Number of solutions to a quadratic equation

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$$ax^2 + bx + c = 0$$

with $a \neq 0$, will have

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- one solution (two equal solutions) if $\Delta = 0$,
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We must have that $a \neq 0$, because if $a = 0$, then the equation is not a quadratic equation and it makes no sense to analyse Δ.

Intodoctory problems

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b) $2x^2 = 3x + 1$
We first move all terms to one side. We get $2x^2 - 3x - 1 = 0$. We have $a = 2, b = -3$ and $c = -1$, so $\Delta = (-3)^2 - 4 \cdot 2 \cdot (-1) = 17$.

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$\Delta = 0$, so there will be **one solution**.

Further examples - example 1

Suppose now that we want to find the possible values of parameter k, for which the equation

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Because we want two distinct real solutions we must have $\Delta > 0$, so we solve:

$$9 + 8k > 0$$

And we get that $k > -\frac{9}{8}$. So for all values of k greater than $-\frac{9}{8}$, the above equation will have two distinct real solutions.

Further examples - example 2

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We have $a = 2, b = -m$ and $c = 6$, so $\Delta = (-m)^2 - 4 \cdot 2 \cdot 6 = m^2 - 48$.

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We have $a = 2, b = -m$ and $c = 6$, so $\Delta = (-m)^2 - 4 \cdot 2 \cdot 6 = m^2 - 48$.

This time we want exactly one real solutions, so we must have $\Delta = 0$, so we solve:

$$m^2 - 48 = 0$$

We get that $m = \pm\sqrt{48} = \pm 4\sqrt{3}$.

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We can factorize the left hand side:

$$(m + 4)(m - 4) > 0$$

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We can factorize the left hand side:

$$(m+4)(m-4) > 0$$

Now we can sketch the function and we can see that it is greater than 0 for $m < -4$ or $m > 4$.

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Find the possible values of p for which the equation:

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We want no real solutions, so we must have $\Delta < 0$, so we solve:

$$p^2 - 10p + 16 < 0$$

We can factorize the left hand side:

$$(p-2)(p-8) < 0$$

Now we can sketch the function and we can see that it is smaller than 0 for $2 < p < 8$.

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Find the possible values of q for which the equation:

$$x^2 - 4(q+1)x + 2q(q-1) = 0$$

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$$\Delta = (-4(q+1))^2 - 4 \cdot 1 \cdot 2q(q-1) = 16(q^2+2q+1) - 8q^2 + 4q = 8q^2 + 36q + 16$$

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We want two distinct real solutions, so we must have $\Delta > 0$, so we solve:

$$8q^2 + 36q + 16 > 0$$

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We want two distinct real solutions, so we must have $\Delta > 0$, so we solve:

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We first divide both sides by 4 and then factorize to get:

$$(2q+1)(q+4) > 0$$

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We want two distinct real solutions, so we must have $\Delta > 0$, so we solve:

$$8q^2 + 36q + 16 > 0$$

We first divide both sides by 4 and then factorize to get:

$$(2q+1)(q+4) > 0$$

Now we do the sketch and we get that we have two solutions to the original equation for $q < -4$ or $q > -\frac{1}{2}$.

If you have any questions you can contact me at t.j.lechowski@gmail.com.