Revision - kinematics + crv *[264 marks]*

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

1a. Find the maximum distance of the particle from O. *[3 marks]*

Markscheme

use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is 16.5 cm (to the left of O) ***A1***

[3 marks]

1b. Find the acceleration of the particle at the instant it first changes direction. *[4 marks]*

Markscheme

attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t– intercept on the graph of s'. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of s' for **their** value of t, $s''(1.51986...)$ ***(M1)***

$= -8.92$ (cm/s^2) ***A1***

[4 marks]

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

2a. Find $P(24.15 < X < 25)$. *[2 marks]*

Markscheme

attempt to use the symmetry of the normal curve ***(M1)***

eg diagram, $0.5 - 0.1446$

$\text{P}(24.15 < X < 25) = 0.3554$ ***A1***

[2 marks]

2b. Find σ, the standard deviation of X. *[3 marks]*

Markscheme

use of inverse normal to find z score ***(M1)***

$z = -1.0598$

correct substitution $\frac{24.15-25}{\sigma} = -1.0598$ ***(A1)***

$\sigma = 0.802$ ***A1***

[3 marks]

2c. Hence, find the probability that a seashell selected at random has a length greater than 26 mm. *[2 marks]*

Markscheme

$\text{P}(X > 26) = 0.106$ ***(M1)A1***

[2 marks]

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

2d. Find $\text{E}(Y)$. *[3 marks]*

Markscheme

recognizing binomial probability ***(M1)***

E(Y) = 10 × 0.10621 ***(A1)***

= 1.06 ***A1***

[3 marks]

2e. Find the probability that exactly three of these seashells have a length greater than 26 mm. *[2 marks]*

Markscheme

P(Y = 3) ***(M1)***

= 0.0655 ***A1***

[2 marks]

2f. A seashell selected at random has a length less than 26 mm. *[3 marks]*

Find the probability that its length is between 24.15 mm and 25 mm.

Markscheme

recognizing conditional probability ***(M1)***

correct substitution ***A1***

$$\frac{0.3554}{1-0.10621}$$

= 0.398 ***A1***

[3 marks]

In a city, the number of passengers, *X*, who ride in a taxi has the following probability distribution.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.60	0.30	0.03	0.05	0.02

After the opening of a new highway that charges a toll, a taxi company introduces a charge for passengers who use the highway. The charge is $ 2.40 per taxi plus $ 1.20 per passenger. Let *T* represent the amount, in dollars, that is charged by the taxi company per ride.

3a. Find E(*T*). *[4 marks]*

Markscheme

METHOD 1

attempting to use the expected value formula ***(M1)***

$\mathrm{E}\left(X\right)=\left(1\times 0.60\right)+\left(2\times 0.30\right)+\left(3\times 0.03\right)+\left(4\times 0.05\right)+\left(5\times 0.02\right)$

$\mathrm{E}\left(X\right)=1.59$($) ***(A1)***

use of $\mathrm{E}\left(1.20X+2.40\right)=1.20\mathrm{E}\left(X\right)+2.40$ ***(M1)***

$\mathrm{E}\left(T\right)=1.20\left(1.59\right)+2.40$

$=4.31$($) ***A1***

METHOD 2

attempting to find the probability distribution for *T* ***(M1)***

t	3.60	4.80	6.00	7.20	8.40
$\mathrm{P}(T=t)$	0.60	0.30	0.03	0.05	0.02

(A1)

attempting to use the expected value formula ***(M1)***

$\mathrm{E}\left(T\right)=\left(3.60\times 0.60\right)+\left(4.80\times 0.30\right)+\left(6.00\times 0.03\right)+\left(7.20\times 0.05\right)+\left(8.40\times\right.$

$=4.31$($) ***A1***

[4 marks]

3b. Given that Var(*X*) = 0.8419, find Var(*T*). *[2 marks]*

Markscheme

METHOD 1

using Var(1.20 *X* + 2.40) = $(1.20)^2$ Var(*X*) with Var(*X*) = 0.8419 ***(M1)***

Var(*T*) = 1.21 ***A1***

METHOD 2

finding the standard deviation for **their** probability distribution found in part (a) ***(M1)***

Var(*T*) = $(1.101\ldots)^2$

= 1.21 ***A1***

Note: Award ***M1A1*** for Var(*T*) = $(1.093\ldots)^2$ = 1.20.

[2 marks]

A particle moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, at time t seconds is given by

$v = 4t^2 - 6t + 9 - 2\sin(4t), 0 \le t \le 1.$

The particle's acceleration is zero at $t = T$.

4a. Find the value of T. *[2 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

attempts either graphical or symbolic means to find the value of t when $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ **(M1)**

$T = 0.465\,(\mathrm{s})$ **A1**

[2 marks]

4b. Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$. *[3 marks]*

Show that $s_2 > s_1$.

Markscheme

attempts to find the value of either $s_1 = \int_0^{0.46494\ldots} v\,\mathrm{d}t$ or $s_2 = \int_{0.46494\ldots}^{1} v\,\mathrm{d}t$ **(M1)**

$s_1 = 3.02758\ldots$ and $s_2 = 3.47892\ldots$ **A1A1**

Note: Award as above for obtaining, for example, $s_2 - s_1 = 0.45133\ldots$ or $\frac{s_2}{s_1} = 1.14907\ldots$

Note: Award a maximum of **M1A1A0FT** for use of an incorrect value of T from part (a).

so $s_2 > s_1$ **AG**

[3 marks]

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

5a. By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. *[4 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$T \sim \mathrm{N}\left(\mu,\ 8.6^2\right)$

$\mathrm{P}(T \le 36.8) = 0.7$ **(A1)**

states a correct equation, for example, $\frac{36.8-\mu}{8.6} = 0.5244\ldots$ **A1**

attempts to solve their equation **(M1)**

$\mu = 36.8 - (0.5244\ldots)(8.6)(= 32.2902\ldots)$ **A1**

the solution to the equation is $\mu = 32.29$, correct to two decimal places **AG**

[4 marks]

Use $\mu = 32.29$ in the remainder of the question.

