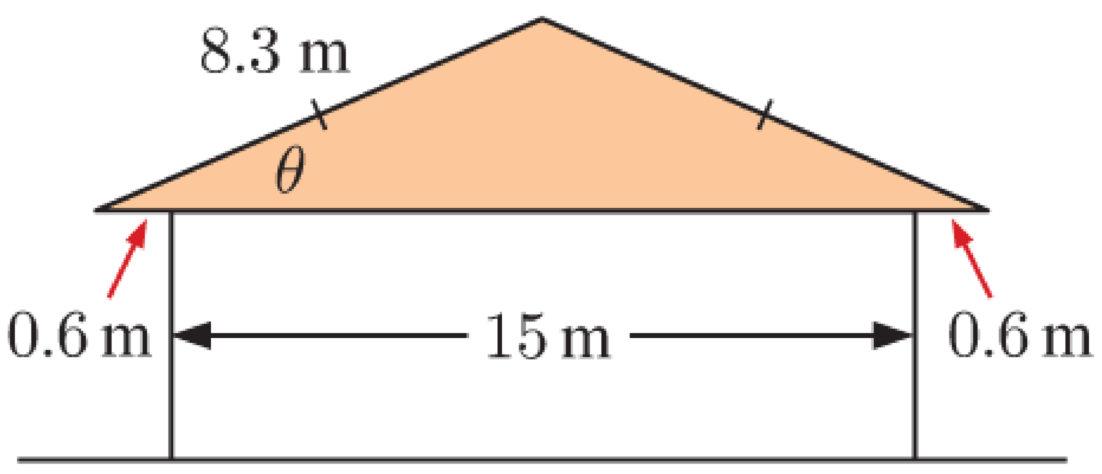
$$\therefore \quad \theta = \cos^{-1}\left(\frac{7.5}{8.7}\right)$$

$$\therefore \quad \theta \approx 30.5^\circ$$

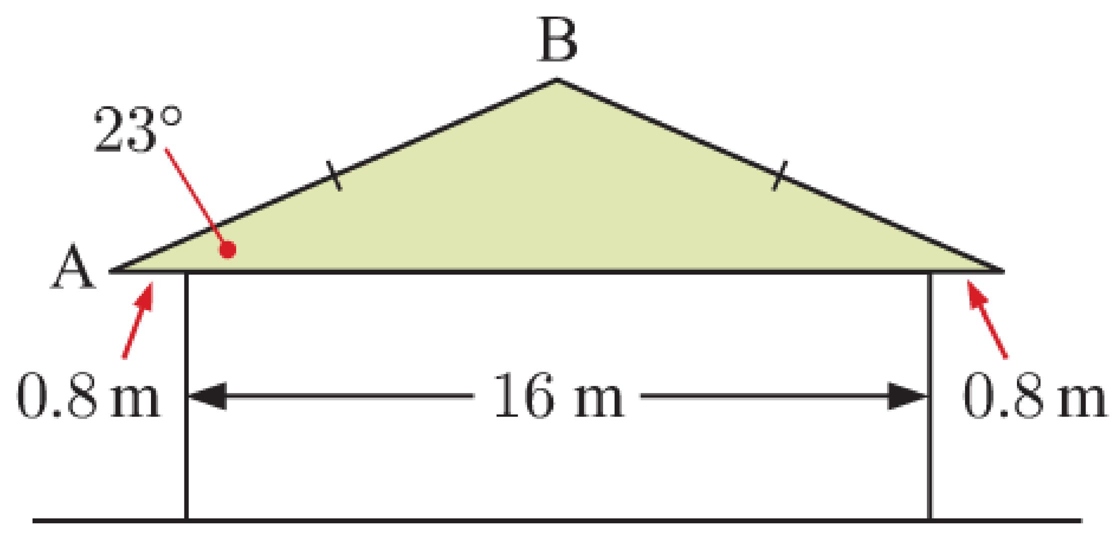
The pitch of the roof is approximately 30.5°.



6 Find θ, the pitch of the roof.

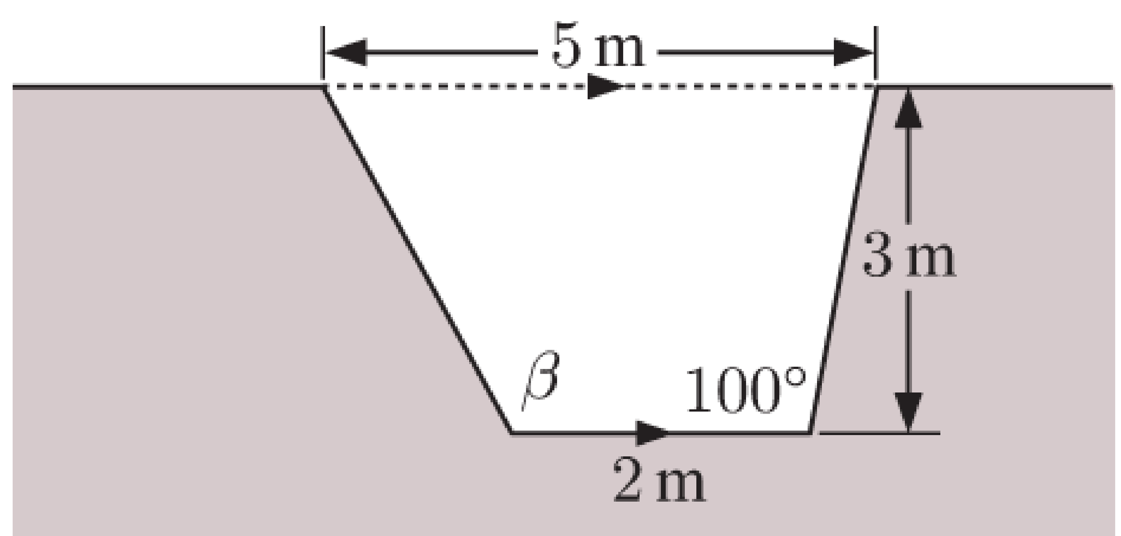


7 The pitch of the given roof is 23°. Find the length of the timber beam [AB].

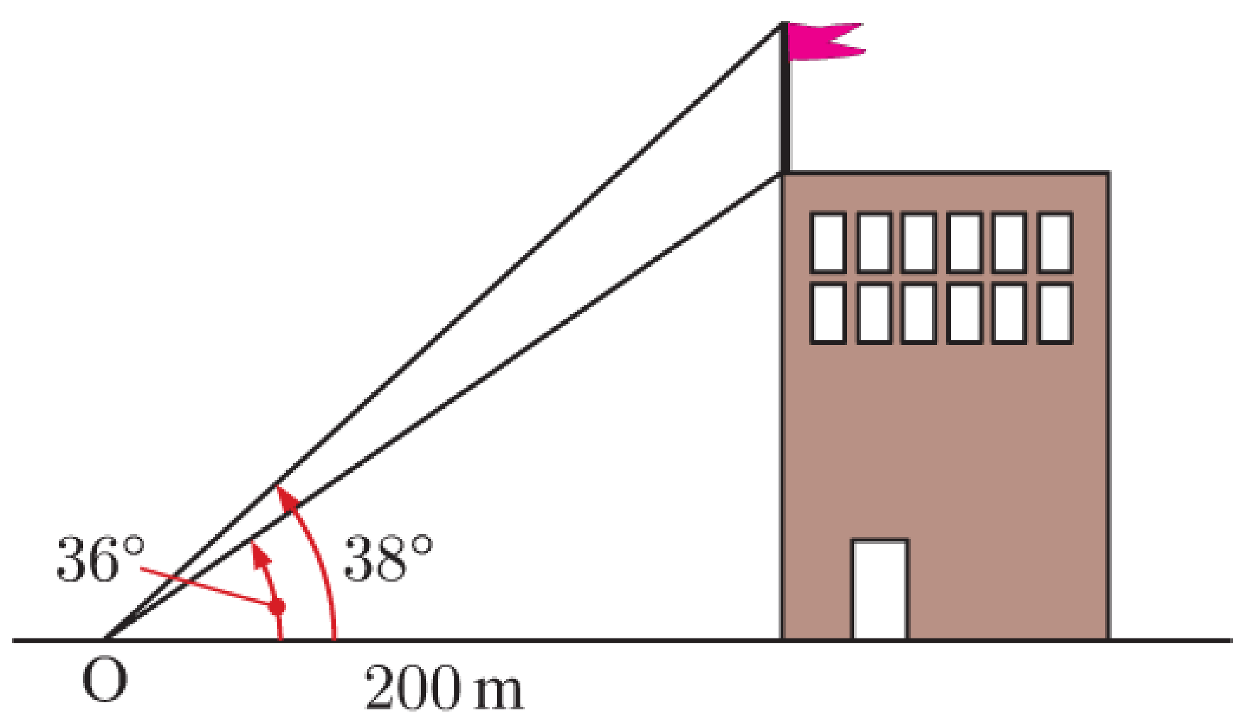


8 A rectangular field is 20 metres longer than it is wide. When Patrick walks from one corner to the opposite corner, he makes an angle of 55° with the shorter side of the field. Find the length of this shorter side.

9 A stormwater drain has the shape illustrated. Determine the angle β where the left hand side meets with the bottom of the drain.



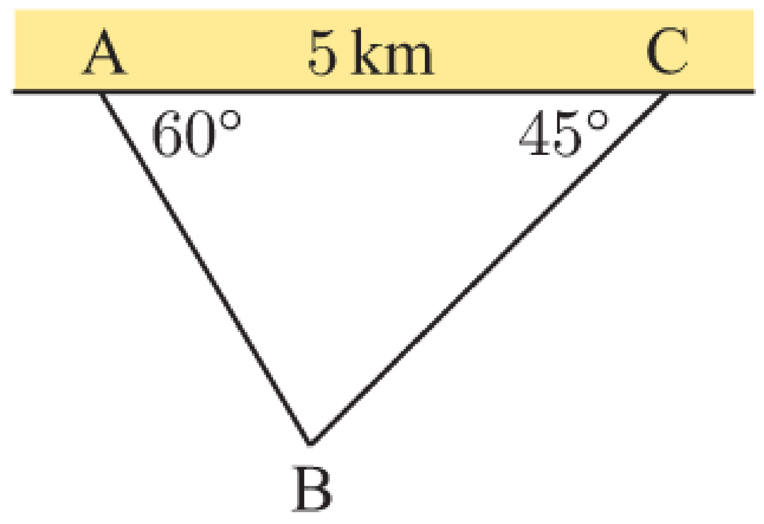
10 From an observer O who is 200 m from a building, the angles of elevation to the bottom and top of a flagpole are 36° and 38° respectively. Find the height of the flagpole.



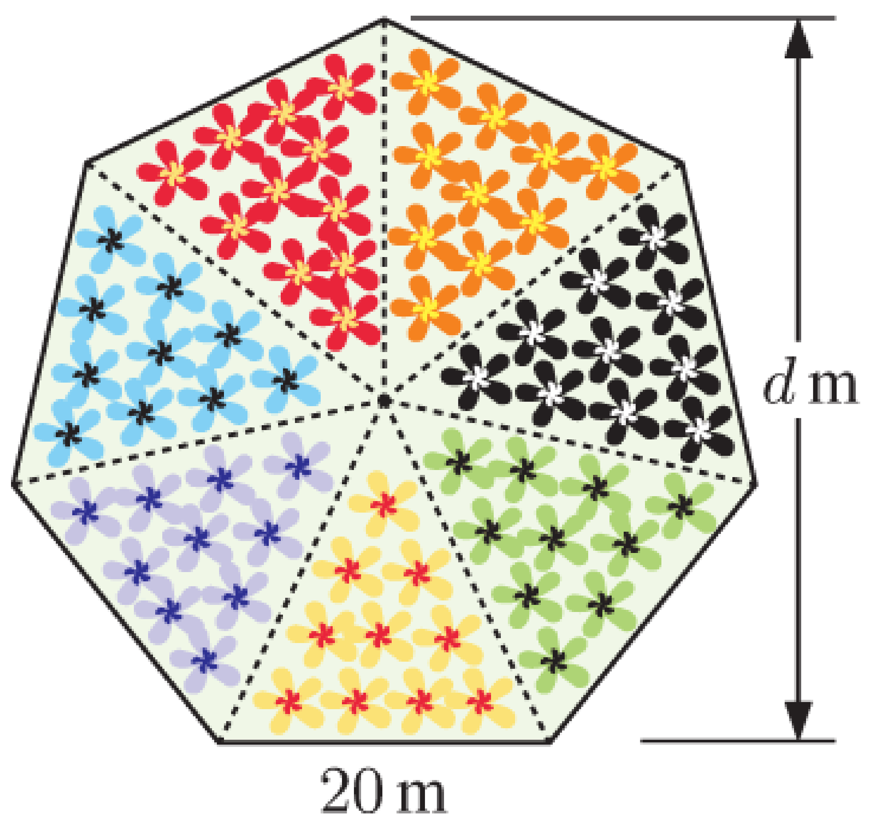
11 The angle of depression from the top of a 15 m high cliff to a boat at sea is $2.7°$. How much closer to the cliff must the boat move for the angle of depression to become $4°$?

12 A helicopter flies horizontally at 100 km h^{-1}. An observer notices that it takes 20 seconds for the helicopter to fly from directly overhead to being at an angle of elevation of $60°$. Find the height of the helicopter above the ground.

13 [AC] is a straight shore line 5 km long. B is a boat out at sea. Find the shortest distance from the boat to the shore.



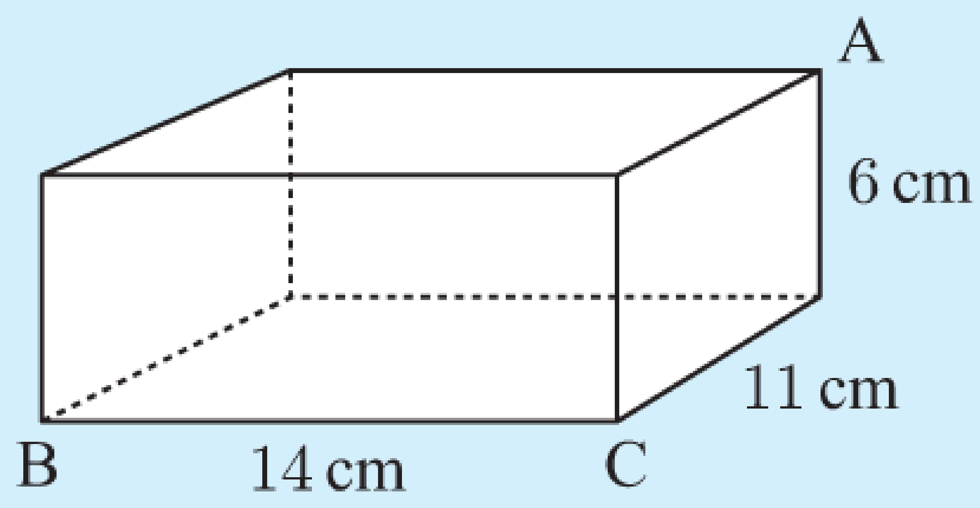
14



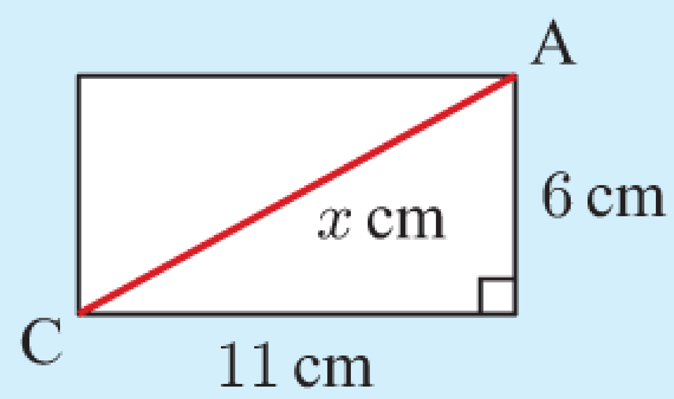
A new feature at the botanical gardens will be a regular heptagonal flower bed with sides of length 20 m.
Find the width of land d m required for the flower bed.

Example 10 | **Self Tutor**

A rectangular prism has the dimensions shown.
Find the measure of $\widehat{ABC}$.



Consider the end of the prism containing points A and C.
Let $AC = x$ cm.

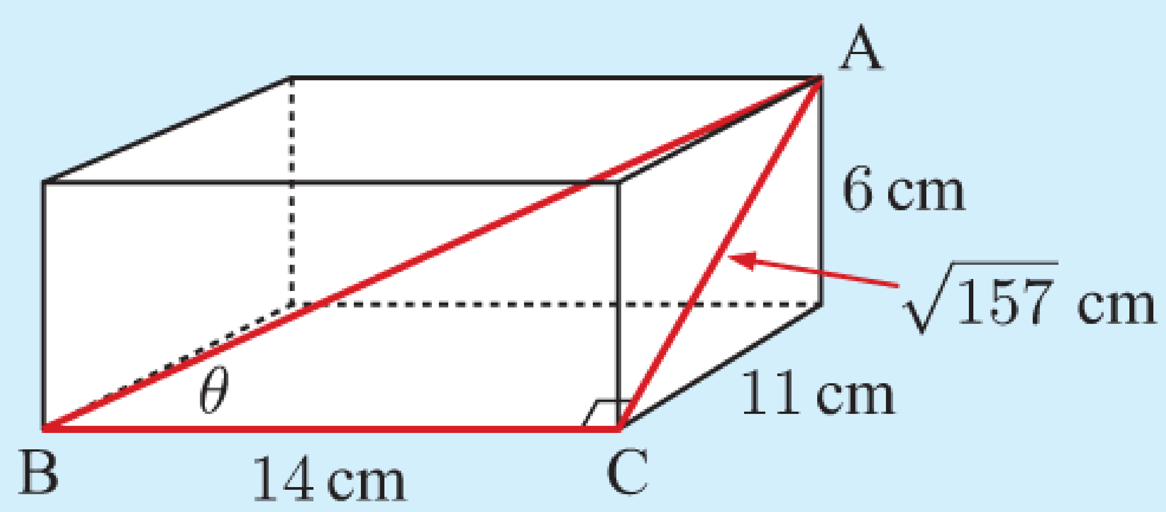


Using Pythagoras, $x^2 = 6^2 + 11^2$

$\therefore\ x^2 = 157$

$\therefore\ x = \sqrt{157}$ $\{\text{as } x > 0\}$

$\triangle$ABC is right angled at C.

