[Maximum mark: 30]

This question asks you to investigate some properties of the graph of the function $f(x) = e^{-x} \sin x$.

Important: In this question zeroes and stationary points are arranged in ascending order with respect to their x-coordinate.

Let $f(x) = e^{-x} \sin x$, with $x \geqslant 0$.

(a) Write down the first three zeroes of the graph of $y = f(x)$. [2]

Let x_n be the n-th zero of $f(x)$.

(b) The sequence $\{x_n\}$ with $n \in \mathbb{Z}^+$ is an arithmetic sequence. State the value of the common difference of this sequence. [1]

(c) Find $f'(x)$. [2]

Let P_n be the n-th stationary point of the graph of $y = f(x)$.

(d) Find the exact value of the x-coordinate of P_1. [2]

(e) Find an expression for the x-coordinate of P_n. [2]

(f) Show that $\sin(\theta + \pi) \equiv -\sin\theta$ [2]

(g) Show that y-coordinates of the sequence of points $\{P_n\}$ form a geometric sequence and find the exact value of the ratio of this sequence. [4]

(h) Sketch the graph of $y = f(x)$ for $0 \leqslant x \leqslant 2\pi$. [2]

Let $A_n = \displaystyle\int_{x_n}^{x_{n+1}} f(x)dx$.

(i) Find the exact value of A_1. [5]

(j) Show that the sequence $\{A_n\}$ with $n \in \mathbb{Z}^+$ is a geometric sequence with common ratio equal to $-e^{-\pi}$. [6]

(l) Hence or otherwise calculate $\displaystyle\int_0^\infty f(x)dx$ [2]