1. The polynomial $f(x) = x^3 + 3x^2 + ax + b$ leaves the same remainder when divided by $(x - 2)$ as when divided by $(x + 1)$. Find the value of a.

(Total 6 marks)

2. When $f(x) = x^4 + 3x^3 + px^2 - 2x + q$ is divided by $(x - 2)$ the remainder is 15, and $(x + 3)$ is a factor of $f(x)$.

Find the values of p and q.

(Total 6 marks)

3. The polynomial $P(x) = x^3 + ax^2 + bx + 2$ is divisible by $(x + 1)$ and by $(x - 2)$.

Find the value of a and of b, where $a, b \in \mathbb{R}$.

(Total 6 marks)

4. When the function $q(x) = x^3 + kx^2 - 7x + 3$ is divided by $(x + 1)$ the remainder is seven times the remainder that is found when the function is divided by $(x + 2)$.

Find the value of k.

(Total 5 marks)

5. Given that $Ax^3 + Bx^2 + x + 6$ is exactly divisible by $(x + 1)(x - 2)$, find the value of A and the value of B.

(Total 5 marks)

6. Given that 2 + i is a root of the equation $x^3 - 6x^2 + 13x - 10 = 0$ find the other two roots.

(Total 5 marks)

7. Consider the polynomial $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where $a, b, c, d \in \mathbb{R}$.

Given that 1 + i and 1 – 2i are zeros of $p(x)$, find the values of a, b, c and d.

(Total 7 marks)

8. Given that $z_1 = 2$ and $z_2 = 1 + i\sqrt{3}$ are roots of the cubic equation $z^3 + bz^2 + cz + d = 0$ where $b, c, d \in \mathbb{R}$,

(a) write down the third root, z_3, of the equation;

(1)

(b) find the values of b, c and d;

(4)

(Total 5 marks)

9. (a) Show that the complex number i is a root of the equation

$$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0.$$

(2)

(b) Find the other roots of this equation.

(4)

(Total 6 marks)

10. Consider the equation $z^3 + az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$. The points in the Argand diagram representing the three roots of the equation form the vertices of a triangle whose area is 9. Given that one root is –1 + 3i, find

(a) the other two roots;

(4)

(b) a, b and c.

(3)

(Total 7 marks)