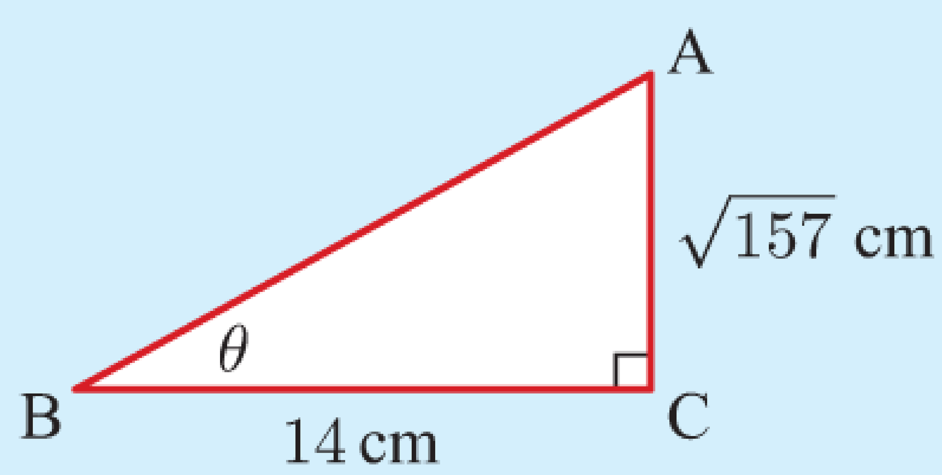


Let $\widehat{ABC} = \theta$

$\therefore\ \tan\theta = \frac{\sqrt{157}}{14}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore\ \theta = \tan^{-1}\left(\frac{\sqrt{157}}{14}\right)$

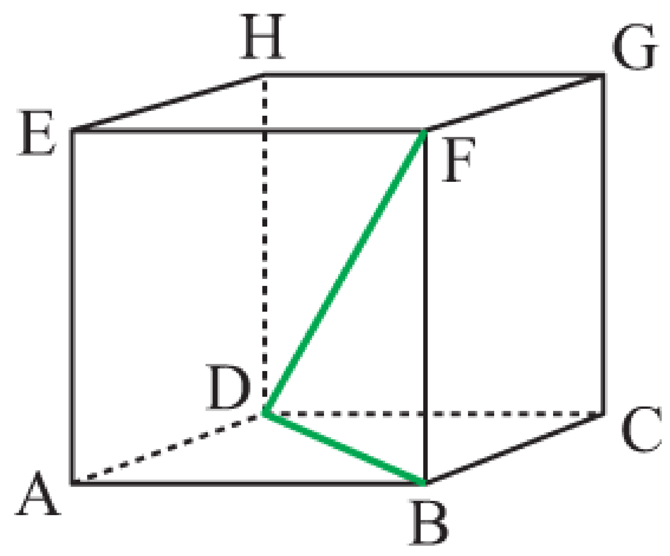
$\therefore\ \theta \approx 41.8°$



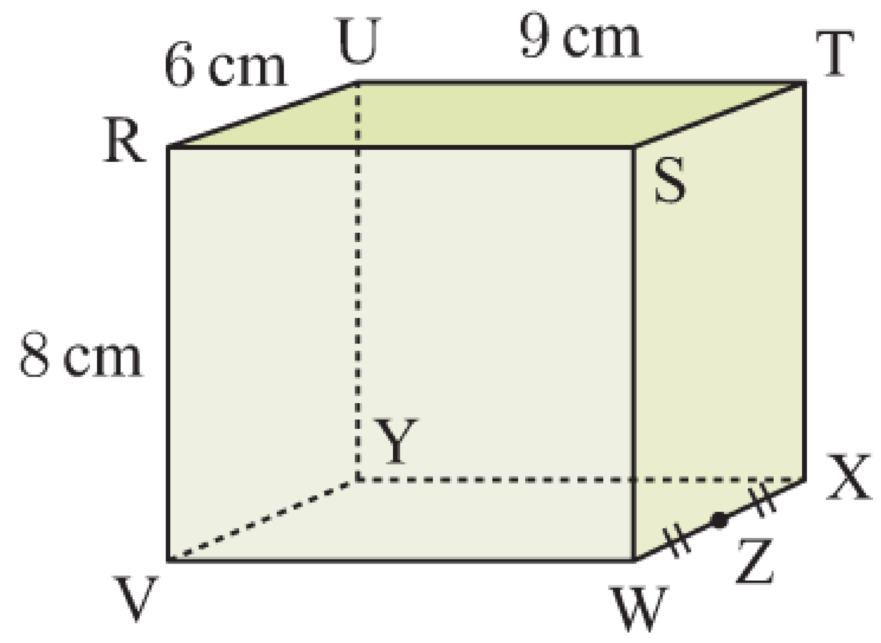
So, $\widehat{ABC} \approx 41.8°$.

15 The cube shown has sides of length 13 cm. Find:

a BD **b** $F\widehat{D}B$.



16 In the rectangular prism shown, Z is the midpoint of [WX]. Find:



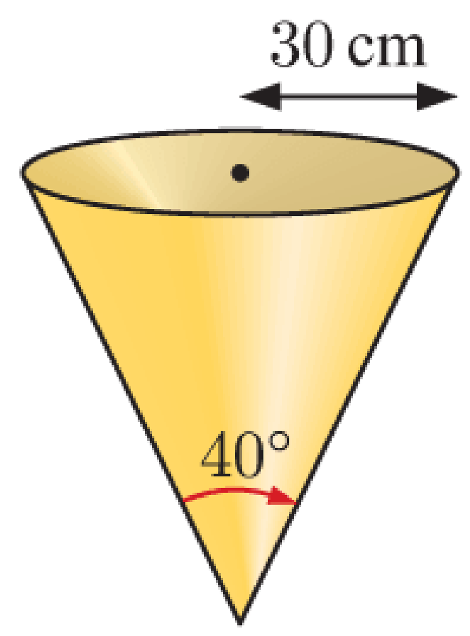
a VX **b** $R\widehat{X}V$

c YZ **d** $Y\widehat{Z}U$.

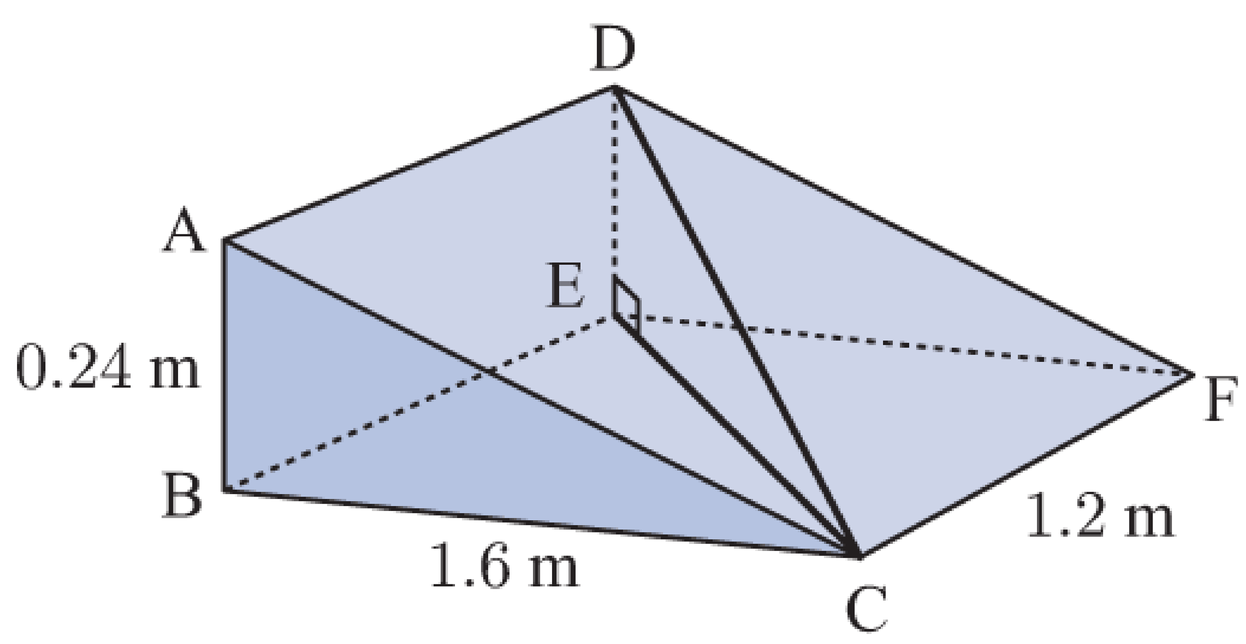
17 An open cone has a vertical angle measuring 40° and a base radius of 30 cm. Find:

a the height of the cone

b the capacity of the cone in litres.



18 A ramp is built as the triangular prism shown.



a Find the length:

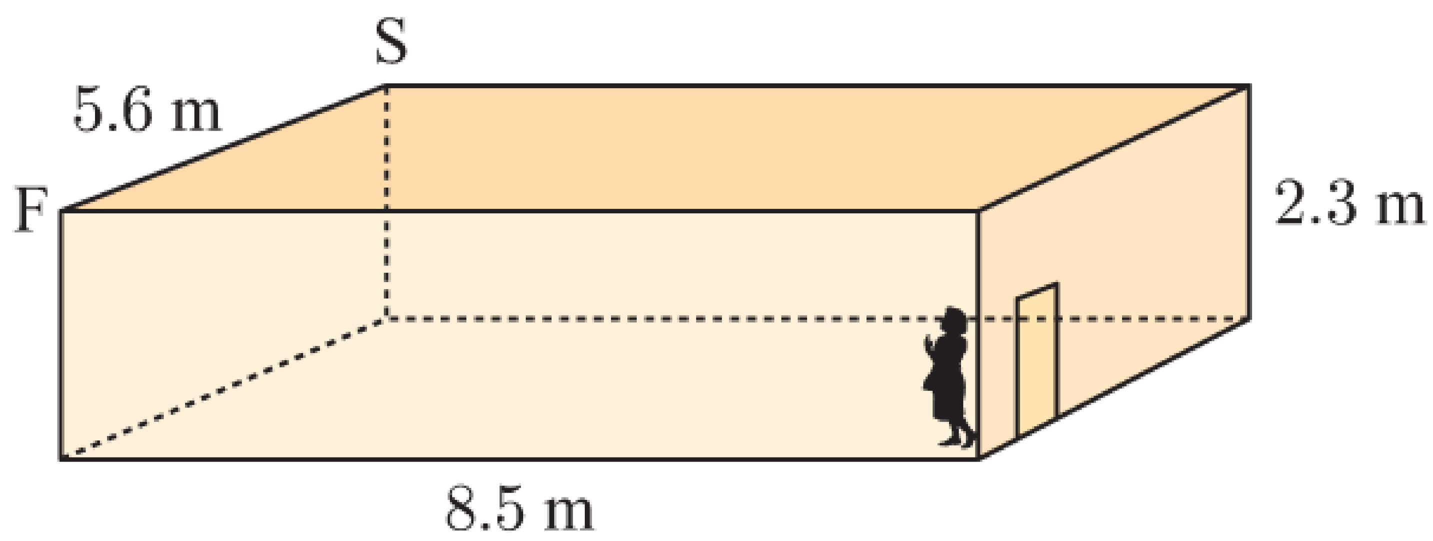
i CE **ii** CD.

b Find $D\widehat{C}E$.

19 Elizabeth is terrified of spiders. When she walks into a room, she notices one in the opposite corner S.

a If Elizabeth is 1.6 m tall, how far is the spider from her head?

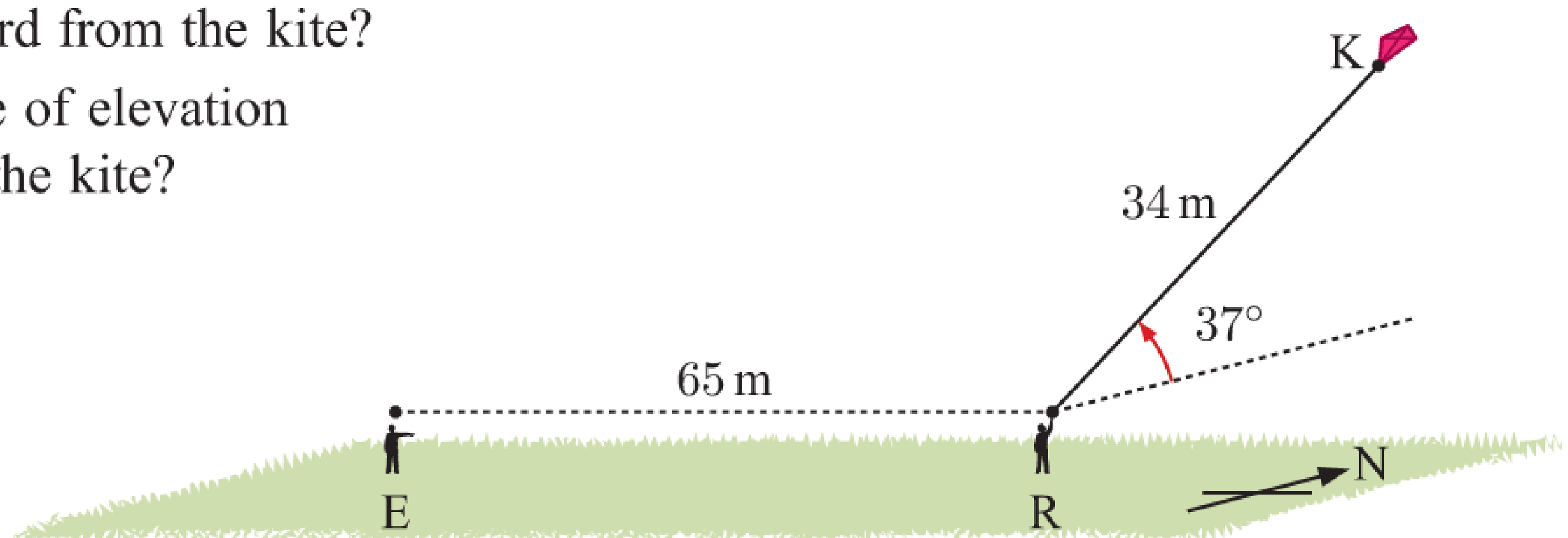
b The spider can see up to an angle of 42° from the direction it is facing. This spider is facing a fly at F. Can it see Elizabeth?



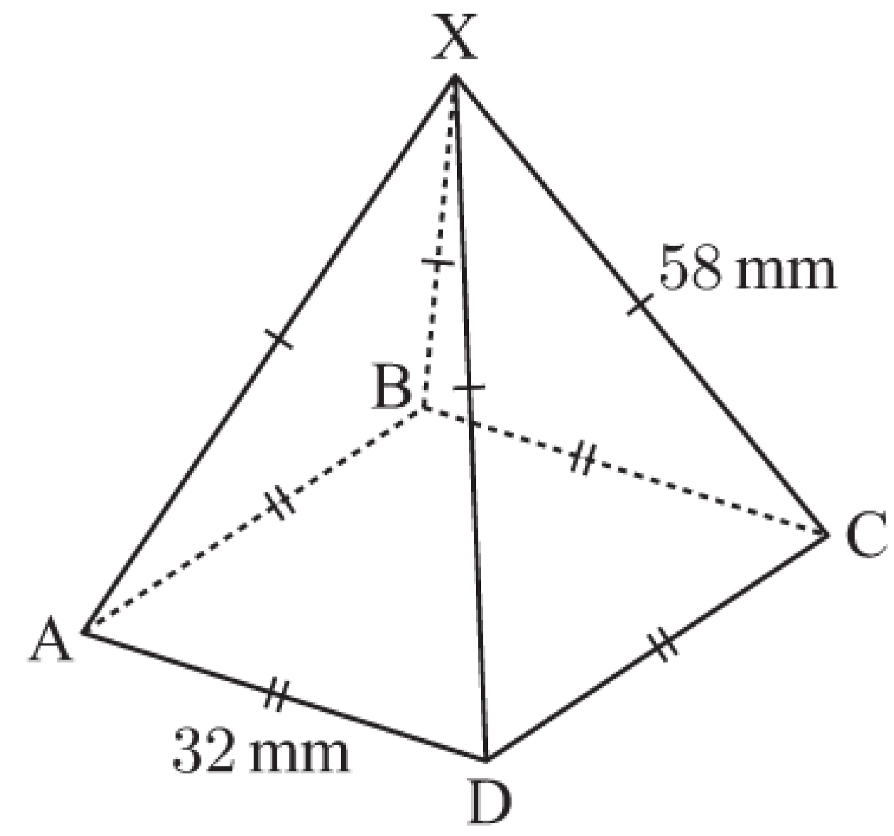
20 Rico is flying his kite with the aid of a southerly wind. He has let out 34 m of string, and the kite is at an angle of elevation of 37°. His friend Edward stands to the west, 65 m away.

a How far is Edward from the kite?

b What is the angle of elevation from Edward to the kite?

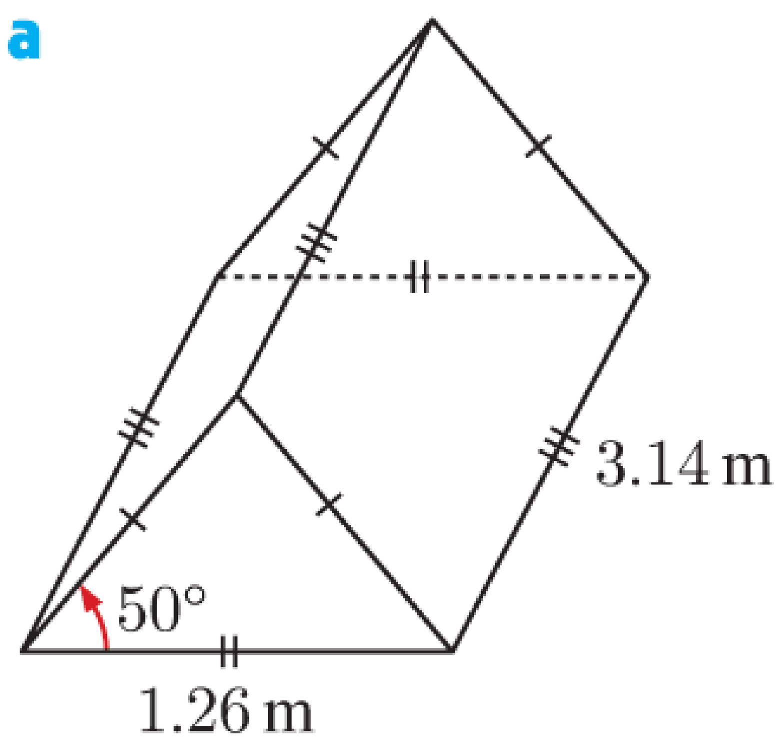


21 Find the angle between the slant edge [AX] and the base diagonal [AC].

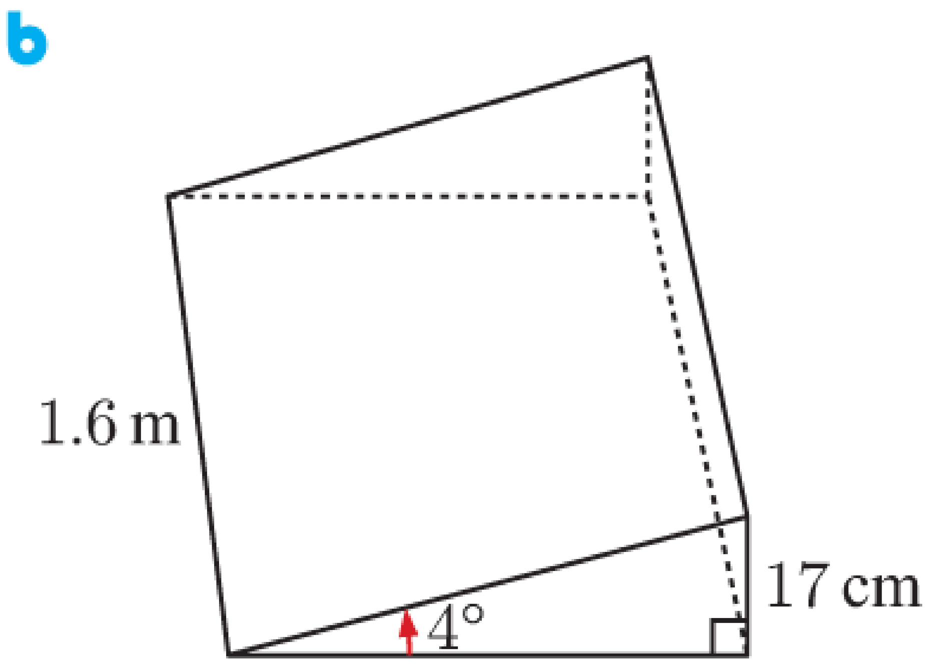


22 Find the volume of each solid:

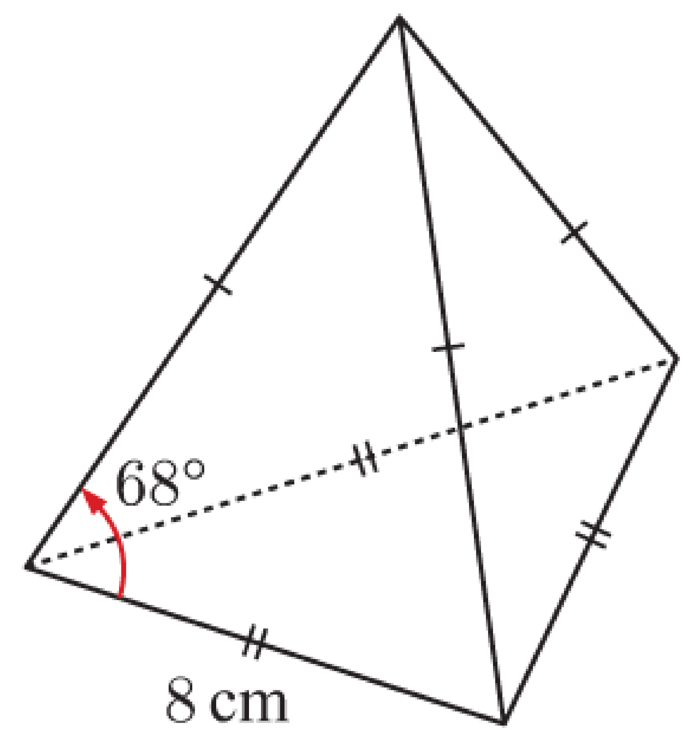
a



b



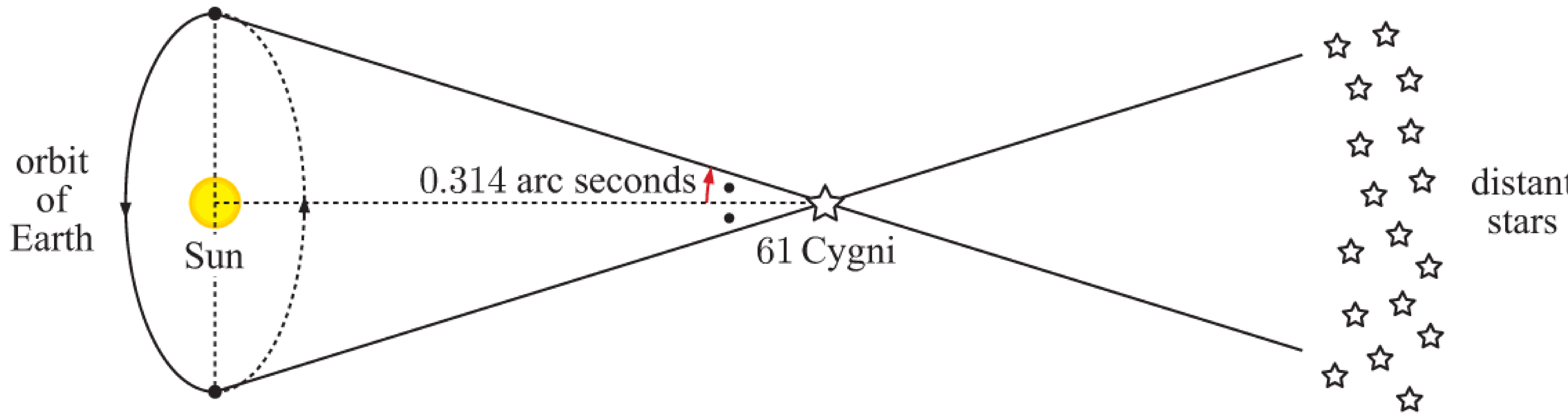
c



23 A **parallax** is the angle through which an object appears to move when viewed from different positions.

In 1838, the Prussian astronomer **Friedrich Wilhelm Bessel** (1784 - 1846) used the telescopes at the Königsberg Observatory to measure the parallax of the star 61 Cygni to be approximately 0.314 arc seconds, where one arc second is $\frac{1}{3600}$ of a degree. He did this by comparing the apparent positions of 61 Cygni during the year, relative to a fixed backdrop of distant stars.