5b. Find the 86th percentile time to complete the jigsaw puzzle. *[2 marks]*

Markscheme

let $t_{0.86}$ be the 86th percentile

attempts to use the inverse normal feature of a GDC to find $t_{0.86}$ **(M1)**

$t_{0.86} = 41.6$ (mins) **A1**

[2 marks]

5c. Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. *[2 marks]*

Markscheme

evidence of identifying the correct area under the normal curve **(M1)**

Note: Award **M1** for a clearly labelled sketch.

$\mathrm{P}(T > 30) = 0.605$ **A1**

[2 marks]

Six randomly chosen people complete the jigsaw puzzle.

5d. Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. *[3 marks]*

Markscheme

let X represent the number of people out of the six who take more than 30 minutes to complete the jigsaw puzzle

$X \sim \mathrm{B}(6, 0.6049\ldots)$ **(M1)**

for example, $\mathrm{P}(X = 5) + \mathrm{P}(X = 6)$ or $1 - \mathrm{P}(X \leq 4)$ **(A1)**

$\mathrm{P}(X \geq 5) = 0.241$ **A1**

[3 marks]

5e. Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle. *[4 marks]*

Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle.

Markscheme

recognizes that $\mathrm{P}\left(T > 30 \middle| T \geq 25\right)$ is required **(M1)**

Note: Award **M1** for recognizing conditional probability.

$= \frac{\mathrm{P}\left(T > 30 \cap T \geq 25\right)}{\mathrm{P}\left(T \geq 25\right)}$ **(A1)**

$= \frac{\mathrm{P}\left(T > 30\right)}{\mathrm{P}\left(T \geq 25\right)} = \frac{0.6049\ldots}{0.8016\ldots}$ **M1**

$= 0.755$ **A1**

[4 marks]

The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{k}{\sqrt{4-3x^2}}, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

6a. Find the value of k. *[4 marks]*

Markscheme

attempt to integrate $\frac{k}{\sqrt{4-3x^2}}$ ***(M1)***

$= k\left[\frac{1}{\sqrt{3}}\arcsin\left(\frac{\sqrt{3}}{2}x\right)\right]$ ***A1***

Note: Award ***(M1)A0*** for $\arcsin\left(\frac{\sqrt{3}}{2}x\right)$.
Condone absence of k up to this stage.

equating their integrand to 1 ***M1***

$k\left[\frac{1}{\sqrt{3}}\arcsin\left(\frac{\sqrt{3}}{2}x\right)\right]_0^1 = 1$

$k = \frac{3\sqrt{3}}{\pi}$ ***A1***

[4 marks]

6b. Find $\mathrm{E}(X)$. *[4 marks]*

Markscheme

$\mathrm{E}\left(X\right) = \frac{3\sqrt{3}}{\pi}\int_0^1 \frac{x}{\sqrt{4-3x^2}}\,\mathrm{d}\,x$ ***A1***

Note: Condone absence of limits if seen at a later stage.

EITHER

attempt to integrate by inspection ***(M1)***

$= \frac{3\sqrt{3}}{\pi} \times -\frac{1}{6}\int -6x\left(4-3x^2\right)^{-\frac{1}{2}}\,\mathrm{d}\,x$

$= \frac{3\sqrt{3}}{\pi}\left[-\frac{1}{3}\sqrt{4-3x^2}\right]_0^1$ ***A1***

Note: Condone the use of k up to this stage.

OR

for example, $u = 4 - 3x^2 \Rightarrow \frac{\mathrm{d}\,u}{\mathrm{d}\,x} = -6x$

Note: Other substitutions may be used. For example $u = -3x^2$.

$= -\frac{\sqrt{3}}{2\pi}\int_4^1 u^{-\frac{1}{2}}\,\mathrm{d}\,u$ ***M1***

Note: Condone absence of limits up to this stage.

$= -\frac{\sqrt{3}}{2\pi}\left[2\sqrt{u}\right]_4^1$ ***A1***

Note: Condone the use of k up to this stage.

THEN

$= \frac{\sqrt{3}}{\pi}$ ***A1***

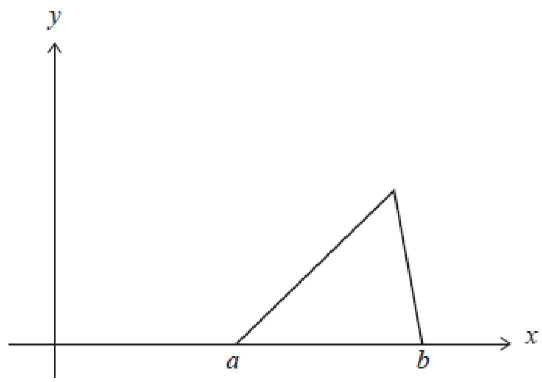
Note: Award ***A0M1A1A0*** for their $k\left[-\frac{1}{3}\sqrt{4-3x^2}\right]$ or $k\left[-2\sqrt{u}\right]$ for working with incorrect or no limits.

[4 marks]

7. A continuous random variable X has the probability density function *[6 marks]*

$$f(x)=\begin{cases} \frac{2}{(b-a)(c-a)}(x-a), & a \le x \le c \\ \frac{2}{(b-a)(b-c)}(b-x), & c < x \le b \\ 0, & \text{otherwise} \end{cases}.$$

The following diagram shows the graph of $y=f(x)$ for $a \le x \le b$.



Given that $c \ge \frac{a+b}{2}$, find an expression for the median of X in terms of $a,$ b and c.

Markscheme

let m be the median

EITHER

attempts to find the area of the required triangle ***M1***

base is $(m-a)$ ***(A1)***

and height is $\frac{2}{(b-a)(c-a)}(m-a)$

area $= \frac{1}{2}(m-a) \times \frac{2}{(b-a)(c-a)}(m-a)$ $\left(= \frac{(m-a)^2}{(b-a)(c-a)}\right)$ ***A1***

OR

attempts to integrate the correct function ***M1***

$$\int_a^m \frac{2}{(b-a)(c-a)}(x-a)\,\mathrm{d}x$$

$= \frac{2}{(b-a)(c-a)}\left[\frac{1}{2}(x-a)^2\right]_a^m$ OR $\frac{2}{(b-a)(c-a)}\left[\frac{x^2}{2} - ax\right]_a^m$ ***A1A1***

Note: Award ***A1*** for correct integration and ***A1*** for correct limits.

