1. *[7 marks]*

A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} \frac{\pi x}{36}\sin\left(\frac{\pi x}{6}\right), & 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}.$$

Find $P(0 \le X \le 3)$.

2a. *[2 marks]*

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \arccos x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The median of this distribution is m.

Determine the value of m.

2b. *[4 marks]*

Given that $P(|X - m| \le a) = 0.3$, determine the value of a.

3a. *[5 marks]*

A continuous random variable X has the probability density function f given by

$$f(x) = \begin{cases} \dfrac{x}{\sqrt{\{x^2 + k^3}} & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

where $k \in \mathbb{R}^+$.

Show that $\sqrt{16 + k} - \sqrt{k} = \sqrt{k}\sqrt{16 + k}$.

3b. *[2 marks]*

Find the value of k.

4a. *[2 marks]*

A continuous random variable X has the probability density function f_n given by

$$f_n(x) = \begin{cases} \{n + 1x^n, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

where $n \in \mathbb{R},\ n \geq 0$.

Show that $\mathrm{E}(X) = \frac{n+1}{n+2}$.

4b. *[4 marks]*

Show that $\mathrm{Var}(X) = \frac{n+1}{(n+2)^2(n+3)}$.

5a. *[1 mark]*

The random variable X has probability density function f given by

$$f(x) = \begin{cases} k(\pi - \arcsin x) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}, \text{ where } k \text{ is a positive constant.}$$

State the mode of X.

5b. *[3 marks]*

Find $\int \arcsin x \, \mathrm{d}x$.

5c. *[3 marks]*

Hence show that $k = \frac{2}{2+\pi}$.

5d. *[4 marks]*

Given that $y = \left(\frac{x^2}{2}\right) \arcsin x - \left(\frac{1}{4}\right) \arcsin x + \left(\frac{x}{4}\right) \sqrt{1 - x^2}$, show that

$\frac{\mathrm{d}y}{\mathrm{d}x} = x \arcsin x$.

5e. *[5 marks]*

$\mathrm{E}(X) = \frac{3\pi}{4(\pi+2)}$.

6a. *[3 marks]*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 3ax & , \quad 0 \le x < 0.5 \\ a(2-x) & , \quad 0.5 \le x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Show that $a = \frac{2}{3}$.

6b. *[3 marks]*

Find $\mathrm{P}(X < 1)$.

6c. *[7 marks]*

Given that $\mathrm{P}(s < X < 0.8) = 2 \times \mathrm{P}(2s < X < 0.8)$, and that $0.25 < s < 0.4$, find the value of s.

7a. *[4 marks]*

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin\left(\frac{\pi x}{6}\right), & 0 \le x \le 6 \\ 0, & \text{otherwise} \end{cases}.$$

Find the value of k.

7b. *[1 mark]*

By considering the graph of f write down the mean of X;

7c. *[1 mark]*

By considering the graph of f write down the median of X;

7d. *[1 mark]*

By considering the graph of f write down the mode of X.

7e. *[4 marks]*

Show that $P(0 \le X \le 2) = \frac{1}{4}$.

7f. *[2 marks]*

Hence state the interquartile range of X.

7g. *[2 marks]*

Calculate $P(X \le 4 | X \ge 3)$.

8a. *[2 marks]*

A continuous random variable T has probability density function f defined by

$$f(t) = \begin{cases} \dfrac{t|\sin 2t|}{\pi}, & 0 \le t \le \pi \\ 0, & \text{otherwise} \end{cases}$$

Sketch the graph of $y = f(t)$.

8b. *[1 mark]*

Use your sketch to find the mode of T.

8c. *[2 marks]*

Find the mean of T.

8d. *[3 marks]*

Find the variance of T.

8e. *[2 marks]*

Find the probability that T lies between the mean and the mode.

8f. *[5 marks]*

(i) Find $\int_0^{\pi} f(t)\mathrm{d}t$ where $0 \le T \le \frac{\pi}{2}$.

(ii) Hence verify that the lower quartile of T is $\frac{\pi}{2}$.