Friedrich Bessel



a Given that the radius of the Earth's orbit is $\approx 1.49 \times 10^{11}$ m and that 1 light-year $\approx 9.461 \times 10^{15}$ m, explain Bessel's calculation that 61 Cygni is about 10.3 light-years away.

b Modern estimates place 61 Cygni at about 11.4 light-years away. Calculate a more accurate value for the parallax of 61 Cygni.

1 parsec or "parallactic second" is the distance to a star with a parallax of 1 arc second measured across the Earth's orbit.
1 parsec $\approx$ 3.26 light-years.

E TRUE BEARINGS

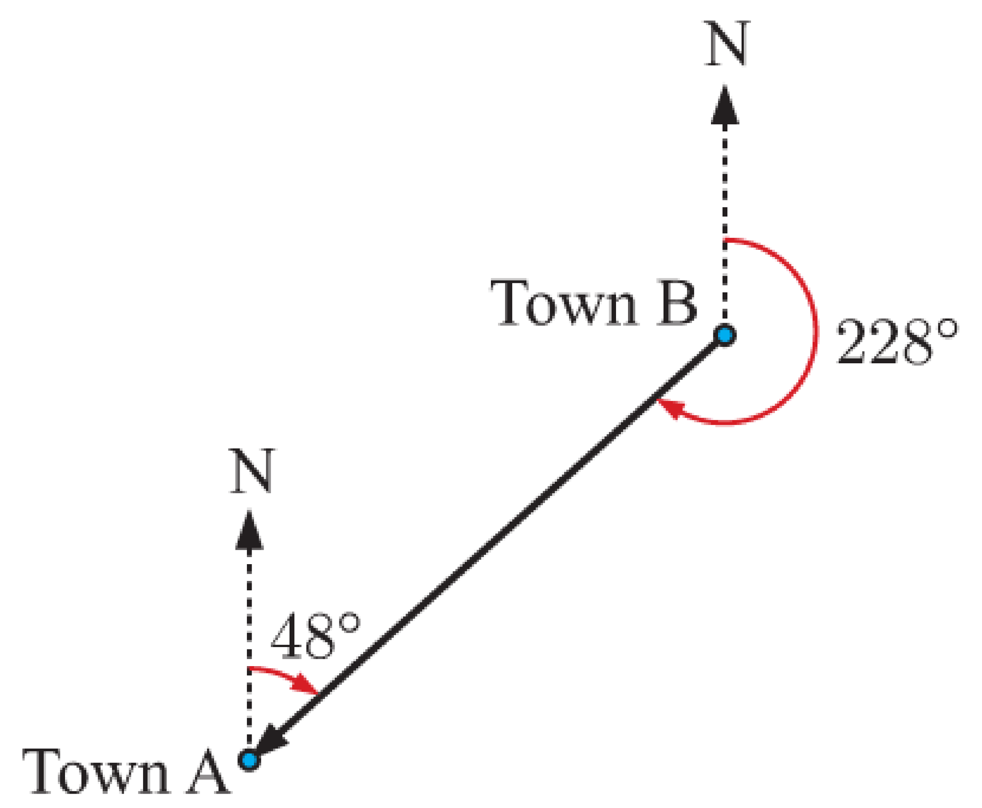
True bearings are used to describe the direction of one object from another. The direction is described by comparing it with the **true north direction**.

> True bearings are measured **clockwise** from true north.
> They are always written with 3 digits, plus decimals if necessary.

Suppose you are in town A and you want to go to town B. If you start facing true north, you need to turn 48° clockwise in order to face town B. We say that the **bearing of B from A** is 048°.

Now suppose you are in town B and want to go to town A. If you start facing true north, you need to turn 228° clockwise in order to face town A. The bearing of A from B is 228°.

Notice that the bearing of B from A and the bearing of A from B differ by 180°.



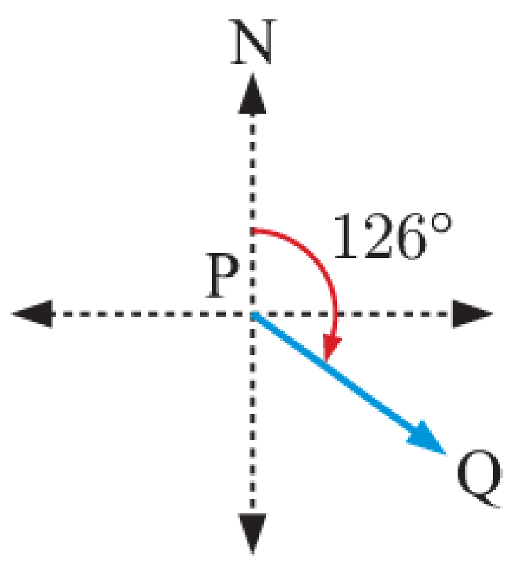
EXERCISE 7E

1 Draw a diagram showing that the bearing of B from A is:

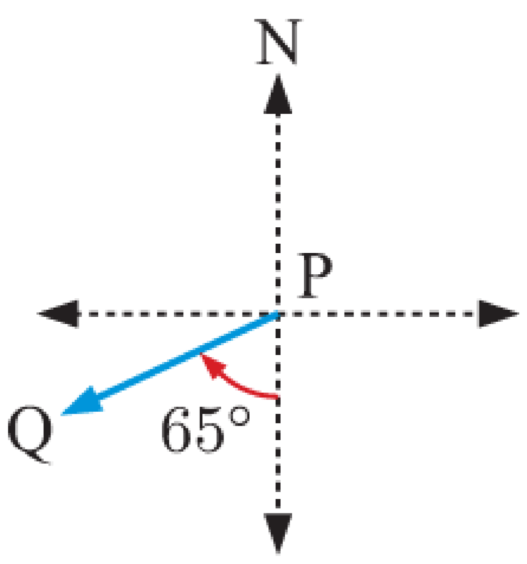
a 075° **b** 205° **c** 110° **d** 325°

2 Find the bearing of Q from P in each diagram:

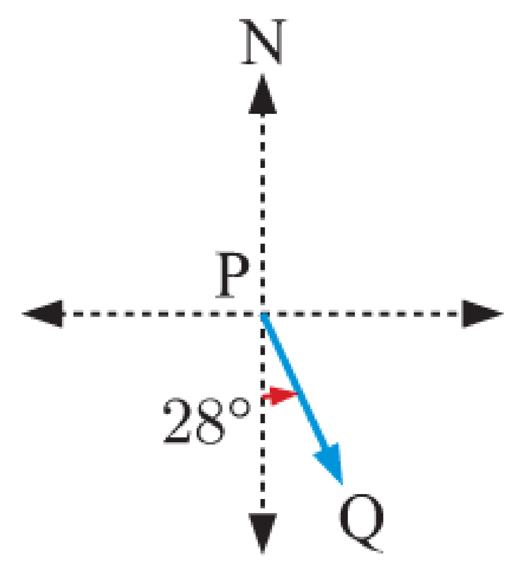
a



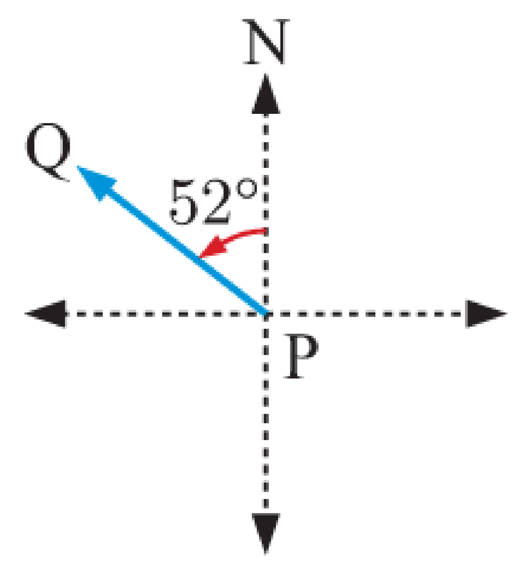
b



c

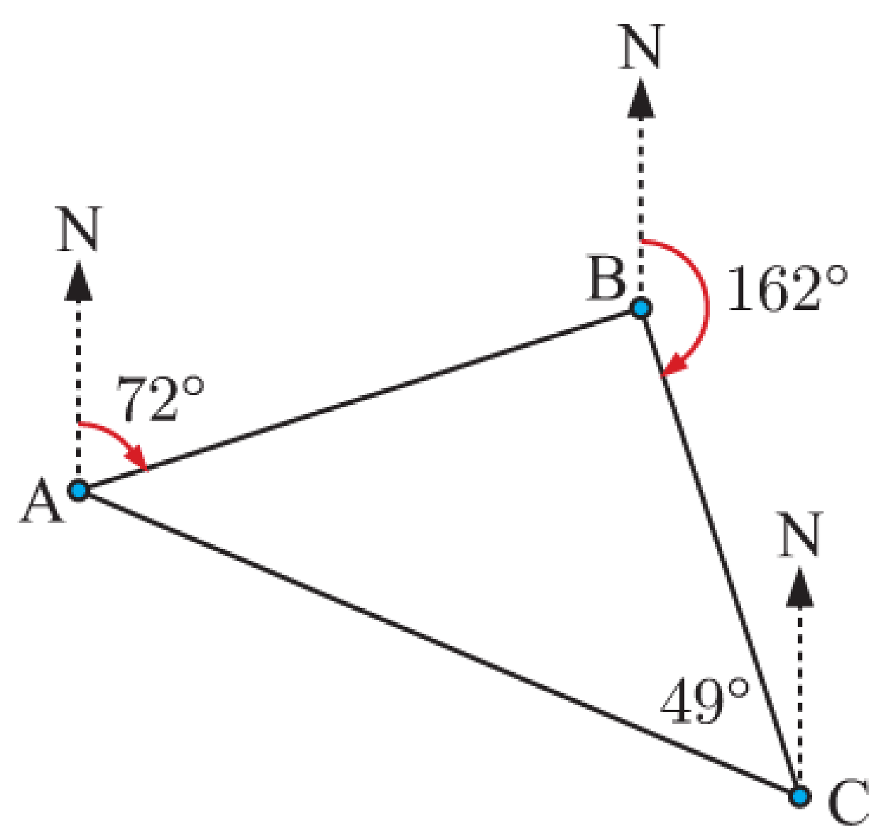


d



3 In the diagram given, find the bearing of:

a B from A **b** A from B **c** C from B
d B from C **e** C from A **f** A from C.



Example 11 **Self Tutor**

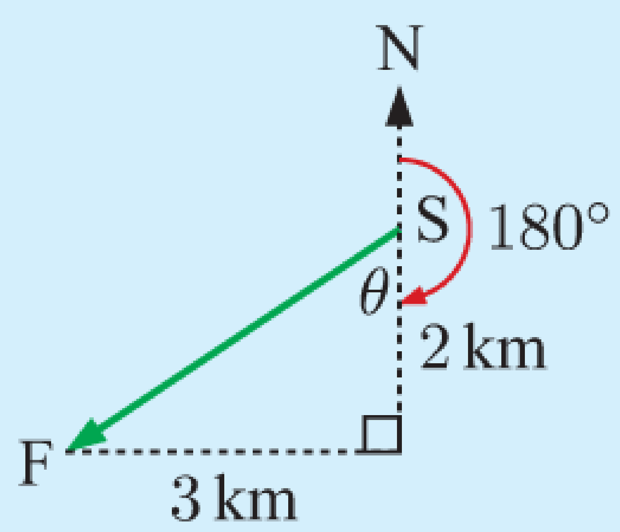
When Samantha goes jogging, she finishes 2 km south and 3 km west of where she started. Find Samantha's bearing from her starting point.

Suppose Samantha starts at S and finishes at F.

$\tan\theta = \frac{3}{2}$

$\therefore \quad \theta = \tan^{-1}\left(\frac{3}{2}\right) \approx 56.3°$

So, the bearing $\approx 180° + 56.3° \approx 236°$



4 When Walter drives to his sports club, he finishes 10 km east and 7 km south of where he started. Find Walter's bearing from his starting point.

5 Julia is swimming in the ocean. A current takes her 200 m north and 100 m west of where she started.

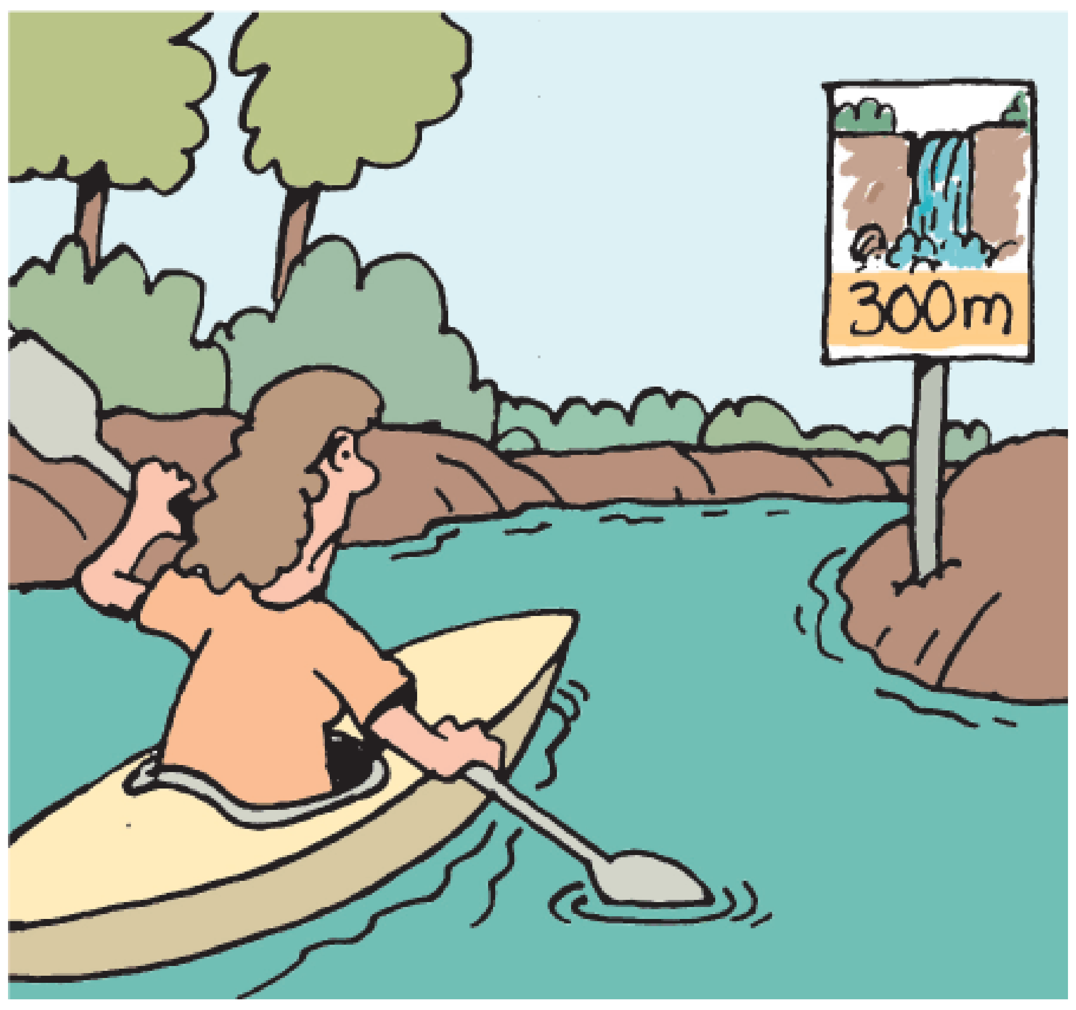
- **a** How far is Julia from her starting point?
- **b** Find Julia's bearing from her starting point.
- **c** In which direction is the starting point from where Julia is now?

6 Paul runs 1.5 km on the bearing 127°.

- **a** Draw a diagram of the situation.
- **b** How far east is Paul from his starting point?
- **c** How far south is Paul from his starting point?

7 Tiffany kayaks 4 km on the bearing 323°. How far west is Tiffany from her starting point?

8 A train travels on the bearing 072° until it is 12 km east of its starting point. How far did the train travel on this bearing?



Example 12 **Self Tutor**

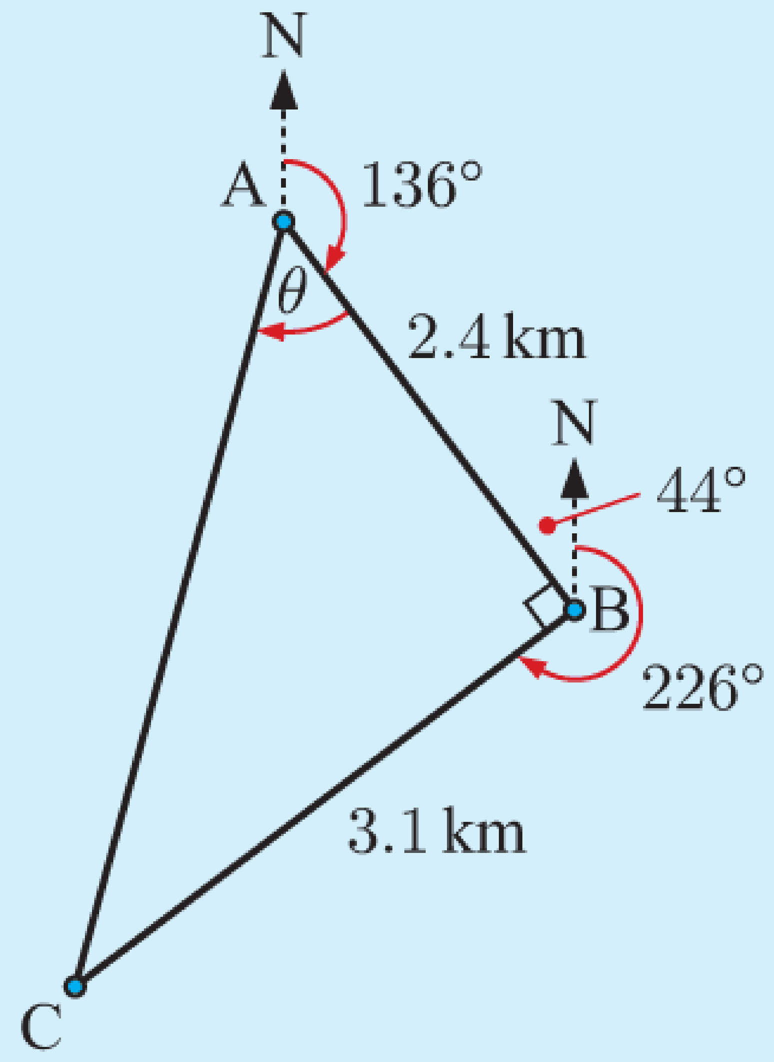
A courier departs from his depot A and drives on a 136° course for 2.4 km to an intersection B. He turns right at the intersection, and drives on a 226° course for 3.1 km to his destination C. Find:

- **a** the distance of C from A
- **b** the bearing of C from A.