THEN

sets up (their) $\int_a^m \frac{2}{(b-a)(c-a)}(x-a)\,\mathrm{d}x$ or area $= \frac{1}{2}$ ***M1***

Note: Award ***M0A0A0M1A0A0*** if candidates conclude that $m > c$ and set up their area or sum of integrals $= \frac{1}{2}$.

$\frac{(m-a)^2}{(b-a)(c-a)} = \frac{1}{2}$

$m = a \pm \sqrt{\frac{(b-a)(c-a)}{2}}$ ***(A1)***

as $m > a$, rejects $m = a - \sqrt{\frac{(b-a)(c-a)}{2}}$

so $m = a + \sqrt{\frac{(b-a)(c-a)}{2}}$ ***A1***

[6 marks]

A particle moves along a straight line so that its velocity, $v\ \mathrm{m\,s^{-1}}$, after t seconds is given by $v(t) = \mathrm{e}^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

8a. Find the value of t when the particle is at rest. *[2 marks]*

Markscheme

recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692\ldots$

$t = 3.35$ (seconds) ***A1***

Note: Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

[2 marks]

8b. Find the acceleration of the particle when it changes direction. *[3 marks]*

Markscheme

recognizing particle changes direction when $v = 0$ OR when $t = 3.34692\ldots$ ***(M1)***

$a = -4.71439\ldots$

$a = -4.71\left(\mathrm{ms}^{-2}\right)$ ***A2***

[3 marks]

8c. Find the total distance travelled by the particle. *[2 marks]*

Markscheme

distance travelled $= \int_0^6 |v| \mathrm{d}t$ OR

$\int_0^{3.34\ldots}\left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t - \int_{3.34\ldots}^{6}\left(\mathrm{e}^{\sin(t)} + 4\sin(t)\right)\mathrm{d}t \ (= 14.3104\ldots + 6.4$ ***(A1)***

$= 20.7534\ldots$

$= 20.8$ (metres) ***A1***

[2 marks]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

9a. Find the probability that a randomly selected chocolate muffin weighs less than 61 g. *[2 marks]*

Markscheme

$\mathrm{P}(C < 61)$ ***(M1)***

$= 0.365112\ldots$

$= 0.365$ ***A1***

[2 marks]

9b. In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. *[2 marks]*

Markscheme

recognition of binomial eg $X \sim \mathrm{B}(12,\ 0.365\ldots)$ ***(M1)***

$\mathrm{P}(X = 5) = 0.213666\ldots$

$= 0.214$ ***A1***

[2 marks]

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

9c. Find the probability that the randomly selected muffin weighs less than 61 g. *[4 marks]*

Markscheme

Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$\mathrm{P}(B < 61) = 0.0197555\ldots$ ***(A1)***

EITHER

$\mathrm{P}(CM) \times \mathrm{P}\left(C < 61 \,\middle|\, CM\right) + \mathrm{P}(BM) \times \mathrm{P}\left(B < 61 \,\middle|\, BM\right)$ (or equivalent in words) ***(M1)***

OR

tree diagram showing two ways to have a muffin weigh < 61 ***(M1)***

THEN

$(0.6 \times 0.365\ldots) + (0.4 \times 0.0197\ldots)$ ***(A1)***

$= 0.226969\ldots$

$= 0.227$ ***A1***

[4 marks]

9d. Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. *[3 marks]*

Markscheme

recognizing conditional probability ***(M1)***

Note: Recognition must be shown in context either in words or symbols, not just $\mathrm{P}\left(A \,\middle|\, B\right)$

$\frac{0.6 \times 0.365112\ldots}{0.226969\ldots}$ ***(A1)***

$= 0.965183\ldots$

$= 0.965$ ***A1***

[3 marks]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

9e. Find the value of σ. *[5 marks]*

Markscheme

METHOD 1

$\mathrm{P}(CM)\times\mathrm{P}\left(C<61 \mid CM\right)\times\mathrm{P}(BM)\times\mathrm{P}\left(B<61 \mid BM\right)=0.157$ ***(M1)***

$(0.6\times\mathrm{P}(C<61))+(0.4\times0.0197555\ldots)=0.157$

$\mathrm{P}(C<61)=0.248496\ldots$ ***(A1)***

attempt to solve for σ using GDC ***(M1)***

Note: Award ***(M1)*** for a graph or table of values to show their $\mathrm{P}(C<61)$ with a variable standard deviation.

$\sigma=1.47225\ldots$

$\sigma=1.47\,(\mathrm{g})$ ***A2***

METHOD 2

$\mathrm{P}(CM)\times\mathrm{P}\left(C<61 \mid CM\right)\times\mathrm{P}(BM)\times\mathrm{P}\left(B<61 \mid BM\right)=0.157$ ***(M1)***

$(0.6\times\mathrm{P}(C<61))+(0.4\times0.0197555\ldots)=0.157$

$\mathrm{P}(C<61)=0.248496\ldots$ ***(A1)***

use of inverse normal to find z score of their $\mathrm{P}(C<61)$ ***(M1)***

$z=-0.679229\ldots$

correct substitution ***(A1)***

$\frac{61-62}{\sigma}=-0.679229\ldots$

$\sigma=1.47225\ldots$

$\sigma=1.47\,(\mathrm{g})$ ***A1***

[5 marks]

A particle moves in a straight line such that its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4},\ 0 \le t \le 3$.

10a. Determine when the particle changes its direction of motion. *[2 marks]*

Markscheme

recognises the need to find the value of t when $v = 0$ ***(M1)***

$t = 1.57079\ldots\ \left(= \frac{\pi}{2}\right)$

$t = 1.57\ \left(= \frac{\pi}{2}\right)$ (s) ***A1***

[2 marks]

10b. Find the times when the particle's acceleration is -1.9 m s^{-2}. *[3 marks]*

Markscheme

recognises that $a(t) = v'(t)$ ***(M1)***

$t_1 = 2.26277\ldots,\ t_2 = 2.95736\ldots$

$t_1 = 2.26,\ t_2 = 2.96$ (s) ***A1A1***

Note: Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \le t \le 3$.

