Trigonometric modelling

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If you understand graphs of trig functions and their transformations, this should all be very easy.

Exercise 17D question 1

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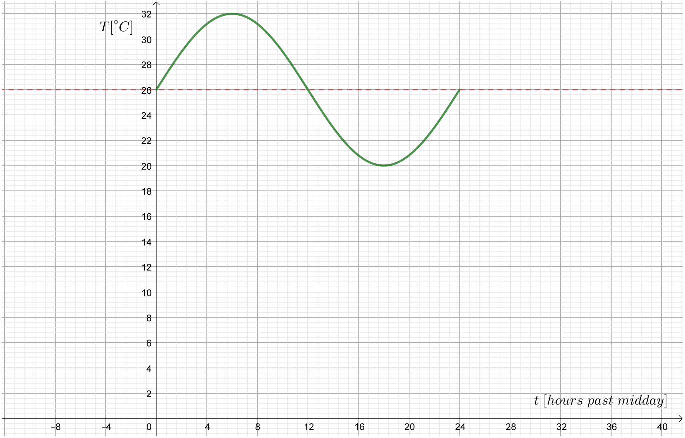
We want to sketch the graph for $0 \leq t \leq 24$, so for one full period.

Exercise 17D question 1 (a)

We get the following graph:



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i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$. We could've also found this temperature from the graph.

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i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$. We could've also found this temperature from the graph.

ii. at 2 pm.: $T(2) = 6\sin(\frac{\pi}{12} \times 2) + 26 = 6\sin\frac{\pi}{6} + 26 = 29^\circ C$.

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The maximum of $\sin x$ is 1, so the maximum of $6\sin x + 26$ is 32. The maximum for the first positive argument occurs when $x = \frac{\pi}{2}$. So the maximum of $6\sin(\frac{\pi}{12}t) + 26$ will occur when

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this gives $t = 6$. So the maximum temperature of $32^\circ C$ occurs at 6 pm.

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$$h(t) = 6\cos\left(\frac{\pi}{6} \times t\right)$$

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$$d(t) = 12\sin(2\pi t)$$

Now we move on to finding trigonometric models based on real-life data.

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Trigonometric model

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Sunrise and sunset by month (Warsaw)

Month	Sunrise	Sunset	Hours of daylight
Januar	07:36 am	03:54 pm	8:18 hours
Februar	06:50 am	04:49 pm	9:59 hours
März	05:47 am	05:42 pm	11:55 hours
April	05:35 am	07:36 pm	14:00 hours
Mai	04:38 am	08:26 pm	15:49 hours
Juni	04:11 am	09:01 pm	16:50 hours
Juli	04:31 am	08:52 pm	16:21 hours
August	05:18 am	08:02 pm	14:44 hours
September	06:09 am	06:52 pm	12:43 hours
Oktober	07:00 am	05:43 pm	10:43 hours
November	06:55 am	03:45 pm	8:50 hours
Dezember	07:37 am	03:25 pm	7:49 hours

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The maximum $D_{max} = 16.8(3)$, the minimum $D_{min} = 7.81(6)$. This gives the principle axis $D = 12.325$ and the amplitude of $4.508(3)$.

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The maximum $D_{max} = 16.8(3)$, the minimum $D_{min} = 7.81(6)$. This gives the principle axis $D = 12.325$ and the amplitude of 4.508(3). The period is of course 12, so we get $b = \frac{\pi}{6}$. Finally we need to figure out the horizontal shift. We will use the maximum value to establish it's value. The maximum of $\sin x$ occurs at $x = \frac{\pi}{2}$, in our case maximum occurs for $m = 6$ (June) so we solve:

$$\frac{\pi}{6}(6 - c) = \frac{\pi}{2}$$

which gives $c = 3$.

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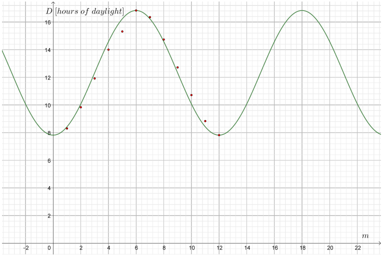

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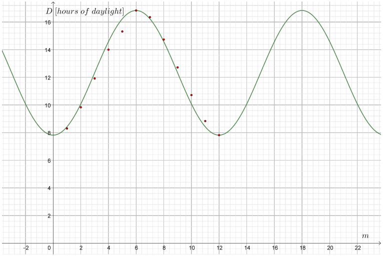

Looks like we could've done a slightly better job with horizontal shift by choosing a different point.

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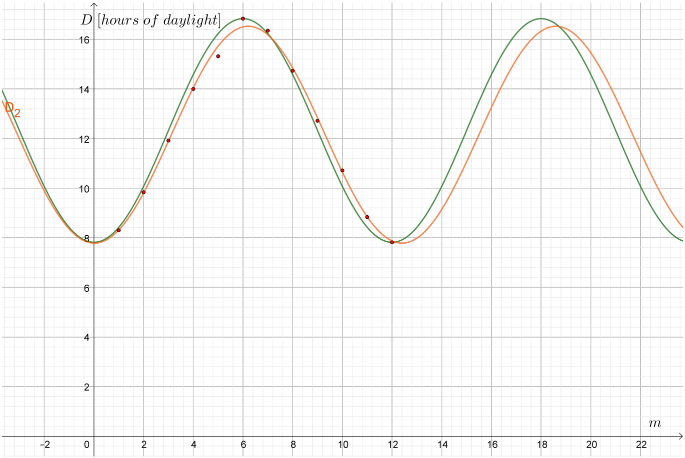
The function we got is:

$$D(m) = 4.3711\sin(0.5075m - 1.5814) + 12.1548$$

Graphs of both functions:



Admittedly the GDC has done a slightly better job.

In case of any questions you can email me at T.J.Lechowski@gmail.com.