$A\widehat{B}N = 180° - 136° = 44°$ {cointerior angles}

$\therefore A\widehat{B}C = 360° - 44° - 226°$ {angles at a point}

$= 90°$



a $AC^2 = 2.4^2 + 3.1^2$ {Pythagoras}

$\therefore AC = \sqrt{2.4^2 + 3.1^2}$ {as $AC > 0$}

≈ 3.92 km

So, C is about 3.92 km from A.

b To find the bearing of C from A, we first need to find θ.

Now $\tan\theta = \frac{3.1}{2.4}$

$\therefore \theta = \tan^{-1}\left(\frac{3.1}{2.4}\right)$

$\therefore \theta \approx 52.3°$

The bearing of C from A is $136° + 52.3° \approx 188°$.

9 An orienteer runs 720 m in the direction 236° to a checkpoint, and then 460 m in the direction 146° to the finish. Find the:

- **a** direct distance from the starting point to the finishing point
- **b** bearing of the finishing point from the starting point.

10 A cruise ship sails from port P in the direction 112° for 13.6 km, and then in the direction 202° for 72 km. Find the distance and bearing of the cruise ship from P.

11 Yachts A and B depart from the same point. Yacht A sails 11 km on the bearing 034°. Yacht B sails 14 km on the bearing 124°. Find the distance and bearing of yacht B from yacht A.

12 An eagle is 2 km away on the bearing 293° from its nest. The eagle flies in a straight line for 5 km, and is now on the bearing 023° from its nest.

a On what bearing did the eagle fly? **b** How far north from its nest is the eagle now?

F THE ANGLE BETWEEN A LINE AND A PLANE

When the sun shines on the *gnomon* of a sundial, it casts a shadow onto the dial beneath it.

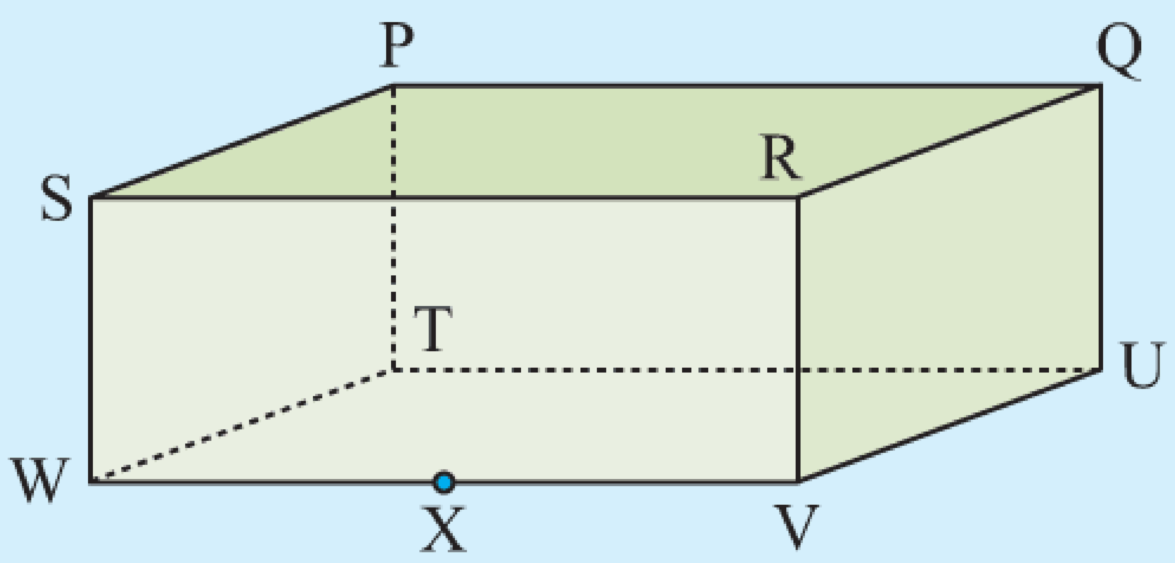
If the sun is directly overhead, its rays are *perpendicular* to the dial. The shadow formed is the **projection** of the gnomon onto the dial.

The **angle between a line and a plane** is the angle between the line and its **projection** on the plane.

Example 13 Self Tutor

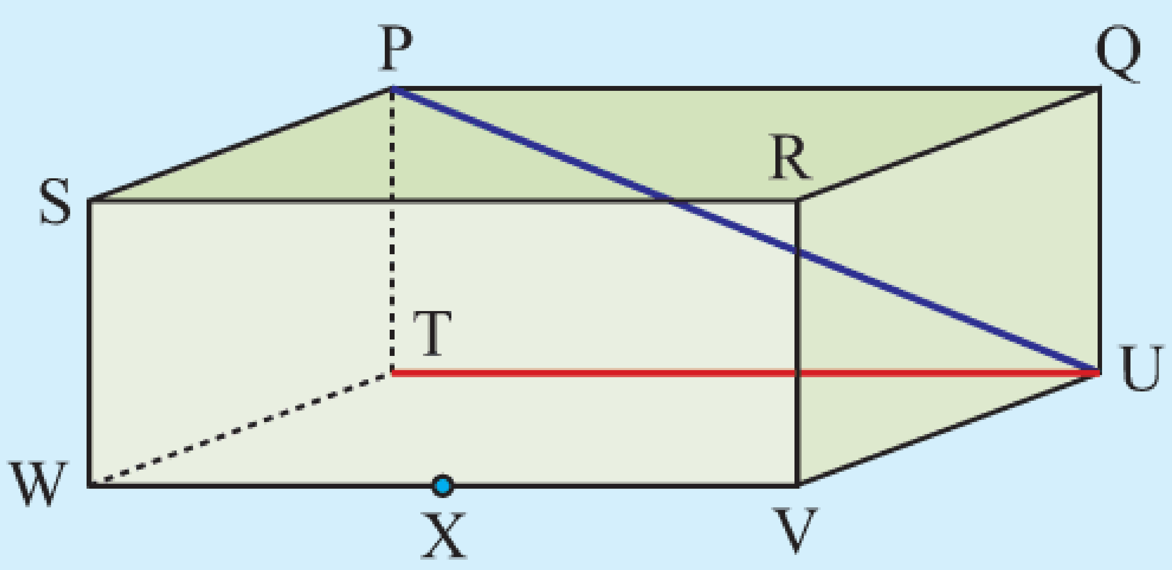
Name the angle between the following line segments and the base plane TUVW:

a [PU] **b** [QX]



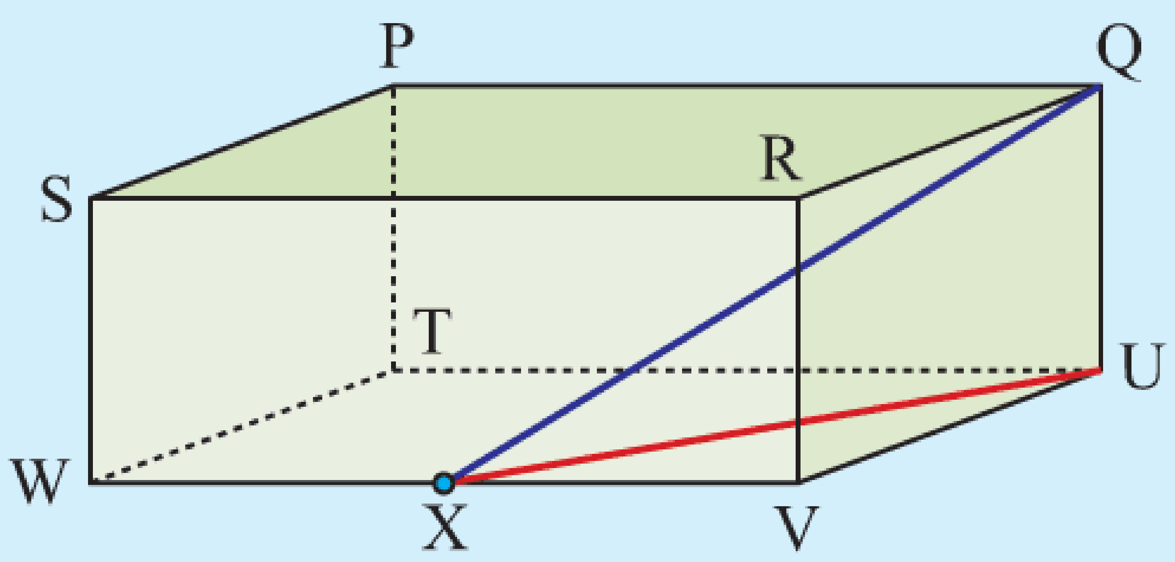
a The projection of [PU] onto the base plane is [TU].

$\therefore$ the required angle is $P\widehat{U}T$.



b The projection of [QX] onto the base plane is [UX].

$\therefore$ the required angle is $Q\widehat{X}U$.

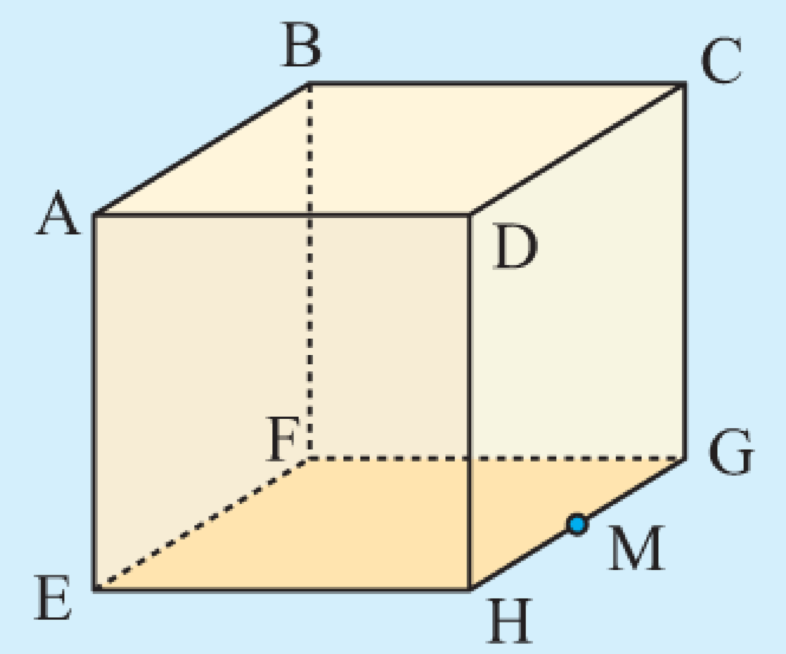


Example 14 **Self Tutor**

The diagram shows a cube with side length 12 cm. M is the midpoint of [GH].

Find the angle between the following line segments and the base plane EFGH:

a [AH] **b** [BM]



a The projection of [AH] onto the base plane is [EH].

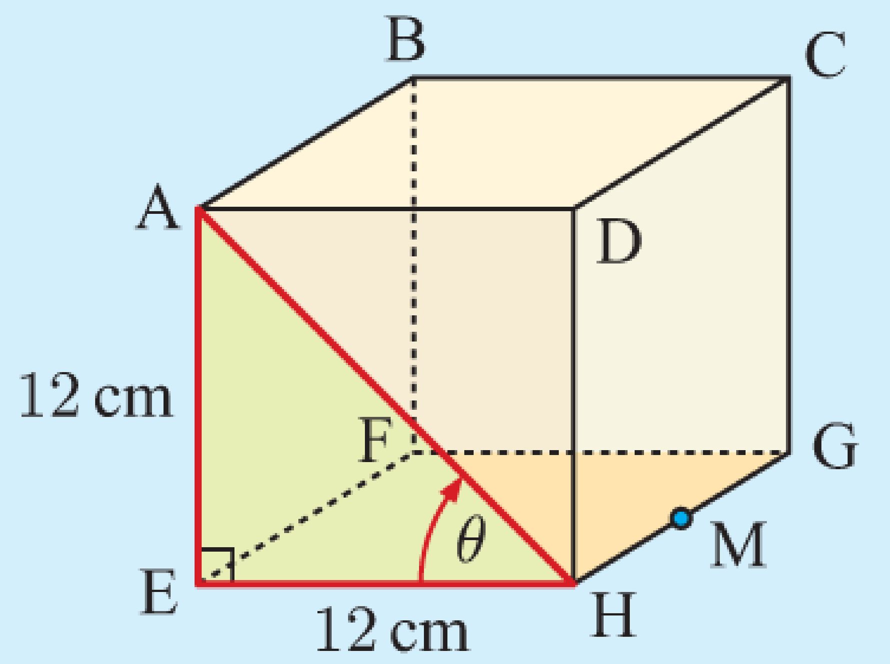
$\therefore$ the required angle is $A\widehat{H}E$.

$\tan\theta = \frac{12}{12}$ $\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore \theta = \tan^{-1} 1$

$\therefore \theta = 45°$

The angle is 45°.



b The projection of [BM] onto the base plane is [FM].

$\therefore$ the required angle is $B\widehat{M}F$.

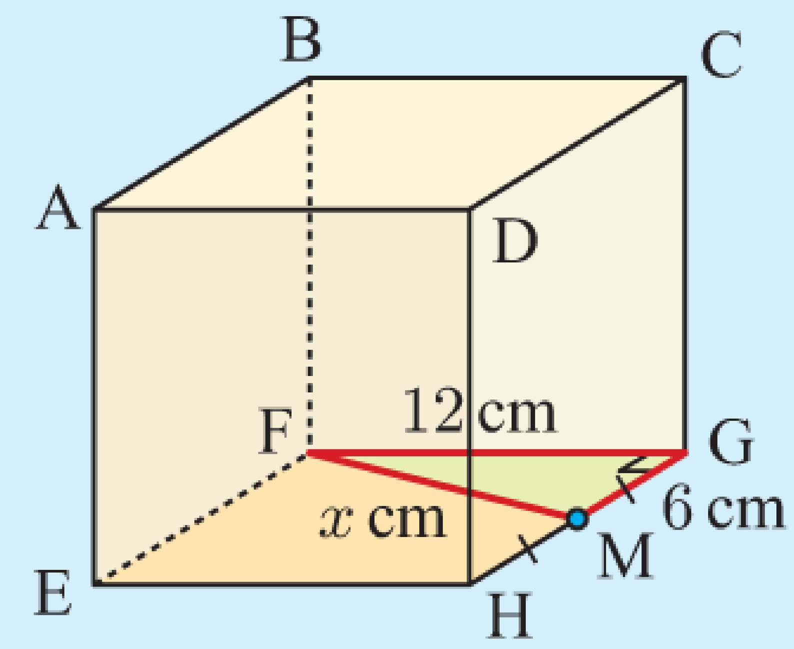
Let FM $= x$ cm.

Using Pythagoras in $\triangle$FGM,

$x^2 = 12^2 + 6^2$

$\therefore x^2 = 180$

$\therefore x = \sqrt{180}$ $\{\text{as } x > 0\}$



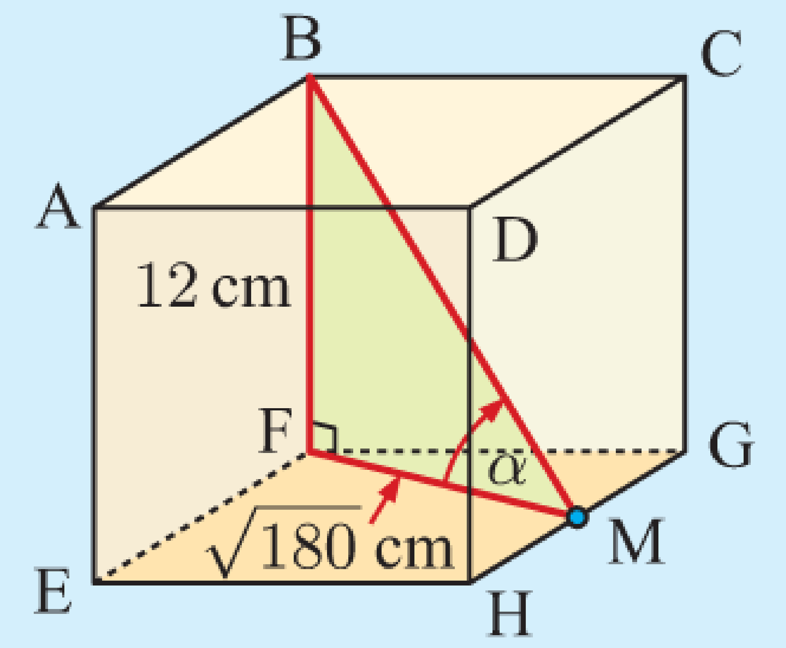
Let $B\widehat{M}F = \alpha$

$\therefore \tan\alpha = \frac{12}{\sqrt{180}}$ $\{\tan\alpha = \frac{\text{OPP}}{\text{ADJ}}\}$

$\therefore \alpha = \tan^{-1}\left(\frac{12}{\sqrt{180}}\right)$

$\therefore \alpha \approx 41.81°$

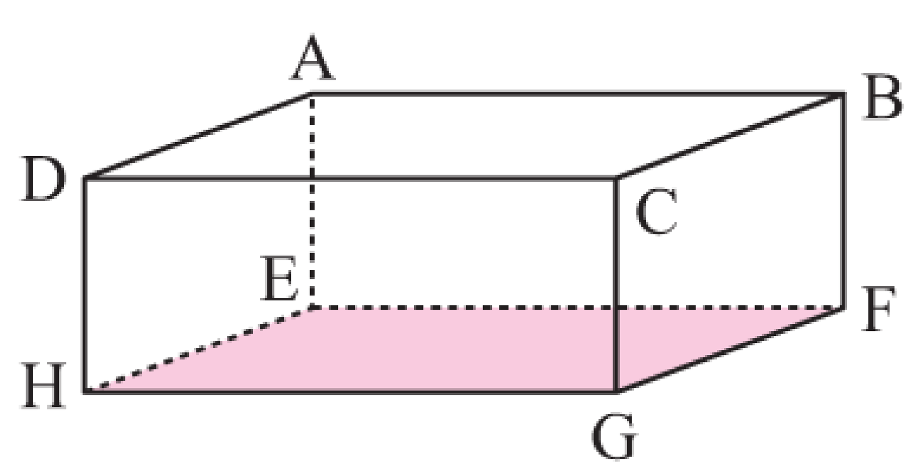
The angle is about 41.8°.