[3 marks]

10c. Find the particle's acceleration when its speed is at its greatest. *[2 marks]*

Markscheme

speed is greatest at $t = 3$ ***(A1)***

$a = -1.83778\ldots$

$a = -1.84\ \left(\text{m}\,\text{s}^{-2}\right)$ ***A1***

[2 marks]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

11a. Find the value of σ. *[4 marks]*

Markscheme

METHOD 1

$T \sim \mathrm{N}\left(35, \sigma^2\right)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

attempt to solve for σ graphically or numerically using the GDC ***(M1)***

graph of normal curve $T \sim \mathrm{N}\left(35, \sigma^2\right)$ for $\mathrm{P}(T > 40)$ and $y = 0.25$ OR
$\mathrm{P}(T < 40)$ and $y = 0.75$
OR table of values for $\mathrm{P}(T < 40)$ or $\mathrm{P}(T > 40)$

$\sigma = 7.413011\ldots$

$\sigma = 7.41$ (min) ***A2***

METHOD 2

$T \sim \mathrm{N}\left(35, \sigma^2\right)$

$\mathrm{P}(T > 40) = 0.25$ or $\mathrm{P}(T < 40) = 0.75$ ***(M1)***

$z = 0.674489\ldots$ ***(A1)***

valid equation using their z-score (clearly identified as z-score and not a probability) ***(M1)***

$\frac{40-35}{\sigma} = 0.674489\ldots$ OR $5 = 0.674489\ldots\sigma$

$7.413011\ldots$

$\sigma = 7.41$ (min) ***A1***

[4 marks]

11b. On a randomly selected day, find the probability that Suzi’s drive to work will take longer than 45 minutes. *[2 marks]*

Markscheme

$\mathrm{P}(T > 45)$ ***(M1)***

$= 0.0886718\ldots$

$= 0.0887$ ***A1***

[2 marks]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

11c. Find the probability that she will be late to work at least one day next week. *[3 marks]*

Markscheme

recognizing binomial probability ***(M1)***

L~$\mathrm{B}(5,\ 0.0886718\ldots)$

$\mathrm{P}(L \geq 1)= 1 - \mathrm{P}(L = 0)$ OR

$\mathrm{P}(L \geq 1)= \mathrm{P}(L = 1)+\mathrm{P}(L = 2)+\mathrm{P}(L = 3)+\mathrm{P}(L = 4)+\mathrm{P}(L = 5)$ ***(M1)***

$0.371400\ldots$

$\mathrm{P}(L \geq 1)= 0.371$ ***A1***

[3 marks]

11d. Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. *[5 marks]*

Markscheme

recognizing conditional probability in context ***(M1)***

finding $\{L < 3\} \cap \{L \geq 1\} = \{L = 1,\ L = 2\}$ (may be seen in conditional probability) ***(A1)***

$\mathrm{P}(L = 1) + \mathrm{P}(L = 2) = 0.36532\ldots$ (may be seen in conditional probability) ***(A1)***

$P\left(L < 3 \middle| L \geq 1\right) = \frac{0.36532\ldots}{0.37140\ldots}$ ***(A1)***

$0.983636\ldots$

0.984 ***A1***

[5 marks]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

11e. Find the probability that Suzi will receive a bonus. *[4 marks]*

Markscheme

METHOD 1

recognizing that Suzi can be late no more than once (in the remaining six days) ***(M1)***

$X \sim B(6,\ 0.0886718\ldots)$, where X is the number of days late ***(A1)***

$P(X \leq 1) = P(X = 0) + P(X = 1)$ ***(M1)***

$= 0.907294\ldots$

$P(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

METHOD 2

recognizing that Suzi must be on time at least five times (of the remaining six days) ***(M1)***

$X \sim B(6,\ 0.911328\ldots)$, where X is the number of days on time ***(A1)***

$P(X \geq 5) = 1 - P(X \leq 4)$ OR $1 - 0.0927052\ldots$ OR $P(X = 5) + (X = 6)$ OR $0.334434\ldots + 0.572860\ldots$ ***(M1)***

$= 0.907294\ldots$

$P(\text{Suzi gets a bonus}) = 0.907$ ***A1***

Note: The first two marks may be awarded independently.

[4 marks]

12. Rachel and Sophia are competing in a javelin-throwing competition. *[7 marks]*

The distances, R metres, thrown by Rachel can be modelled by a normal distribution with mean 56.5 and standard deviation 3.

The distances, S metres, thrown by Sophia can be modelled by a normal distribution with mean 57.5 and standard deviation 1.8.

In the first round of competition, each competitor must have five throws. To qualify for the next round of competition, a competitor must record at least one throw of 60 metres or greater in the first round.

Find the probability that only one of Rachel or Sophia qualifies for the next round of competition.

Markscheme

Rachel: $R \sim \mathrm{N}\left(56.5, 3^2\right)$

$\mathrm{P}(R \geq 60) = 0.1216\ldots$ ***(A1)***

Sophia: $S \sim \mathrm{N}\left(57.5, 1.8^2\right)$

$\mathrm{P}(S \geq 60) = 0.0824\ldots$ ***(A1)***

recognises binomial distribution with $n = 5$ ***(M1)***

let N_R represent the number of Rachel's throws that are longer than 60 metres

$N_R \sim \mathrm{B}(5, 0.1216\ldots)$

either $\mathrm{P}(N_R \geq 1) = 0.4772\ldots$ or $\mathrm{P}(N_R = 0) = 0.5227\ldots$ ***(A1)***

let N_S represent the number of Sophia's throws that are longer than 60 metres

$N_S \sim \mathrm{B}(5, 0.0824\ldots)$

either $\mathrm{P}(N_S \geq 1) = 0.3495\ldots$ or $\mathrm{P}(N_S = 0) = 0.6504\ldots$ ***(A1)***

EITHER

uses $\mathrm{P}(N_R \geq 1)\mathrm{P}(N_S = 0) + \mathrm{P}(N_S \geq 1)\mathrm{P}(N_R = 0)$ ***(M1)***

$\mathrm{P}(\text{one of Rachel or Sophia qualify}) = (0.4772\ldots \times 0.6504) + (0.3495\ldots \times 0.52$

OR

uses $\mathrm{P}(N_R \geq 1) + \mathrm{P}(N_S \geq 1) - 2 \times \mathrm{P}(N_R \geq 1) \times \mathrm{P}(N_S \geq 1)$ ***(M1)***

$\mathrm{P}(\text{one of Rachel or Sophia qualify}) = 0.4772\ldots + 0.3495\ldots - 2 \times 0.4772\ldots \times$

THEN

$= 0.4931\ldots$

$= 0.493$ ***A1***

Note: ***M*** marks are not dependent on the previous ***A*** marks.

