1. Given that $\frac{z}{z+2} = 2 - \mathrm{i}$, $z \in \mathbb{C}$, find z in the form $a + \mathrm{i}b$.

(Total 4 marks)

2. Given that $(a + b\mathrm{i})^2 = 3 + 4\mathrm{i}$ obtain a pair of simultaneous equations involving a and b. Hence find the two square roots of $3 + 4\mathrm{i}$.

(Total 7 marks)

3. Solve the simultaneous equations

$$\mathrm{i}z_1 + 2z_2 = 3$$
$$z_1 + (1 - \mathrm{i})z_2 = 4$$

giving z_1 and z_2 in the form $x + \mathrm{i}y$, where x and y are real.

(Total 9 marks)

4. Consider the complex numbers $z = 1 + 2\mathrm{i}$ and $w = 2 + a\mathrm{i}$, where $a \in \mathbb{R}$.

Find a when

(a) $|w| = 2|z|$;

(3)

(b) $\mathrm{Re}\,(zw) = 2\,\mathrm{Im}(zw)$.

(3)

(Total 6 marks)

5. If z is a non-zero complex number, we define $L(z)$ by the equation

$$L(z) = \ln|z| + \mathrm{i}\arg(z),\ 0 \le \arg(z) < 2\pi.$$

(a) Show that when z is a positive real number, $L(z) = \ln z$.

(2)

(b) Use the equation to calculate

(i) $L(-1)$;

(ii) $L(1 - \mathrm{i})$;

(iii) $L(-1 + \quad \mathrm{i})$.

(5)

(c) Hence show that the property $L(z_1 z_2) = L(z_1) + L(z_2)$ does not hold for all values of z_1 and z_2.

(2)
(Total 9 marks)

6. Given that $|z| = \sqrt{10}$, solve the equation $5z + \dfrac{10}{z^*} = 6 - 18\mathrm{i}$, where z^* is the conjugate of z.

(Total 7 marks)