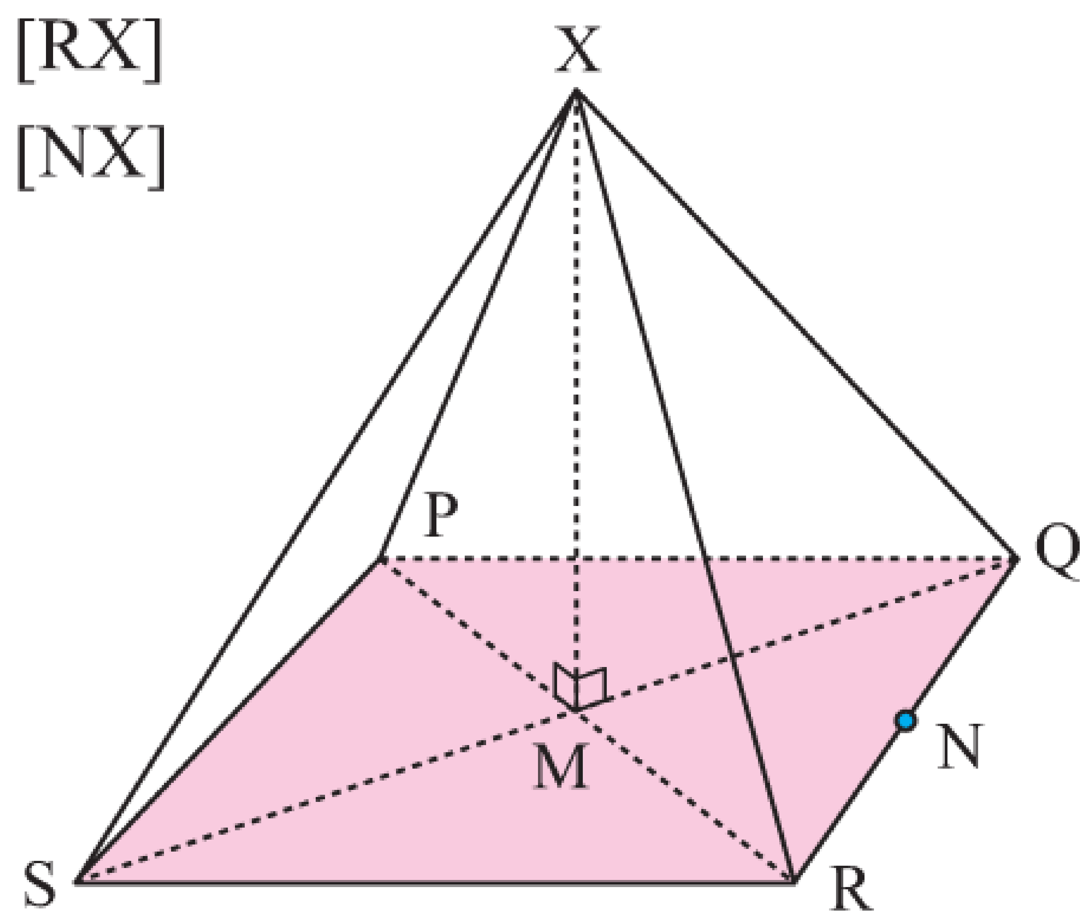
EXERCISE 7F

1 Name the projection of each line segment onto the base plane of the given figure:

a **i** [AH]
ii [BE]
iii [AG]
iv [BH]

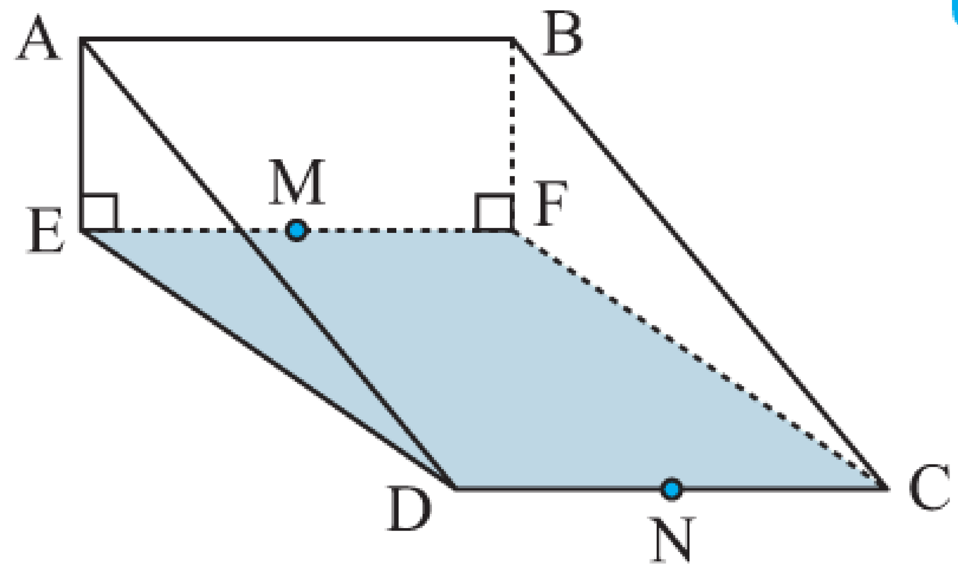


b **i** [RX]
ii [NX]

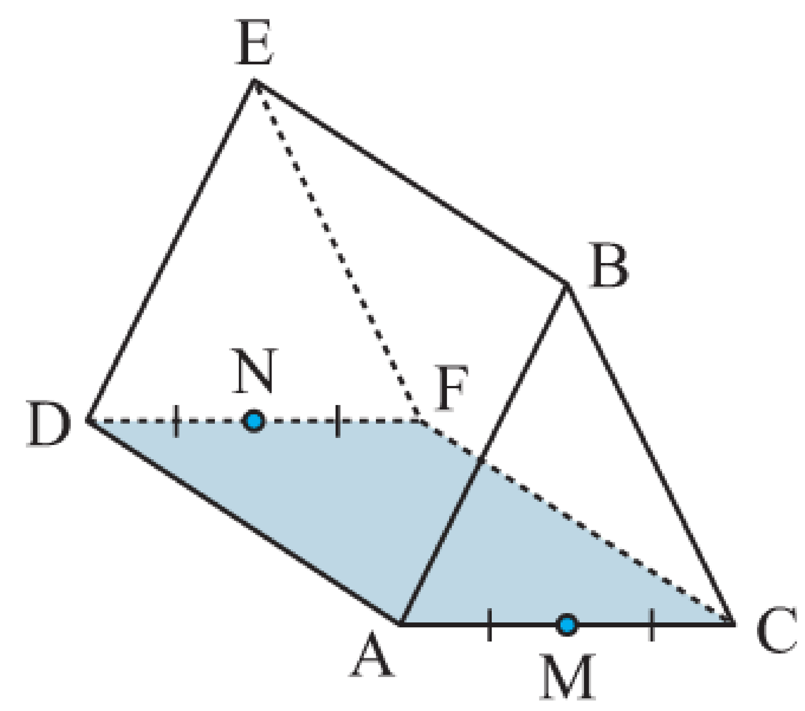


2 Name the angle between the following line segments and the base plane of the figure:

a **i** [AF]
ii [BM]
iii [AD]
iv [BN]

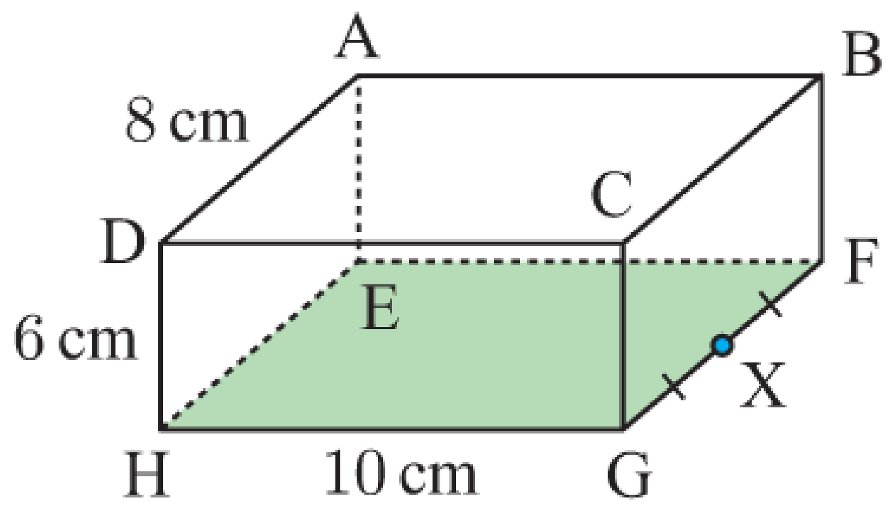


b **i** [AB]
ii [BN]
iii [AE]

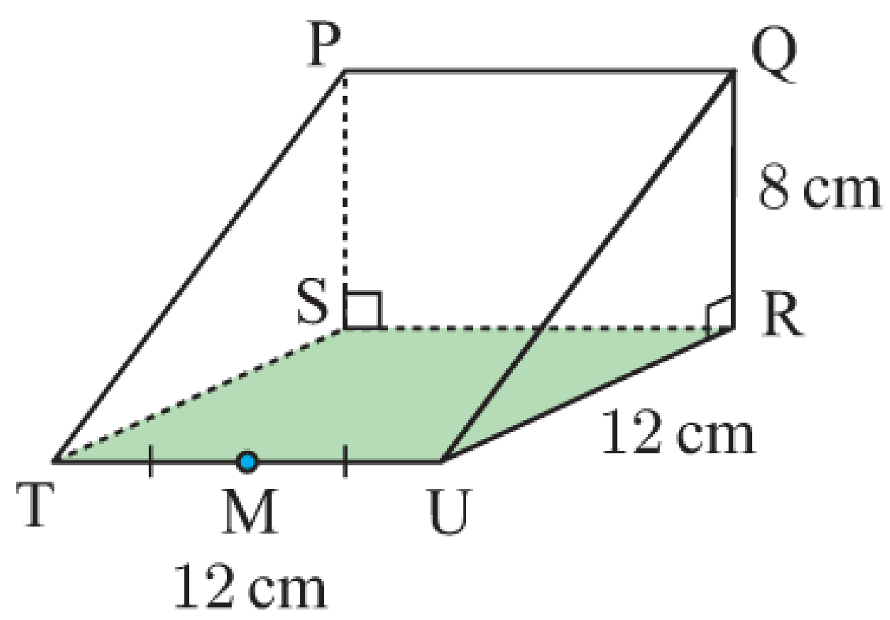


3 Find the angle between the following line segments and the base plane of the figure:

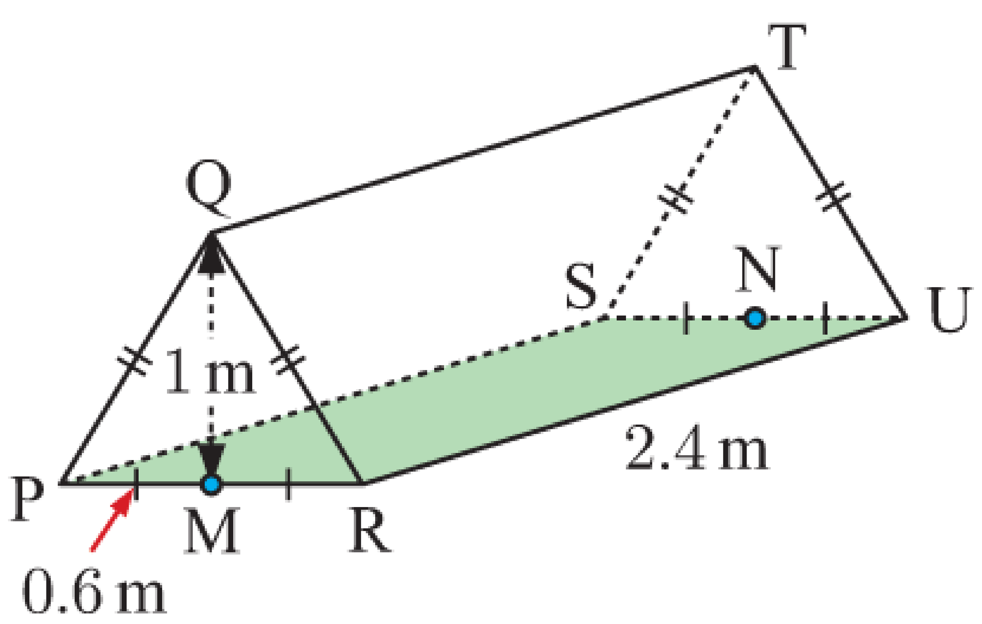
a **i** [CF]
ii [AG]
iii [BX]
iv [DX]



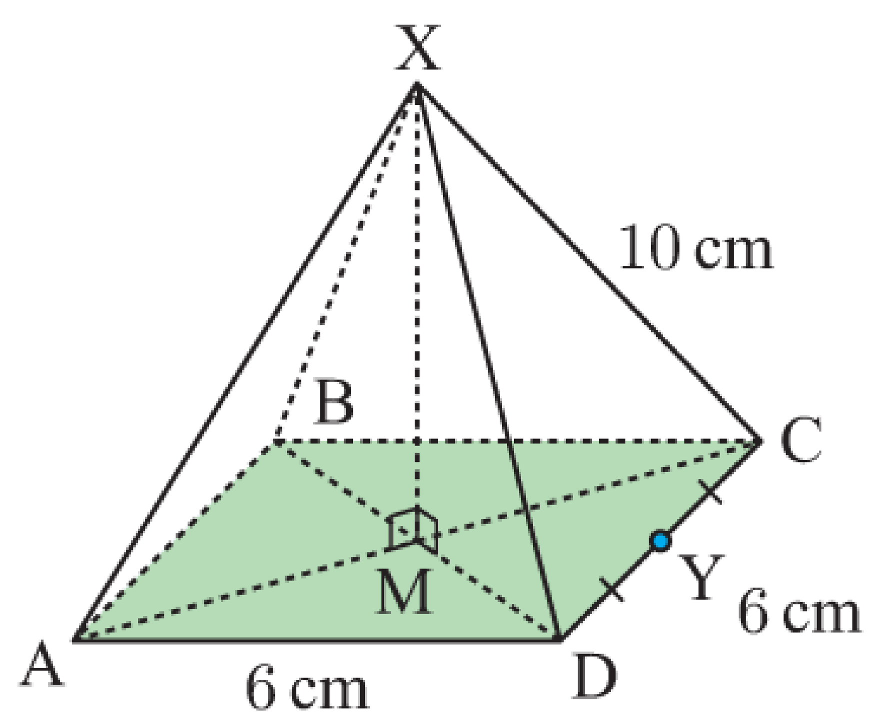
b **i** [PR]
ii [QU]
iii [PU]
iv [QM]



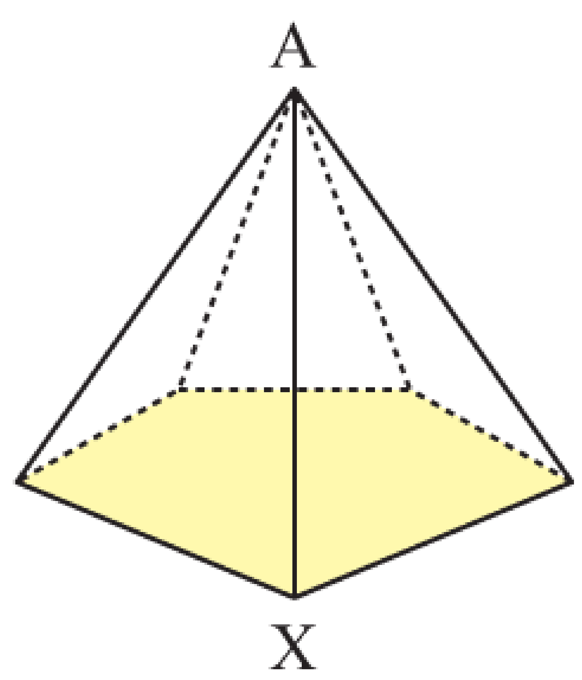
c **i** [QR]
ii [QU]
iii [QN]



d **i** [AX]
ii [XY]



4 The base of this pyramid is a regular pentagon. All sides of the pyramid have the same length.
Find the angle between [AX] and the base plane.



RESEARCH — SUNDIALS

1 Who invented sundials?

2 The gnomon of a sundial is the tilted arm that extends above the dial.

a At what time of day will the shadow of the gnomon be its mathematical projection on the dial?

b Why does the longitude of a sundial matter?

c How does the shadow help us tell the time at other times of the day?

3 Visit www.shadowspro.com to investigate sundials further. What other types of sundials are there? How do they differ?

4 In the 2018 ITV documentary *The Queen's Green Planet*, host Sir David Attenborough commented to Queen Elizabeth II of England that her sundial in the Buckingham Palace garden had been "neatly planted in the shade". The Queen asked her head gardener "Hadn't we thought of that? It wasn't in the shade originally, I'm sure. Maybe we could move it." But it was Sir David who responded "Depends if you want to tell the time or not!"

Jokes aside, the conversation highlights a very real problem that people faced for centuries: When the sky is overcast, one could neither use a sundial to tell the time, nor the stars to navigate.

What other inventions were made to help with timekeeping and navigation?

RESEARCH — ASTROLABES

The **astrolabe** was invented around 200 BC. The Greek astonomer Hipparchus is often credited with its invention.

An astrolabe is an astronomical model of the celestial sphere. It was used primarily to take astronomical measurements such as the altitudes of astronomical bodies, but philosophers, astrologers, and sailors found many other uses for it.

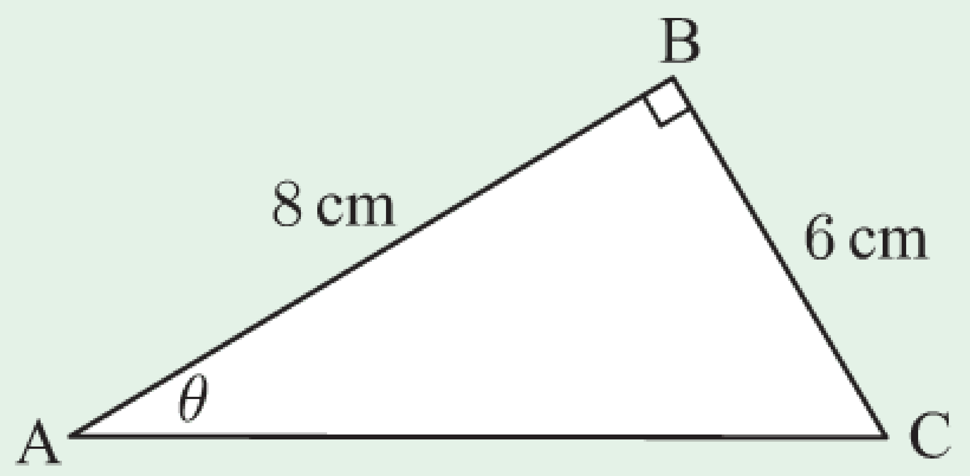
The astrolabe provided accurate measurements of the entire sky, such as the position of the sun, moon, and other heavenly bodies, and accurate times for sunrises, sunsets, and phases of the moon.

On land, we may be concerned about where we need to go, but at sea we also need to know where we are now. The astrolabe could be used to determine latitude and longitude, and measure altitude.

1. How was an astrolabe made?
2. How exactly does an astrolabe work?

REVIEW SET 7A

1



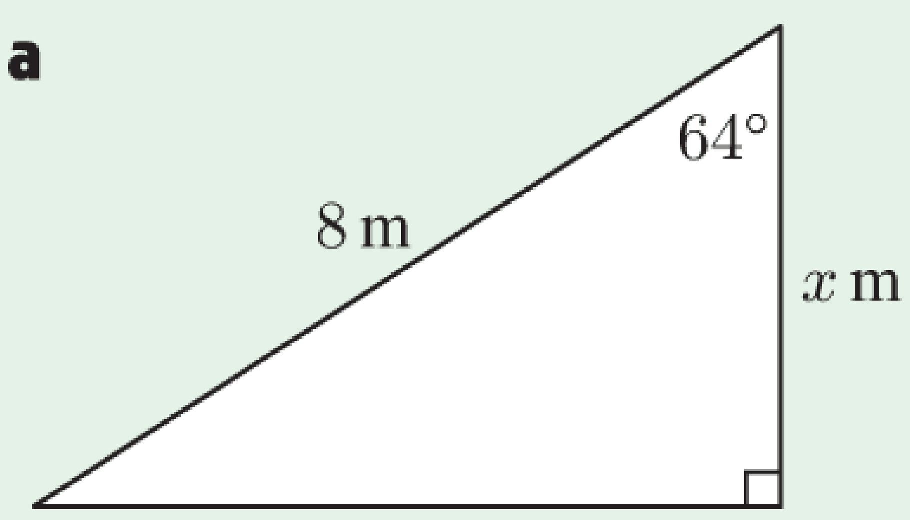
For the triangle alongside, find:

a the length of the hypotenuse

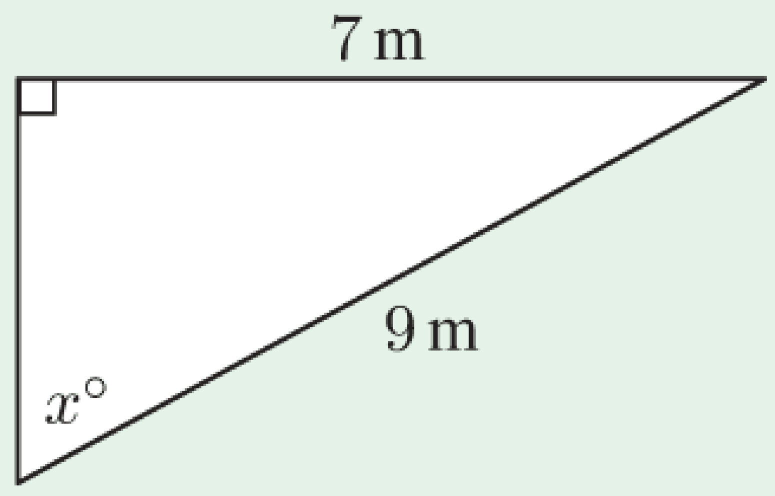
b $\sin\theta$ **c** $\cos\theta$ **d** $\tan\theta$.