[7 marks]

A particle P moves along the x-axis. The velocity of P is $v \text{ m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O.

13a. Find the value of t when P reaches its maximum velocity. *[2 marks]*

Markscheme

valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

[2 marks]

13b. Show that the distance of P from O at this time is $\frac{88}{27}$ metres. *[5 marks]*

Markscheme

attempt to integrate v ***(M1)***

$\int v \, \mathrm{d}t = \int (4 + 4t - 3t^2) \, \mathrm{d}t = 4t + 2t^2 - t^3 (+c)$ ***A1A1***

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

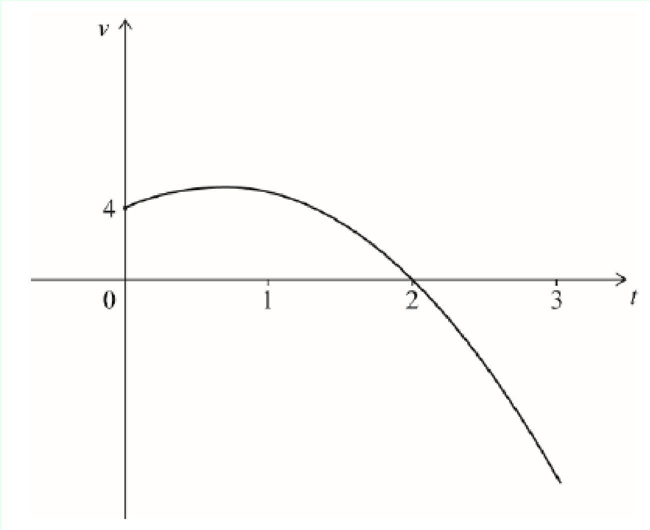
$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

$= \frac{88}{27}$ (m) ***AG***

[5 marks]

13c. Sketch a graph of v against t, clearly showing any points of intersection with the axes. *[4 marks]*

Markscheme



valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\frac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\frac{2}{3}$ and $v>4$ ***A1***

[4 marks]

13d. Find the total distance travelled by P. *[5 marks]*

Markscheme

recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4+4t-3t^2\right| \, \mathrm{d}t$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \, \mathrm{d}t$

$= 8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \, \mathrm{d}t$

$= -5$ ***A1***

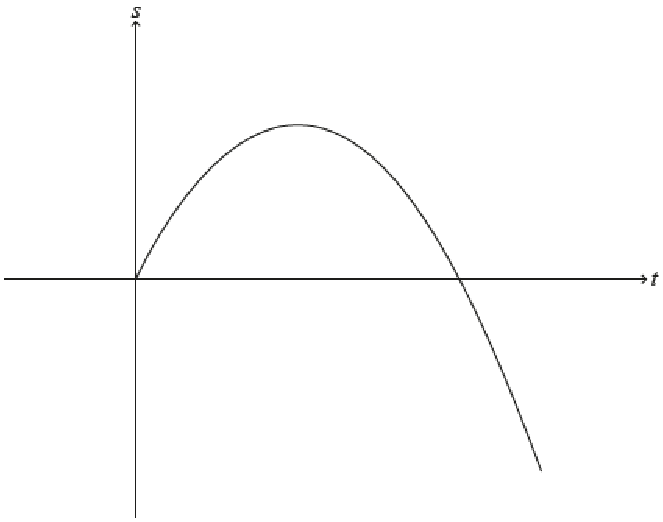
valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, \mathrm{d}t - \int_2^3 v \, \mathrm{d}t$ OR $\int_0^2 v \, \mathrm{d}t + \left|\int_2^3 v \, \mathrm{d}t\right|$

total distance travelled $= 13$ (m) ***A1***

[5 marks]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s\left(t\right) = 8t - t^2$, for $0 \le t \le 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

14a. Find the value of p. *[2 marks]*

Markscheme

setting $s(t) = 0$ ***(M1)***

$8t - t^2 = 0$

$t(8 - t) = 0$

$p = 8$ (accept $t = 8, (8,\ 0)$) ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $p = 0,\ 8$).

[2 marks]

Particle A changes direction when $t = q$.

14b. Find the value of q. *[2 marks]*

Markscheme

recognition that when particle changes direction $v = 0$ OR local maximum on graph of s OR vertex of parabola ***(M1)***

$q = 4$ (accept $t = 4$) ***A1***

[2 marks]

14c. Find the displacement of particle A from the origin when $t = q$. *[2 marks]*

Markscheme

substituting their value of q into $s(t)$ OR integrating $v(t)$ from $t = 0$ to $t = 4$ ***(M1)***

$\text{displacement} = 16\ (\text{m})$ ***A1***

[2 marks]

14d. Find the distance of particle A from the origin when $t = 10$. *[2 marks]*

Markscheme

$s(10) = -20$ OR $\text{distance} = |\text{s}(t)|$ OR integrating $v(t)$ from $t = 0$ to $t = 10$ ***(M1)***

$\text{distance} = 20\ (\text{m})$ ***A1***

[2 marks]

The total distance travelled by particle A is given by d.

14e. Find the value of d. *[2 marks]*

Markscheme

16 forward $+36$ backward OR $16+16+20$ OR $\int_0^{10} |v(t)| \mathrm{d}\,t$ ***(M1)***

$d = 52\,(\mathrm{m})$ ***A1***

[2 marks]

14f. A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$. *[4 marks]*

When $t = k$, the distance travelled by particle B is equal to d.

Find the value of k.