2 Find x:

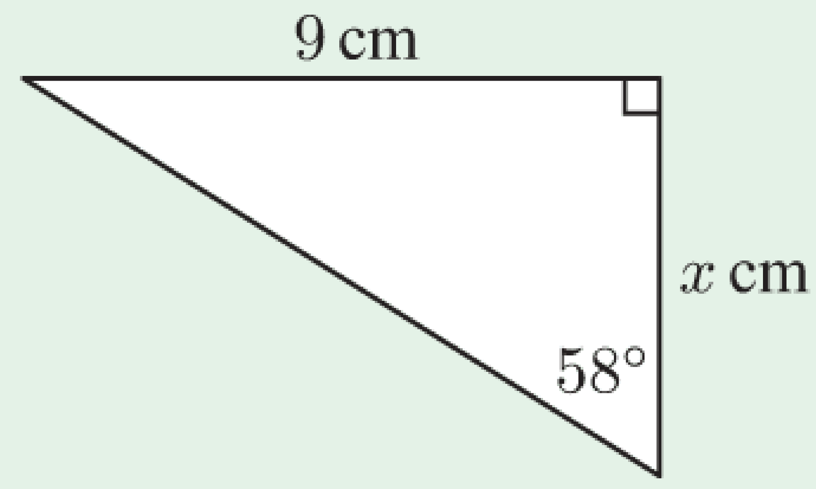
a



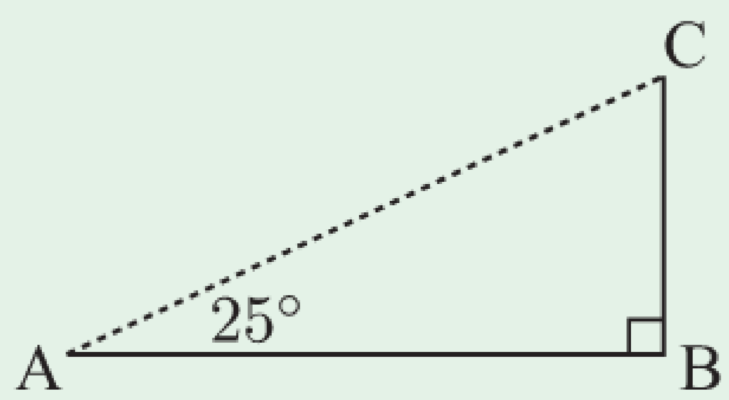
b



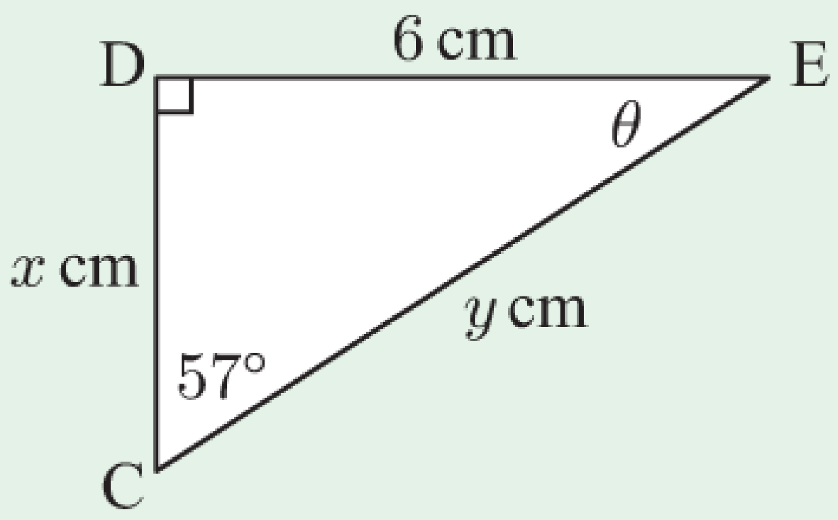
c



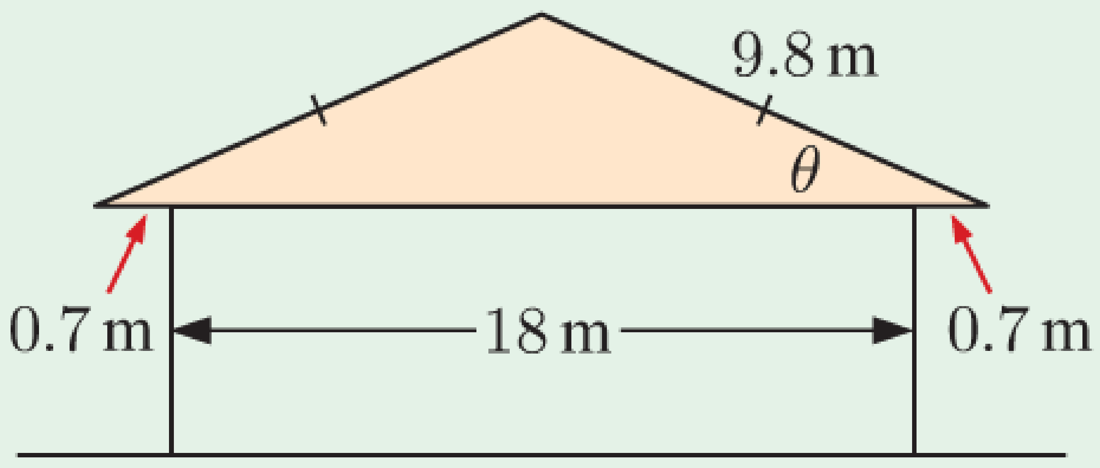
3 A 20 cm piece of wire was bent at B as shown alongside.
Find the area of triangle ABC.



4 Find the measure of all unknown sides and angles in triangle CDE:



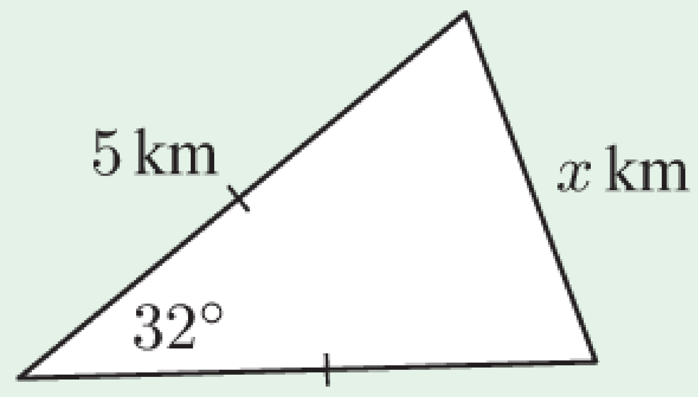
5 Find θ, the pitch of the roof.



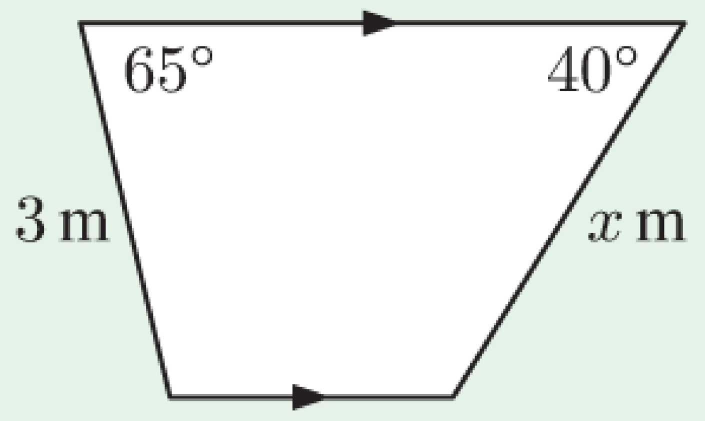
6 A rhombus has diagonals of length 15 cm and 8 cm. Find the larger angle of the rhombus.

7 Find, correct to 2 significant figures, the value of x:

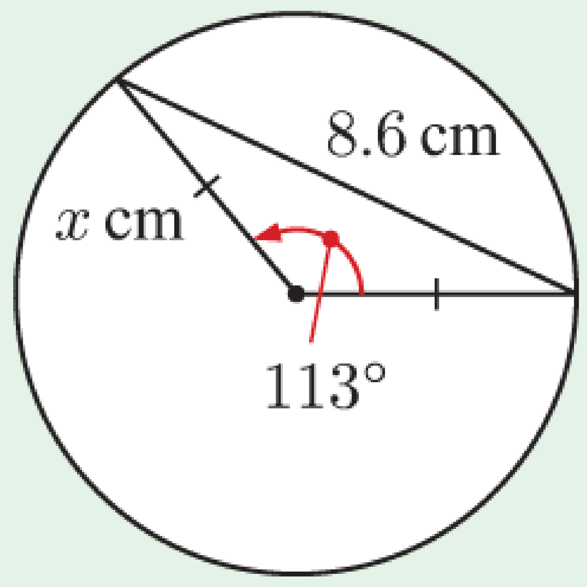
a



b



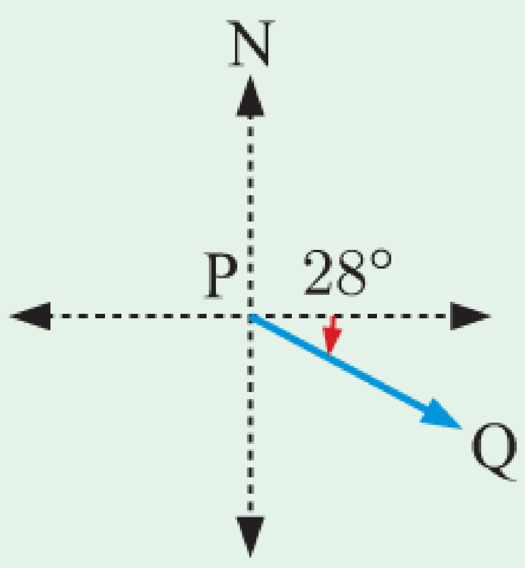
c



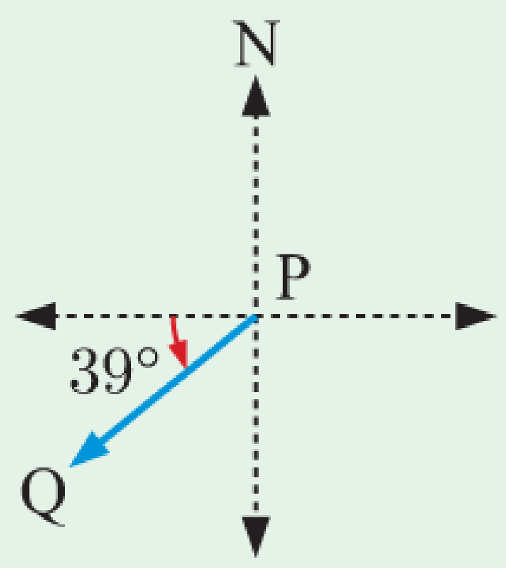
8 From a point 20 m horizontally from the base of a cylindrical lighthouse, the angle of elevation to the top of the lighthouse is 34°. Find the height of the lighthouse.

9 Find the bearing of Q from P in each diagram:

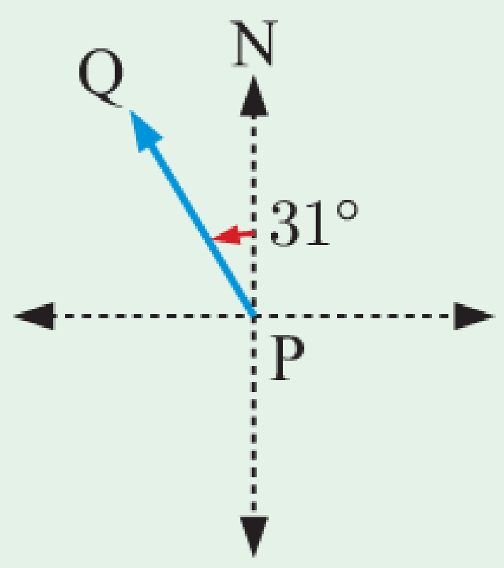
a



b

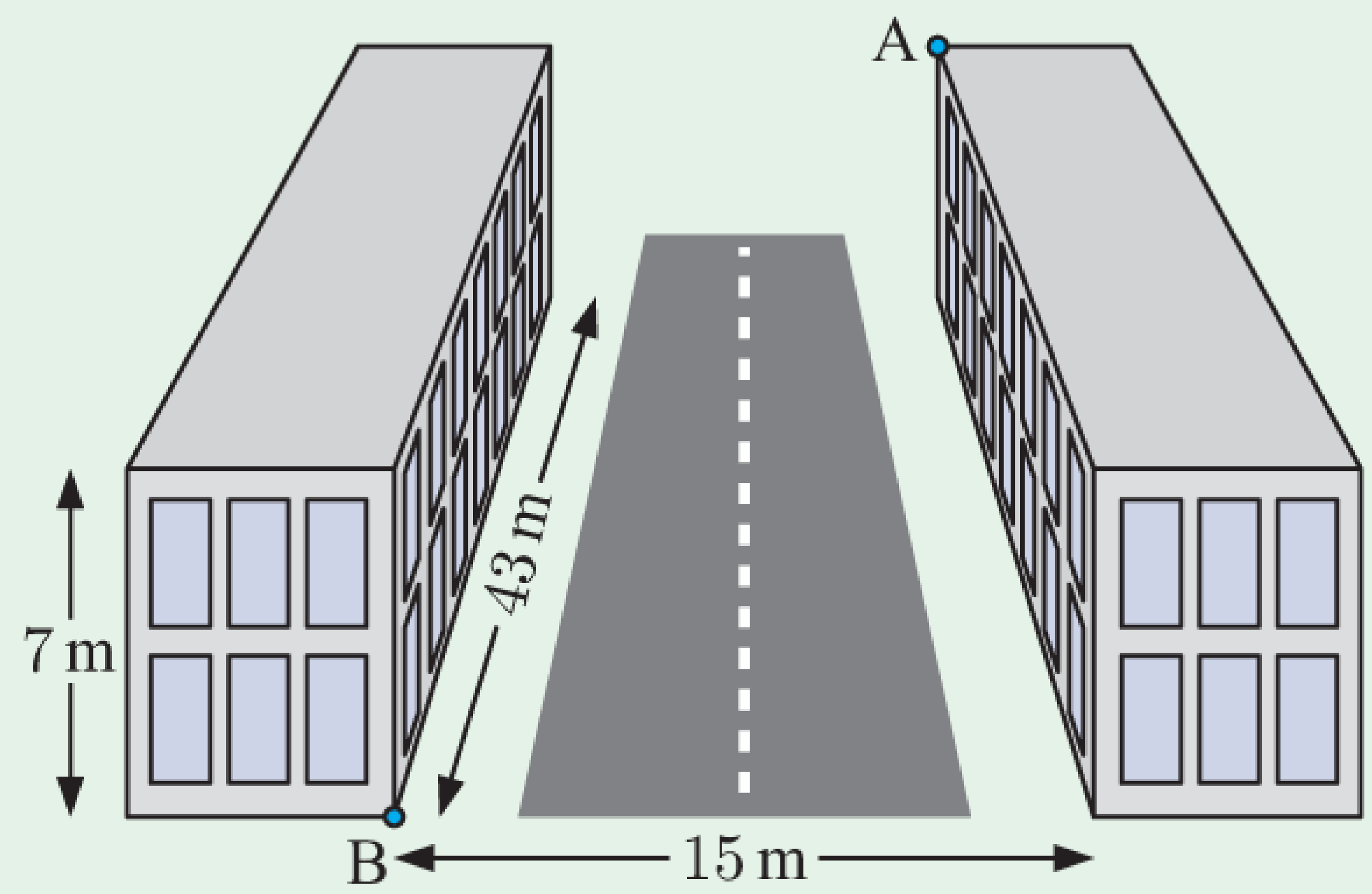


c

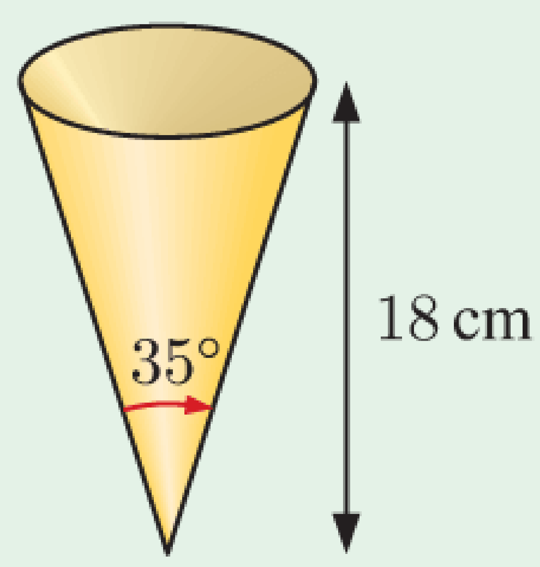


10 After a short flight, a helicopter is 12 km south and 5 km west of its helipad. Find the helicopter's distance and bearing from the helipad.

11 Two identical buildings stand parallel on opposite sides of a road. Find the angle of depression from A to B.

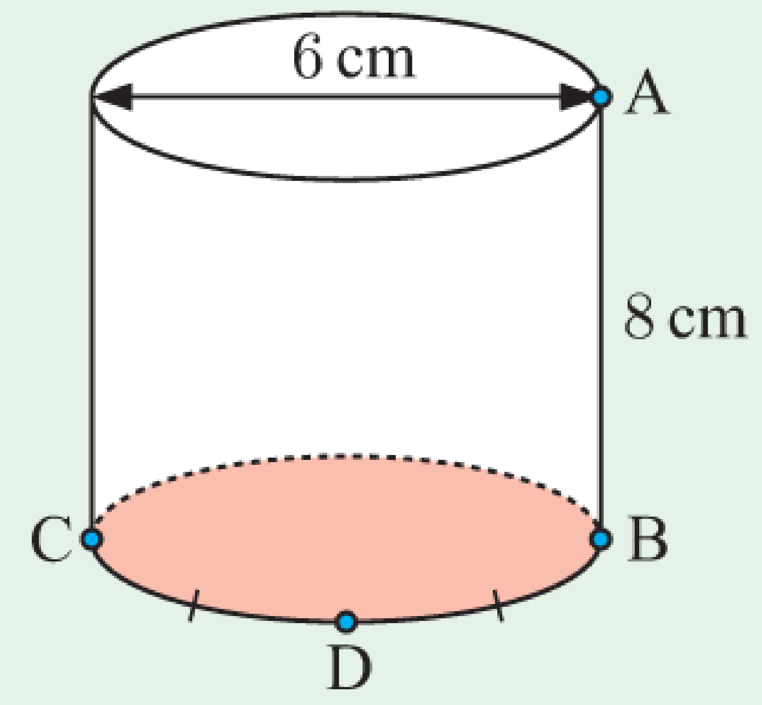


12 An open cone has a vertical angle measuring 35° and a height of 18 cm. Find the capacity of the cone in litres.



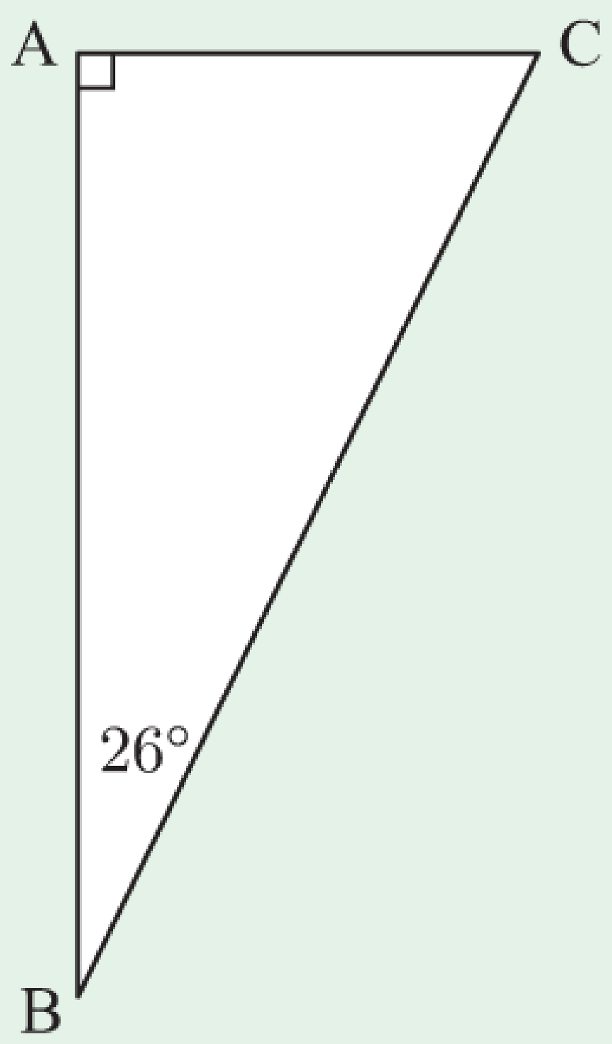
13 Find the angle between the following line segments and the base plane of the figure:

a [AC] **b** [AD]



REVIEW SET 7B

1



The right angled triangle alongside contains an angle of 26°.

a Use a ruler to measure the length of each side, to the nearest millimetre.

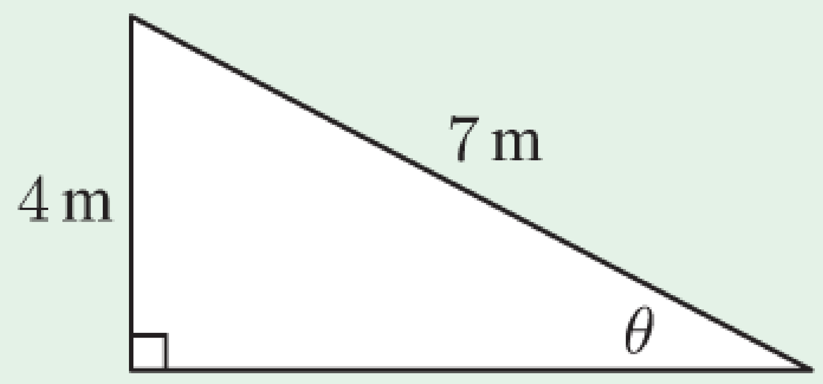
b Hence estimate the value, to 2 decimal places, of:

i $\sin 26°$ **ii** $\cos 26°$ **iii** $\tan 26°$.

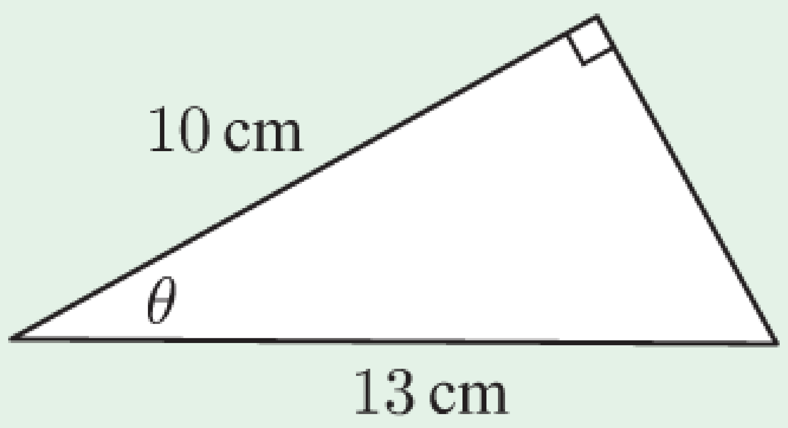
c Check your answers using a calculator.

2 Find the angle marked θ:

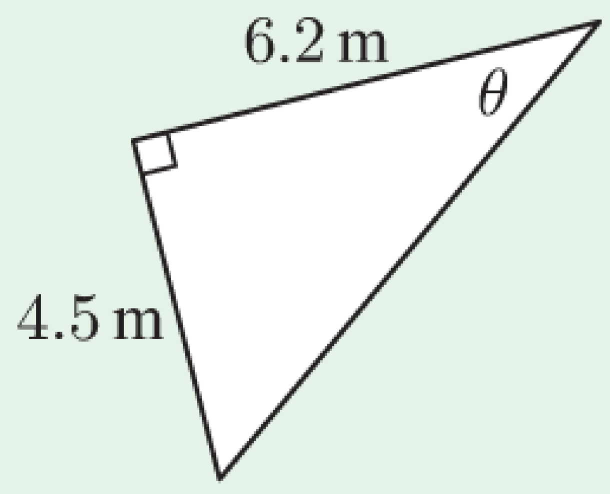
a



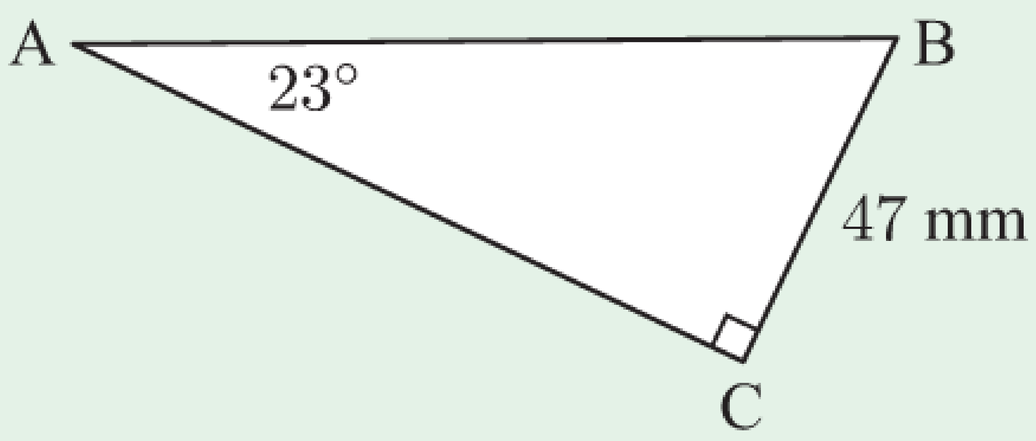
b



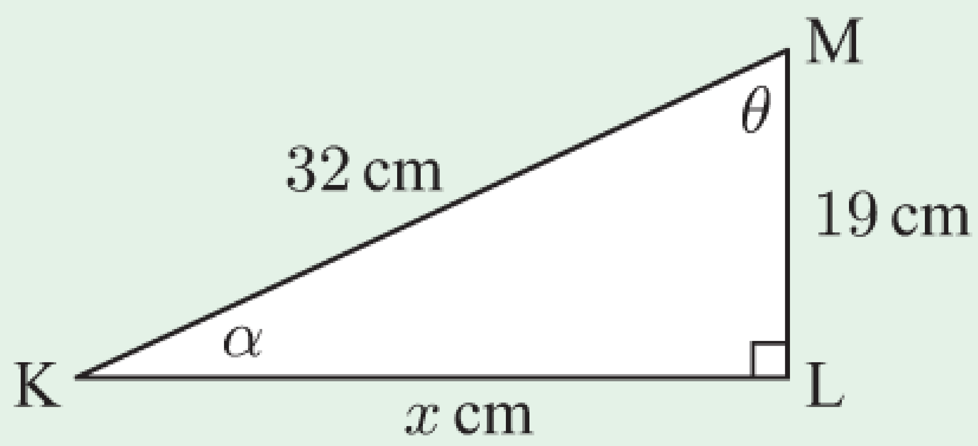
c



3 Find the lengths of the unknown sides:



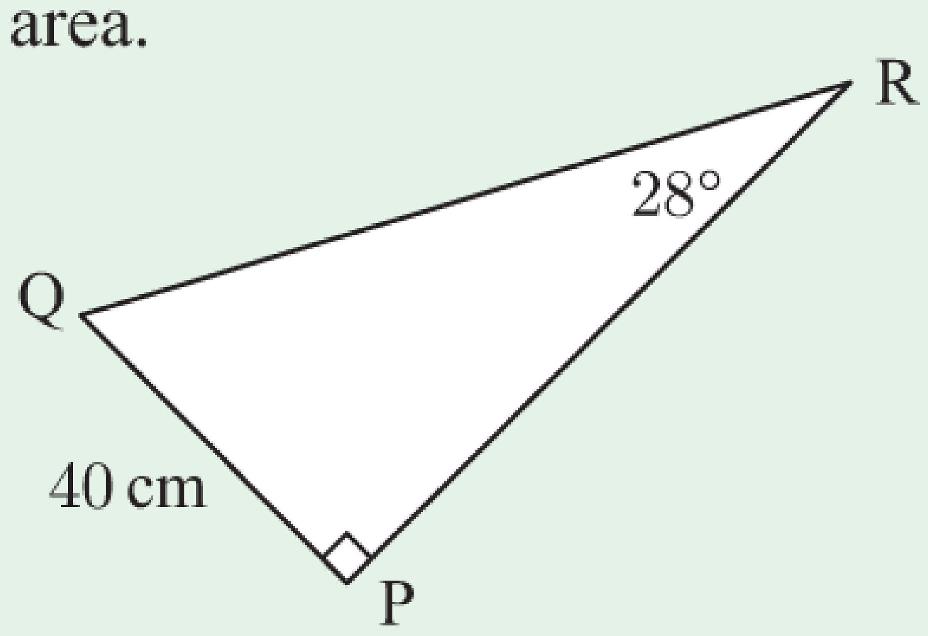
4 Find the measure of all unknown sides and angles in triangle KLM:



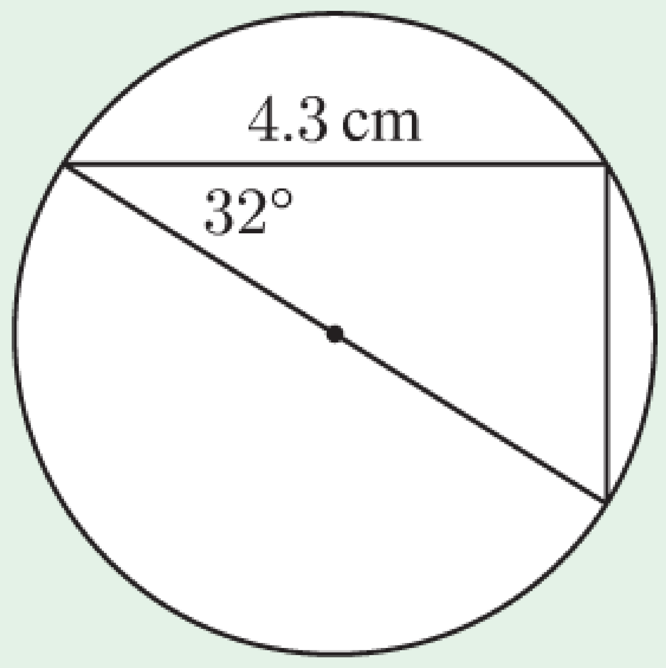
5 For the triangle PQR shown, find the:

a perimeter

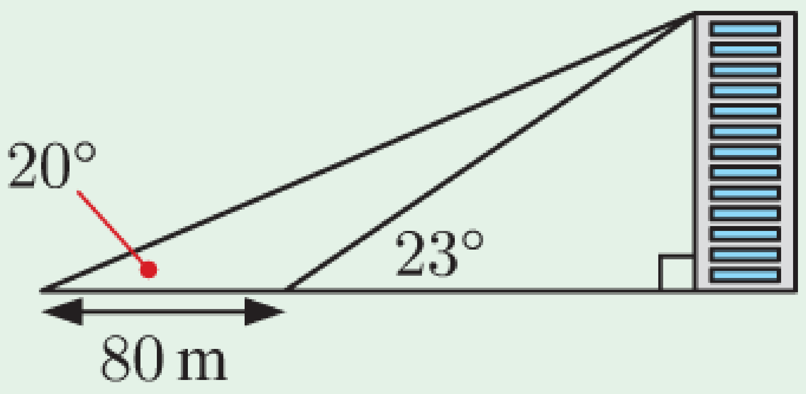
b area.



6 Find the radius of the circle:

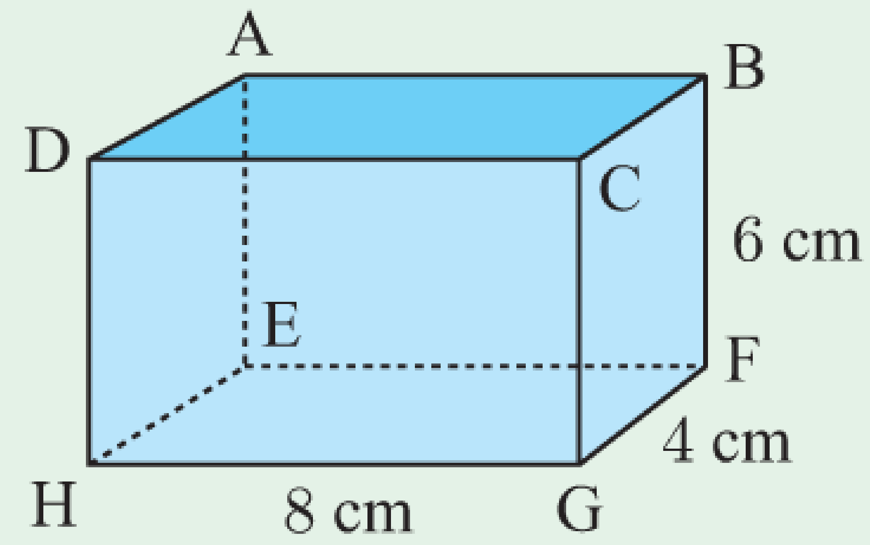


7 From a given point, the angle of elevation to the top of a tall building is $20°$. After walking 80 m towards the building, the angle of elevation is now $23°$. How tall is the building?



8 For the rectangular prism shown, find:

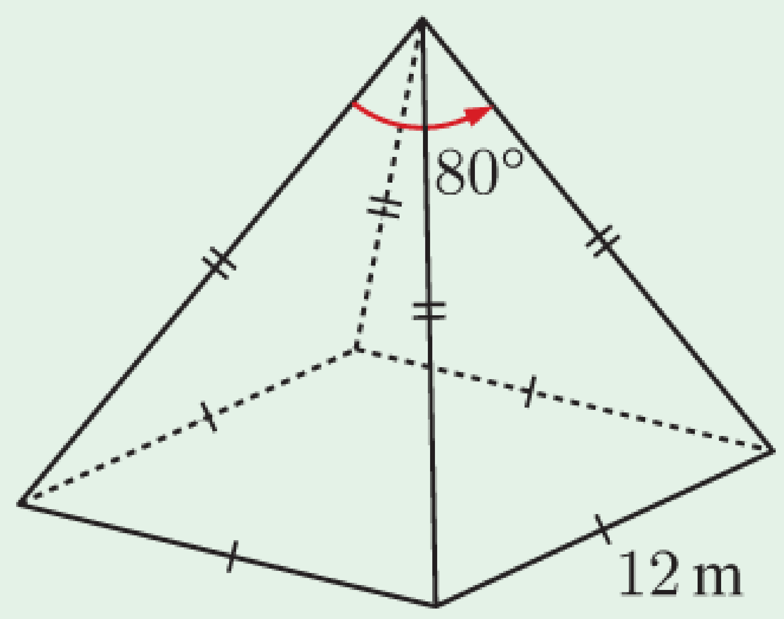
a $\widehat{AHG}$ **b** $\widehat{DFH}$.



9 Aaron ran 3 km on the bearing $213°$, then 2.5 km on the bearing $303°$. Find Aaron's distance and bearing from his starting point.

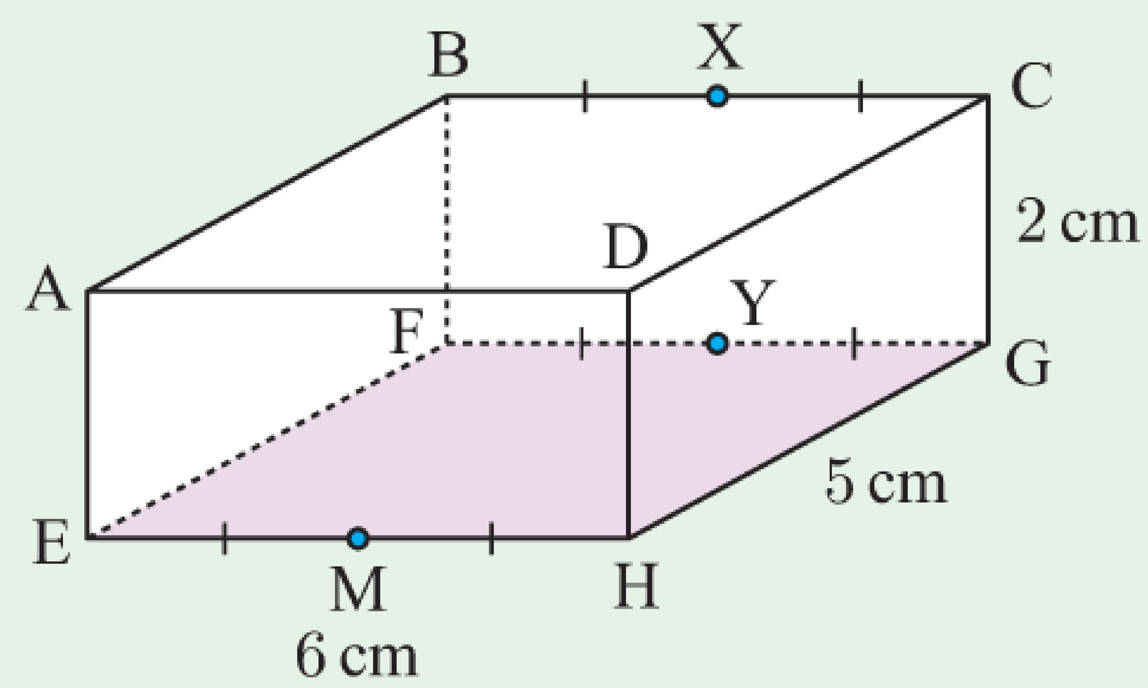
10 Joggers Amelia and Kristos depart from the same position at the same time. Amelia jogs in the direction $074°$, and Kristos jogs in the direction $164°$. After 30 minutes, Amelia has jogged 2 km further than Kristos, and her bearing from Kristos is $044°$.
How far apart are the joggers at this time?

11 Find the volume of this pyramid:



12 Find the angle between the following line segments and the base plane of the given figure:

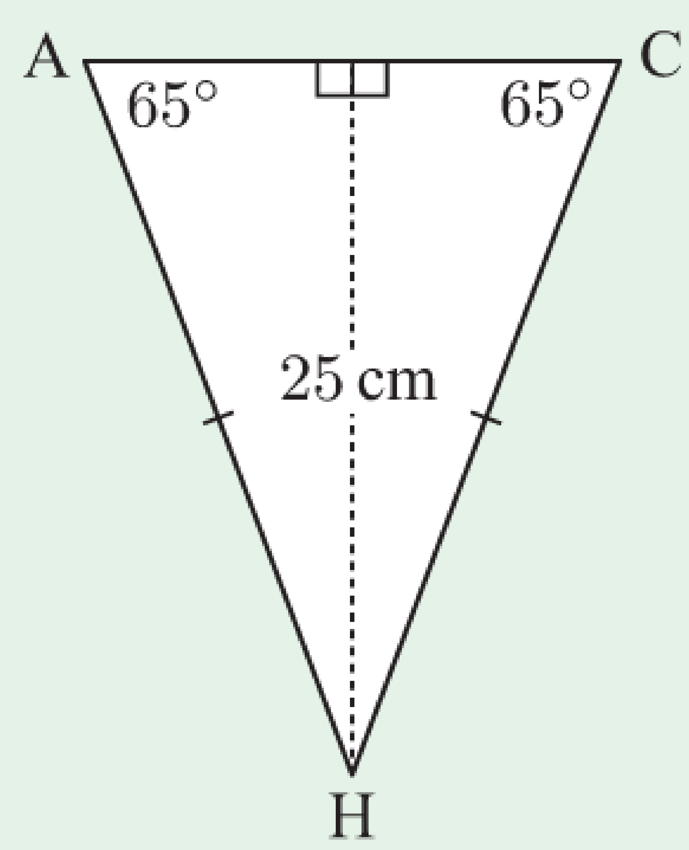
a [BH] **b** [CM] **c** [XM]

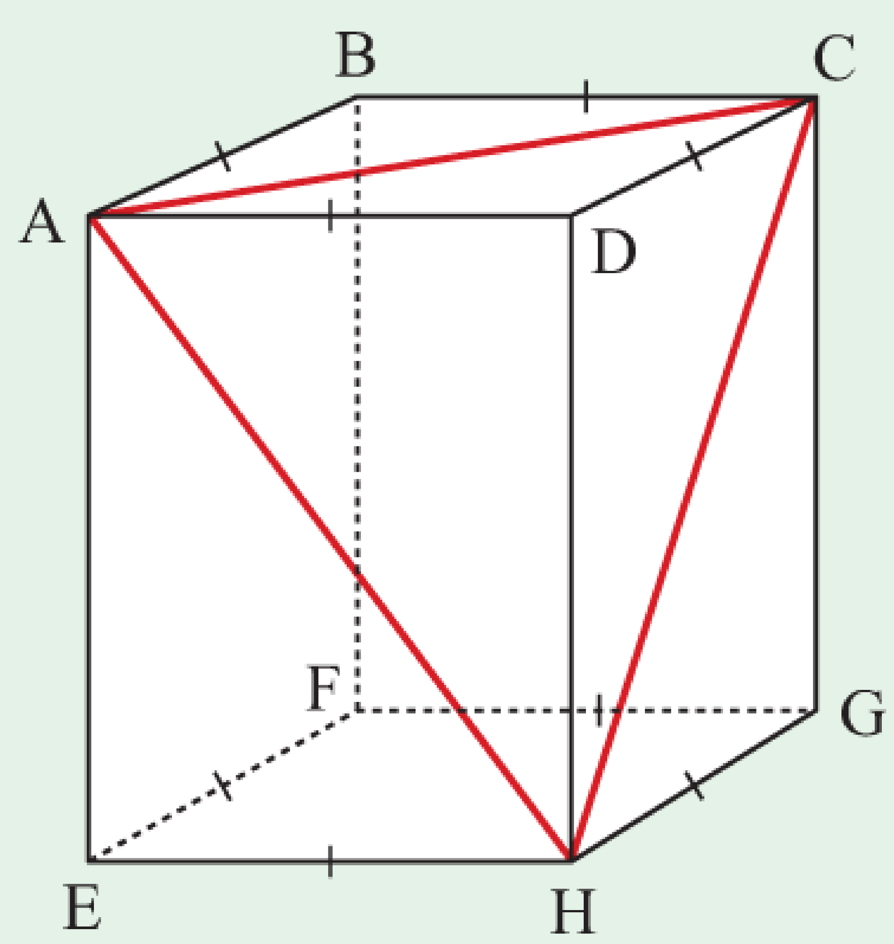


13 Triangle ACH is isosceles with altitude 25 cm and base angles $\widehat{HAC} = \widehat{HCA} = 65°$.

a Find the length:

i AH **ii** AC.





b Triangle ACH lies within the square-based rectangular prism shown. Find the volume of this prism.