Markscheme

METHOD 1

graphical method with triangles on $v(t)$ graph ***M1***

$49 + \left(\frac{x\,(2x)}{2}\right)$ ***(A1)***

$49 + x^2 = 52, x = \sqrt{3}$ ***(A1)***

$k = 7 + \sqrt{3}$ ***A1***

METHOD 2

recognition that distance $= \int |v(t)| \mathrm{d}\,t$ ***M1***

$\int_0^7 (14-2t) \mathrm{d}\,t + \int_7^k (2t-14) \mathrm{d}\,t$

$\left[14t - t^2\right]_0^7 + \left[t^2 - 14t\right]_7^k$ ***(A1)***

$14(7) - 7^2 + \left(\left(k^2 - 14k\right) - \left(7^2 - 14(7)\right)\right) = 52$ ***(A1)***

$k = 7 + \sqrt{3}$ ***A1***

[4 marks]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

15a. Find the probability that a bag selected at random is rejected. *[2 marks]*

Markscheme

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let $X =$ mass of a bag of sugar

evidence of identifying the correct area ***(M1)***

$\mathrm{P}(X < 995) = 0.0765637\ldots$

$= 0.0766$ ***A1***

[2 marks]

15b. Estimate the number of bags which will be rejected from a random sample of 100 bags. *[1 mark]*

Markscheme

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let $X =$ mass of a bag of sugar

0.0766×100

≈ 8 ***A1***

Note: Accept 7.66.

[1 mark]

15c. Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. *[3 marks]*

Markscheme

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let $X =$ mass of a bag of sugar

recognition that $\mathrm{P}\left(X > 1005 \middle| X \geq 995\right)$ is required ***(M1)***

$\frac{\mathrm{P}\left(X \geq 995 \cap X > 1005\right)}{\mathrm{P}\left(X \geq 995\right)}$

$\frac{\mathrm{P}\left(X > 1005\right)}{\mathrm{P}\left(X \geq 995\right)}$ ***(A1)***

$\frac{0.0765637\ldots}{1-0.0765637\ldots}\left(=\frac{0.0765637\ldots}{0.923436\ldots}\right)$

$= 0.0829$ ***A1***

[3 marks]

A particle P moves in a straight line such that after time t seconds, its velocity, v in $\mathrm{m\,s}^{-1}$, is given by $v = \mathrm{e}^{-3t}\sin 6t$, where $0 < t < \frac{\pi}{2}$.

16a. Find the times when P comes to instantaneous rest. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\pi}{6}(= 0.524)$ ***A1***

$\frac{\pi}{3}(= 1.05)$ ***A1***

[2 marks]

At time t, P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

16b. Find an expression for s in terms of t. *[7 marks]*

Markscheme

attempt to use integration by parts ***M1***

$s = \int e^{-3t} \sin 6t \, dt$

EITHER

$= -\frac{e^{-3t}\sin 6t}{3} - \int -2e^{-3t} \cos 6t \, dt$ ***A1***

$= -\frac{e^{-3t}\sin 6t}{3} - \left(\frac{2e^{-3t}\cos 6t}{3} - \int -4e^{-3t} \sin 6t \, dt\right)$ ***A1***

$= -\frac{e^{-3t}\sin 6t}{3} - \left(\frac{2e^{-3t}\cos 6t}{3} + 4s\right)$

$5s = \frac{-3e^{-3t}\sin 6t - 6e^{-3t}\cos 6t}{9}$ ***M1***

OR

$= -\frac{e^{-3t}\cos 6t}{6} - \int \frac{1}{2} e^{-3t} \cos 6t \, dt$ ***A1***

$= -\frac{e^{-3t}\cos 6t}{6} - \left(\frac{e^{-3t}\sin 6t}{12} + \int \frac{1}{4} e^{-3t} \sin 6t \, dt\right)$ ***A1***

$= -\frac{e^{-3t}\cos 6t}{6} - \left(\frac{e^{-3t}\sin 6t}{12} + \frac{1}{4}s\right)$

$\frac{5}{4}s = \frac{-2e^{-3t}\cos 6t - e^{-3t}\sin 6t}{12}$ ***M1***

THEN

$s = -\frac{e^{-3t}(\sin 6t + 2\cos 6t)}{15} (+c)$ ***A1***

at $t = 0, \ s = 0 \Rightarrow 0 = -\frac{2}{15} + c$ ***M1***

$c = \frac{2}{15}$ ***A1***

$s = \frac{2}{15} - \frac{e^{-3t}(\sin 6t + 2\cos 6t)}{15}$

[7 marks]

16c. Find the maximum displacement of P, in metres, from its initial position. *[2 marks]*

Markscheme

EITHER

substituting $t = \frac{\pi}{6}$ into their equation for s ***(M1)***

$\left(s = \frac{2}{15} - \frac{e^{-\frac{\pi}{2}}(\sin\pi + 2\cos\pi)}{15}\right)$

OR

using GDC to find maximum value ***(M1)***

OR

evaluating $\int_0^{\frac{\pi}{6}} v\mathrm{d}t$ ***(M1)***

THEN

$= 0.161\left(= \frac{2}{15}\left(1 + e^{-\frac{\pi}{2}}\right)\right)$ ***A1***

[2 marks]

16d. Find the total distance travelled by P in the first 1.5 seconds of its motion. *[2 marks]*

Markscheme

METHOD 1

EITHER

distance required $= \int_0^{1.5} \left|e^{-3t} \sin 6t\right| \, dt$ ***(M1)***

OR

distance required $= \int_0^{\frac{\pi}{6}} e^{-3t} \sin 6t \, dt + \left|\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} e^{-3t} \sin 6t \, dt\right| + \int_{\frac{\pi}{3}}^{1.5} e^{-3t} \sin 6t \, dt$

(M1)

$(= 0.16105\ldots + 0.033479\ldots + 0.006806\ldots)$

THEN

$= 0.201 \text{ (m)}$ ***A1***

METHOD 2

using successive minimum and maximum values on the displacement graph ***(M1)***

$0.16105\ldots + (0.16105\ldots - 0.12757\ldots) + (0.13453\ldots - 0.12757\ldots)$

$= 0.201 \text{ (m)}$ ***A1***

[2 marks]

At successive times when the acceleration of P is 0 m s^{-2}, the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1, t_2, t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1, v_2, v_3.

16e. Show that, at these times, $\tan 6t = 2$. *[2 marks]*

Markscheme

valid attempt to find $\frac{\mathrm{d}v}{\mathrm{d}t}$ using product rule and set $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ ***M1***

$\frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{e}^{-3t} 6 \cos 6t - 3\mathrm{e}^{-3t} \sin 6t$ ***A1***

$\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow \tan 6t = 2$ ***AG***

[2 marks]

16f. Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -\mathrm{e}^{-\frac{\pi}{2}}$. *[5 marks]*

Markscheme

attempt to evaluate $t_1,\ t_2,\ t_3$ in exact form ***M1***

$6t_1 = \arctan 2 \left(\Rightarrow t_1 = \frac{1}{6} \arctan 2\right)$

$6t_2 = \pi + \arctan 2 \left(\Rightarrow t_2 = \frac{\pi}{6} + \frac{1}{6} \arctan 2\right)$

$6t_3 = 2\pi + \arctan 2 \left(\Rightarrow t_3 = \frac{\pi}{3} + \frac{1}{6} \arctan 2\right)$ ***A1***

Note: The ***A1*** is for any two consecutive correct, or showing that $6t_2 = \pi + 6t_1$ or $6t_3 = \pi + 6t_2$.

showing that $\sin 6t_{n+1} = -\sin 6t_n$

eg $\tan 6t = 2 \Rightarrow \sin 6t = \pm\frac{2}{\sqrt{5}}$ ***M1A1***

showing that $\frac{\mathrm{e}^{-3t_{n+1}}}{\mathrm{e}^{-3t_n}} = \mathrm{e}^{-\frac{\pi}{2}}$ ***M1***

eg $\mathrm{e}^{-3\left(\frac{\pi}{6}+k\right)} \div \mathrm{e}^{-3k} = \mathrm{e}^{-\frac{\pi}{2}}$

Note: Award the ***A1*** for any two consecutive terms.

$\frac{v_3}{v_2} = \frac{v_2}{v_1} = -\mathrm{e}^{-\frac{\pi}{2}}$ ***AG***

[5 marks]

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\ \mathrm{g}$ and standard deviation $8\ \mathrm{g}$.

17a. Find the probability that a randomly selected packet has a weight less than $100\ \mathrm{g}$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$X \sim \mathrm{N}\left(102,\ 8^2\right)$

$\mathrm{P}(X < 100) = 0.401$ ***(M1)A1***

[2 marks]

17b. The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. *[2 marks]*

Markscheme

$\mathrm{P}(X > w) = 0.444$ ***(M1)***

$\Rightarrow w = 103\ (\mathrm{g})$ ***A1***

[2 marks]

17c. A packet is randomly selected. Given that the packet has a weight greater than $105\ \mathrm{g}$, find the probability that it has a weight greater than $110\ \mathrm{g}$. *[3 marks]*

Markscheme

$\mathrm{P}\left(X > 100 \,\middle|\, X > 105\right) = \frac{\mathrm{P}(X>100 \cap X>105)}{\mathrm{P}(X>105)}$ ***(M1)***

$= \frac{\mathrm{P}(X>100)}{\mathrm{P}(X>105)}$ ***(A1)***

$= \frac{0.15865...}{0.35383...}$

$= 0.448$ ***A1***

[3 marks]

17d. From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. *[3 marks]*

Markscheme

EITHER

$\mathrm{P}(90 < X < 114) = 0.866...$ ***(A1)***

OR

$\mathrm{P}(-1.5 < Z < 1.5) = 0.866...$ ***(A1)***

THEN

$0.866... \times 500$ ***(M1)***

$= 433$ ***A1***

[3 marks]

17e. Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than 95 g. *[4 marks]*

Markscheme

$p = \mathrm{P}(X < 95) = 0.19078\ldots$ ***(A1)***

recognising $Y \sim \mathrm{B}(80,\ p)$ ***(M1)***

now using $Y \sim \mathrm{B}(80,\ 0.19078\ldots)$ ***(M1)***

$\mathrm{P}(Y \geq 20) = 0.116$ ***A1***

[4 marks]

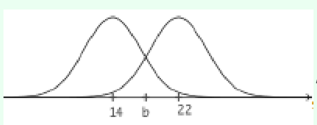
Let X and Y be normally distributed with $X \sim \mathrm{N}\left(14, a^2\right)$ and $Y \sim \mathrm{N}\left(22, a^2\right)$, $a > 0$.

18a. Find b so that $\mathrm{P}\left(X > b\right) = \mathrm{P}\left(Y < b\right)$. *[2 marks]*

Markscheme

METHOD 1

recognizing that b is midway between the means of 14 and 22. ***(M1)***

eg 

, $b = \frac{14+22}{2}$

$b = 18$ ***A1 N2***

METHOD 2

valid attempt to compare distributions ***(M1)***

eg $\frac{b-14}{a} = \frac{b-22}{a}$, $b - 14 = 22 - b$

$b = 18$ ***A1 N2***

[2 marks]

18b. It is given that $\mathrm{P}\left(X > 20\right) = 0.112$. *[4 marks]*

Find $\mathrm{P}\left(16 < Y < 28\right)$.

Markscheme

valid attempt to compare distributions (seen anywhere) ***(M1)***

eg Y is a horizontal translation of X of 8 units to the right,

$\mathrm{P}\left(16 < Y < 28\right) = \mathrm{P}\left(8 < X < 20\right), \mathrm{P}\left(Y > 22 + 6\right) = \mathrm{P}\left(X > 14 + 6\right)$

valid approach using symmetry ***(M1)***

eg

$1 - 2\mathrm{P}\left(X > 20\right), 1 - 2\mathrm{P}\left(Y < 16\right), 2 \times \mathrm{P}\left(14 < x < 20\right), \mathrm{P}\left(X < 8\right) = \mathrm{P}\left(X >\right.$

correct working ***(A1)***

eg $1 - 2\left(0.112\right), 2 \times \left(0.5 - 0.112\right), 2 \times 0.388, 0.888 - 0.112$

$\mathrm{P}\left(16 < Y < 28\right) = 0.776$ ***A1 N3***

[4 marks]

A rocket is travelling in a straight line, with an initial velocity of $140\,\mathrm{m\,s^{-1}}$. It accelerates to a new velocity of $500\,\mathrm{m\,s^{-1}}$ in two stages.