8 **a** ≈ 252 mL **b** **i** ≈ 189 mL **ii** 3.25 cm

9 35 truck loads

10 **a** $\approx 110\,000$ mm^3

b The external surface area and internal surface area of a container may be different.

c **i** $1\,870\,000$ mm^3 **ii** 1.87 L **iii** $\approx 502\,000$ mm^3

REVIEW SET 6A

1 **a** ≈ 18.3 cm **b** ≈ 38.3 cm **c** ≈ 91.6 cm^2

2 ≈ 10.4 cm

3 **a** ≈ 377.0 cm^2 **b** ≈ 339.8 cm^2 **c** ≈ 201.1 cm^2

4 **a** 71 m^2 **b** \$239.25

5 **a** ≈ 4.99 m^3 **b** 853 cm^3 **c** ≈ 0.452 m^3

6 ≈ 3.22 m^3 **7** $\approx 82\,400$ cm^3 **8** ≈ 1470 m^3

9 **a** 734.44 mL **b** ≈ 198 L **10** ≈ 68.4 mm

11 **a** height $= 3.3$ m $- 1.8$ m $- 0.8$ m $= 0.7$ m $= 70$ cm

b ≈ 1.06 m **c** ≈ 15.7 m^2

d **Hint:** Volume of silo
= volume of hemisphere + volume of cylinder
+ volume of cone

e ≈ 5.2 kL

REVIEW SET 6B

1 **a** $\theta° \approx 76.6°$ **b** ≈ 14.3 cm^2

2 **a** ≈ 29.1 cm **b** ≈ 25.1 cm^2

3 **a** ≈ 84.7 cm^2 **b** ≈ 7110 mm^2 **c** ≈ 8.99 m^2

4 ≈ 23.5 m^2 **5** ≈ 434 cm^2

6 **a** ≈ 164 cm^3 **b** 120 m^3 **c** $\approx 10\,300$ mm^3

7 **a** 0.52 m^3 **b** 5.08 m^2 **8** ≈ 5680 L **9** ≈ 1.03 m

10 **a** $\approx 6.08 \times 10^{18}$ m^2 **b** $\approx 1.41 \times 10^{27}$ m^3

11 **a** ≈ 56.5 cm^3 **b** 3 cm **c** ≈ 96.5 cm^2

EXERCISE 7A

1 **a** **i** $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$

b **i** $\frac{5}{8}$ **ii** $\frac{\sqrt{39}}{8}$ **iii** $\frac{5}{\sqrt{39}}$

c **i** $\frac{7}{\sqrt{65}}$ **ii** $\frac{4}{\sqrt{65}}$ **iii** $\frac{7}{4}$

d **i** $\frac{5}{\sqrt{61}}$ **ii** $\frac{6}{\sqrt{61}}$ **iii** $\frac{5}{6}$

2 **a** XY ≈ 4.9 cm, XZ ≈ 3.3 cm, YZ ≈ 5.9 cm

b **i** ≈ 0.83 **ii** ≈ 0.56 **iii** ≈ 1.48

3 **a** **Hint:** Base angles of an isosceles triangle are equal, and sum of all angles in a triangle is $180°$.

b AB $= \sqrt{2} \approx 1.41$ m

c **i** $\frac{1}{\sqrt{2}} \approx 0.707$ **ii** $\frac{1}{\sqrt{2}} \approx 0.707$ **iii** 1

4 The OPP and ADJ sides will always be smaller than the HYP. So, the sine and cosine ratios will always be less than or equal to 1.

5 **a** **i** $\frac{a}{c}$ **ii** $\frac{b}{c}$ **iii** $\frac{a}{b}$ **iv** $\frac{b}{c}$ **v** $\frac{a}{c}$ **vi** $\frac{b}{a}$

b $A = 90° - B$

c **i** $\sin\theta = \cos(90° - \theta)$ **ii** $\cos\theta = \sin(90° - \theta)$

iii $\tan\theta = \dfrac{1}{\tan(90° - \theta)}$

6 **a** ≈ 7.50 m **b** ≈ 7.82 cm **c** ≈ 4.82 cm

d ≈ 5.17 m **e** ≈ 6.38 m **f** ≈ 4.82 cm

7 **a** $x \approx 3.98$ **b** **i** $y \approx 4.98$ **ii** $y \approx 4.98$

8 **a** $x \approx 2.87$, $y \approx 4.10$ **b** $x \approx 16.40$, $y \approx 18.25$

c $x \approx 10.77$, $y \approx 14.50$

9 **a** perimeter ≈ 23.2 cm, area ≈ 22.9 cm^2

b perimeter ≈ 17.0 cm, area ≈ 10.9 cm^2

10 ≈ 21.7 cm

EXERCISE 7B

1 **a** $\theta \approx 53.1°$ **b** $\theta \approx 45.6°$ **c** $\theta \approx 13.7°$

d $\theta \approx 52.4°$ **e** $\theta \approx 76.1°$ **f** $\theta \approx 36.0°$

2 **a** $\theta \approx 56.3°$ **b** **i** $\phi \approx 33.7°$ **ii** $\phi \approx 33.7°$

3 **a** $\theta \approx 39.7°$, $\phi \approx 50.3°$ **b** $\alpha \approx 38.9°$, $\beta \approx 51.1°$

c $\theta \approx 61.5°$, $\phi \approx 28.5°$

4 **a** The triangle cannot be drawn with the given dimensions.

b The triangle cannot be drawn with the given dimensions.

c The result is not a triangle, but a straight line of length 9.3 m.

5 **a** $x \approx 2.65$, $\theta \approx 37.1°$

b $x \approx 6.16$, $\theta \approx 50.3°$, $y \approx 13.0$

6 $\approx 135°$ **7** $\alpha \approx 6.92$

EXERCISE 7C

1 **a** $x \approx 4.13$ **b** $\alpha \approx 75.5°$ **c** $\beta \approx 41.0°$

d $x \approx 6.29$ **e** $\theta \approx 51.9°$ **f** $x \approx 12.6$

2 $\approx 22.4°$ **3** ≈ 11.8 cm

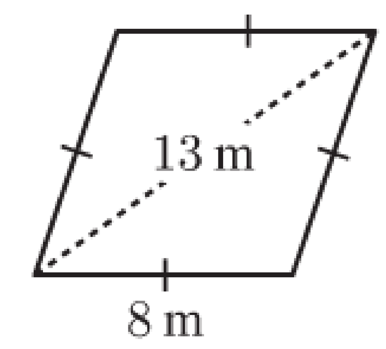
4 **a** ≈ 27.2 cm^2 **b** ≈ 153 m^2 **5** $\approx 119°$

6 ≈ 36.5 cm **7** **a** $x \approx 45.4$ **b** $x \approx 2.24$

8 **a** $x \approx 3.44$ **b** $\alpha \approx 51.5°$

9 **a** ≈ 12.3 cm^2 **b** ≈ 14.3 cm^2

10 **a**



b ≈ 9.33 m

c $\approx 71.3°$

11 **a** ≈ 2.59 cm **b** ≈ 8.46 cm

12 **a** $\theta \approx 36.9°$ **b** $r \approx 11.3$ **c** $\alpha \approx 61.9°$

13 ≈ 7.99 cm **14** $\approx 89.2°$ **15** $\approx 47.2°$ **16** ≈ 6.78 cm^2

EXERCISE 7D

1 ≈ 18.3 m **2** **a** ≈ 46.4 m **b** ≈ 259 m

3 $\approx 1.58°$ **4** **a** $\approx 26.4°$ **b** $\approx 26.4°$

5 ≈ 142 m **6** $\theta \approx 12.6°$ **7** ≈ 9.56 m

8 ≈ 46.7 m **9** $\beta \approx 129°$ **10** ≈ 10.9 m

11 ≈ 104 m **12** ≈ 962 m **13** ≈ 3.17 km

14 ≈ 43.8 m **15** **a** ≈ 18.4 cm **b** $\approx 35.3°$

16 **a** ≈ 10.8 cm **b** $\approx 36.5°$ **c** ≈ 9.49 cm **d** $\approx 40.1°$

17 **a** ≈ 82.4 cm **b** ≈ 77.7 L

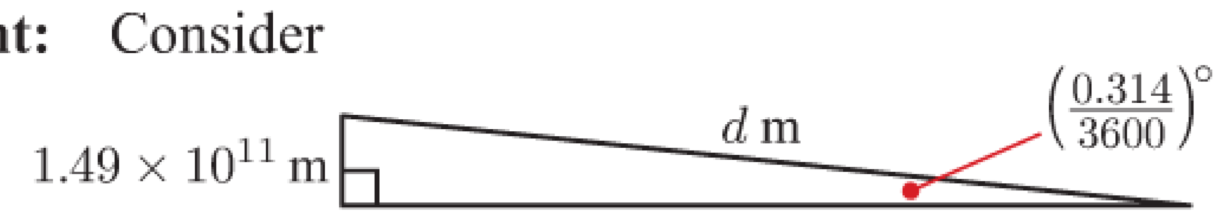
18 **a** **i** 2 m **ii** ≈ 2.01 m **b** $\approx 6.84°$

19 **a** ≈ 10.2 m **b** no **20** **a** ≈ 73.4 m **b** $\approx 16.2°$

21 $\approx 67.0°$

22 **a** ≈ 1.49 m^3 **b** ≈ 0.331 m^3 **c** ≈ 88.9 cm^3

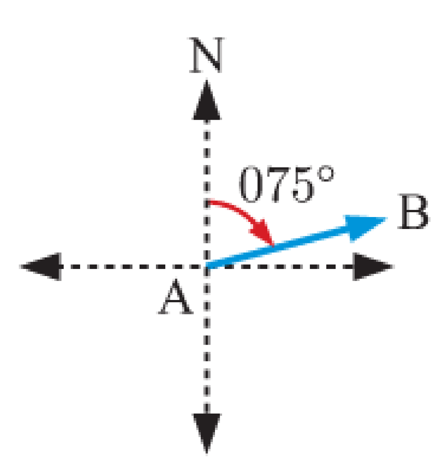
23 **a** **Hint:** Consider



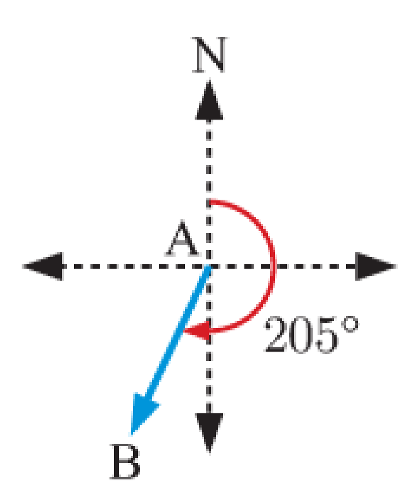
b ≈ 0.285 arc seconds

EXERCISE 7E

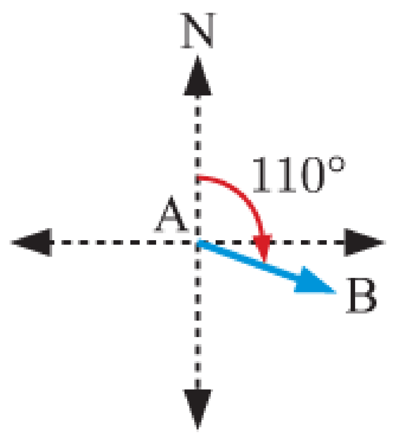
1 **a**



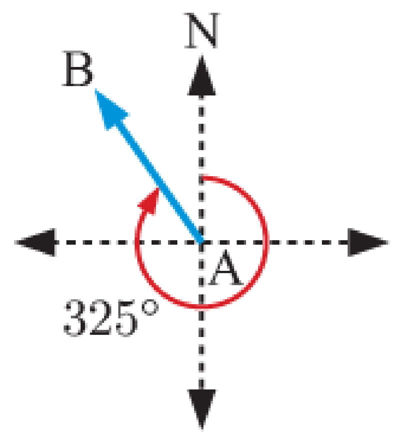
b



c



d

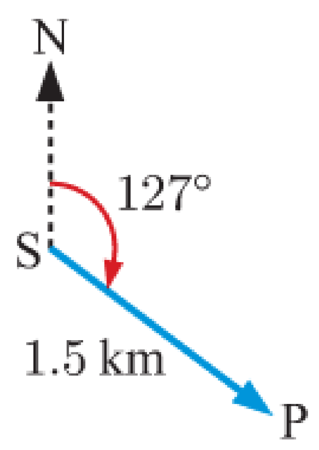


2 **a** 126° **b** 245° **c** 152° **d** 308°

3 **a** 072° **b** 252° **c** 162° **d** 342°
e 113° **f** 293°

4 $\approx 125°$ **5** **a** ≈ 224 m **b** $\approx 333°$ **c** $\approx 153°$

6 **a**



b ≈ 1.20 km **c** ≈ 0.903 km

7 ≈ 2.41 km **8** ≈ 12.6 km

9 **a** ≈ 854 m **b** $\approx 203°$

10 ≈ 73.3 km on the bearing $\approx 191°$

11 ≈ 17.8 km on the bearing $\approx 162°$

12 **a** $\approx 046.6°$ **b** ≈ 4.22 km

EXERCISE 7F

1 **a** **i** [EH] **ii** [EF] **iii** [EG] **iv** [FH]
b **i** [MR] **ii** [MN]

2 **a** **i** $\widehat{\text{AFE}}$ **ii** $\widehat{\text{BMF}}$ **iii** $\widehat{\text{ADE}}$ **iv** $\widehat{\text{BNF}}$
b **i** $\widehat{\text{BAM}}$ **ii** $\widehat{\text{BNM}}$ **iii** $\widehat{\text{EAN}}$

3 **a** **i** $\approx 36.9°$ **ii** $\approx 25.1°$ **iii** $\approx 56.3°$ **iv** $\approx 29.1°$
b **i** $\approx 33.7°$ **ii** $\approx 33.7°$ **iii** $\approx 25.2°$ **iv** $\approx 30.8°$
c **i** $\approx 59.0°$ **ii** $\approx 22.0°$ **iii** $\approx 22.6°$
d **i** $\approx 64.9°$ **ii** $\approx 71.7°$

4 $\approx 31.7°$

REVIEW SET 7A

1 **a** 10 cm **b** $\frac{6}{10} = \frac{3}{5}$ **c** $\frac{8}{10} = \frac{4}{5}$ **d** $\frac{6}{8} = \frac{3}{4}$

2 **a** $x \approx 3.51$ **b** $x \approx 51.1$ **c** $x \approx 5.62$

3 ≈ 43.4 cm^2 **4** $\theta = 33°$, $x \approx 3.90$, $y \approx 7.15$

5 $\theta \approx 8.19°$ **6** $\approx 124°$

7 **a** $x \approx 2.8$ **b** $x \approx 4.2$ **c** $x \approx 5.2$

8 ≈ 13.5 m **9** **a** 118° **b** 231° **c** 329°

10 13 km on the bearing $\approx 203°$ from the helipad.

11 $\approx 8.74°$ **12** ≈ 0.607 L **13** **a** $\approx 53.1°$ **b** $\approx 62.1°$

REVIEW SET 7B

1 **a** AB ≈ 4.5 cm, AC ≈ 2.2 cm, BC ≈ 5.0 cm
b **i** ≈ 0.44 **ii** ≈ 0.90 **iii** ≈ 0.49

2 **a** $\theta \approx 34.8°$ **b** $\theta \approx 39.7°$ **c** $\theta \approx 36.0°$

3 AB ≈ 120 mm, AC ≈ 111 mm

4 $x \approx 25.7$, $\theta \approx 53.6°$, $\alpha \approx 36.4°$

5 **a** ≈ 200 cm **b** ≈ 1500 cm^2 **6** ≈ 2.54 cm

7 ≈ 204 m **8** **a** 90° **b** $\approx 33.9°$

9 ≈ 3.91 km on the bearing $\approx 253°$ from his starting point.

10 ≈ 5.46 km **11** ≈ 485 m^3

12 **a** $\approx 14.4°$ **b** $\approx 18.9°$ **c** $\approx 21.8°$

13 **a** **i** ≈ 27.6 cm **ii** ≈ 23.3 cm **b** ≈ 6010 cm^3

EXERCISE 8A

1 **a** $\frac{\pi}{2}$ **b** $\frac{\pi}{3}$ **c** $\frac{\pi}{6}$ **d** $\frac{\pi}{10}$ **e** $\frac{\pi}{20}$
f $\frac{3\pi}{4}$ **g** $\frac{5\pi}{4}$ **h** $\frac{3\pi}{2}$ **i** 2π **j** 4π
k $\frac{7\pi}{4}$ **l** 3π **m** $\frac{\pi}{5}$ **n** $\frac{4\pi}{9}$ **o** $\frac{23\pi}{18}$

2 **a** $\approx 0.641^c$ **b** $\approx 2.39^c$ **c** $\approx 5.55^c$ **d** $\approx 3.83^c$
e $\approx 6.92^c$

3 **a** 36° **b** 108° **c** 135° **d** 10° **e** 20°
f 140° **g** 18° **h** 27° **i** 210° **j** 22.5°

4 **a** $\approx 114.59°$ **b** $\approx 87.66°$ **c** $\approx 49.68°$
d $\approx 182.14°$ **e** $\approx 301.78°$

5 **a**

Degrees	0	45	90	135	180	225	270	315	360
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Deg.	0	30	60	90	120	150	180	210	240	270	300	330	360
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 8B

1 **a** 7 cm **b** 12 cm **c** ≈ 13.0 m

2 **a** 6 cm^2 **b** 48 cm^2 **c** ≈ 8.21 cm^2

3 **a** arc length ≈ 49.5 cm, area ≈ 223 cm^2
b arc length ≈ 23.0 cm, area ≈ 56.8 cm^2

4 **a** $\approx 0.686^c$ **b** 0.6^c

5 **a** $\theta = 0.75^c$, area $= 24$ cm^2
b $\theta = 1.68^c$, area $= 21$ cm^2
c $\theta \approx 2.32^c$, area $= 126.8$ cm^2

6 **a** ≈ 3.15 m **b** ≈ 9.32 m^2

7 **a** ≈ 5.91 cm **b** ≈ 18.9 cm

8 **a** $\alpha \approx 0.3218^c$ **b** $\theta \approx 2.498^c$ **c** ≈ 387 m^2

9 **a** ≈ 11.7 cm **b** $r \approx 11.7$ **c** ≈ 37.7 cm **d** $\theta \approx 3.23^c$

10 ≈ 25.9 cm **11** **b** ≈ 2 h 24 min **12** ≈ 227 m^2

13 **a** $\alpha \approx 5.739$ **b** $\theta \approx 168.5$ **c** $\phi \approx 191.5$
d ≈ 71.62 cm

14 **a** 4 cm **b** **i** ≈ 2.16 cm^2 **ii** ≈ 29.3 cm^2

15 **a** **Hint:** Let the largest circle have radius r_1, and use a right angled triangle to show that $\sin\frac{\pi}{6} = \dfrac{r_1}{10 - r_1}$.
b $\frac{25\pi}{2}$ units^2 **c** $\frac{3}{4}$

EXERCISE 8C

1

θ (degrees)	0°	90°	180°	270°	360°	450°
θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
sine	0	1	0	-1	0	1
cosine	1	0	-1	0	1	0
tangent	0	undef.	0	undef.	0	undef.

2 **a** **i** A(cos 26°, sin 26°), B(cos 146°, sin 146°), C(cos 199°, sin 199°)
ii A(0.899, 0.438), B(-0.829, 0.559), C(-0.946, -0.326)
b **i** A(cos 123°, sin 123°), B(cos 251°, sin 251°), C(cos($-35°$), sin($-35°$))
ii A(-0.545, 0.839), B(-0.326, -0.946), C(0.819, -0.574)