During the first stage its acceleration, $a\,\mathrm{m\,s^{-2}}$, after t seconds is given by $a\left(t\right) = 240\sin\left(2t\right)$, where $0 \leqslant t \leqslant k$.

19a. Find an expression for the velocity, $v\,\mathrm{m\,s^{-1}}$, of the rocket during the first stage. *[4 marks]*

Markscheme

recognizing that $v = \int a$ ***(M1)***

correct integration ***A1***

eg $-120\cos\left(2t\right) + c$

attempt to find c using their $v\left(t\right)$ ***(M1)***

eg $-120\cos\left(0\right) + c = 140$

$v\left(t\right) = -120\cos\left(2t\right) + 260$ ***A1 N3***

[4 marks]

The first stage continues for k seconds until the velocity of the rocket reaches 375 m s^{-1}.

19b. Find the distance that the rocket travels during the first stage. *[4 marks]*

Markscheme

evidence of valid approach to find time taken in first stage ***(M1)***

eg graph, $-120\cos(2t) + 260 = 375$

$k = 1.42595$ ***A1***

attempt to substitute **their** v and/or **their** limits into distance formula ***(M1)***

eg $\int_0^{1.42595} |v|$, $\int 260 - 120\cos(2t)$, $\int_0^k (260 - 120\cos(2t))\, \mathrm{d}t$

353.608

distance is 354 (m) ***A1 N3***

[4 marks]

19c. During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage. *[6 marks]*

Find the total time taken for the two stages.

Markscheme

recognizing velocity of second stage is linear (seen anywhere) ***R1***

eg graph, $s = \frac{1}{2}h(a+b)$, $v = mt + c$

valid approach ***(M1)***

eg $\int v = 353.608$

correct equation ***(A1)***

eg $\frac{1}{2}h(375 + 500) = 353.608$

time for stage two $= 0.808248$ (0.809142 from 3 sf) ***A2***

2.23420 (2.23914 from 3 sf)

2.23 seconds (2.24 from 3 sf) ***A1 N3***

[6 marks]

SpeedWay airline flies from city A to city B. The flight time is normally distributed with a mean of 260 minutes and a standard deviation of 15 minutes.

A flight is considered late if it takes longer than 275 minutes.

20a. Calculate the probability a flight is **not** late. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}\left(X < 275\right),\ 1 - 0.158655$

0.841344

0.841 ***A1 N2***

[2 marks]

The flight is considered to be **on time** if it takes between m and 275 minutes. The probability that a flight is on time is 0.830.

20b. Find the value of m. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}\left(X < 275\right) - \mathrm{P}\left(X < m\right) = 0.830$

correct working ***(A1)***

eg $\mathrm{P}\left(X < m\right) = 0.0113447$

225.820

226 (minutes) ***A1 N3***

[3 marks]

During a week, SpeedWay has 12 flights from city A to city B. The time taken for any flight is independent of the time taken by any other flight.

20c. Calculate the probability that at least 7 of these flights are **on time**. *[3 marks]*

Markscheme

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***

eg $\quad {}_nC_a \times p^a \times q^{n-a},\ \mathrm{B}\left(n,\, p\right)$

evidence of summing probabilities from 7 to 12 ***(M1)***

eg $\quad \mathrm{P}\left(X = 7\right) + \mathrm{P}\left(X = 8\right) + \ldots + \mathrm{P}\left(X = 12\right),\ 1 - \mathrm{P}\left(X \leqslant 6\right)$

0.991248

0.991 ***A1 N2***

[3 marks]

20d. Given that at least 7 of these flights are on time, find the probability that exactly 10 flights are on time. *[4 marks]*

Markscheme

finding $\mathrm{P}\left(X = 10\right)$ (seen anywhere) ***A1***

eg $\quad \begin{pmatrix} 12 \\ 10 \end{pmatrix} \times 0.83^{10} \times 0.17^2\ \left(= 0.295952\right)$

recognizing conditional probability ***(M1)***

eg $\quad \mathrm{P}\left(A\,|\,B\right),\ \mathrm{P}\left(X = 10\,|\,X \geqslant 7\right),\ \frac{\mathrm{P}\left(X=10 \cap X \geqslant 7\right)}{\mathrm{P}\left(X \geqslant 7\right)}$

correct working ***(A1)***

eg $\quad \frac{0.295952}{0.991248}$

0.298565

0.299 ***A1 N1***

Note: Exception to the *FT* rule: if the candidate uses an incorrect value for the probability that a flight is on time in (i) and working shown, award full ***FT*** in (ii) as appropriate.

[4 marks]

20e. SpeedWay increases the number of flights from city A to city B to 20 flights each week, and improves their efficiency so that more flights are on time. The probability that at least 19 flights are on time is 0.788. *[3 marks]*

A flight is chosen at random. Calculate the probability that it is on time.

Markscheme

correct equation ***(A1)***

eg $\binom{20}{19}p^{19}(1-p)+p^{20}=0.788$

valid attempt to solve ***(M1)***

eg graph

0.956961

0.957 ***A1 N1***

[3 marks]

21. Runners in an athletics club have season's best times for the 100 m, which can be modelled by a normal distribution with mean 11.6 seconds and standard deviation 0.8 seconds. To qualify for a particular competition a runner must have a season's best time of under 11 seconds. A runner from this club who has qualified for the competition is selected at random. Find the probability that he has a season's best time of under 10.7 seconds. *[6 marks]*

Markscheme

$T \sim \mathrm{N}\left(11.6,\, 0.8^2\right)$

$\mathrm{P}\left(T < 10.7 \mid T < 11\right)$ ***(M1)***

$= \frac{\mathrm{P}(T<10.7 \cap T<11)}{\mathrm{P}(T<11)}$ ***(M1)***

$= \frac{\mathrm{P}(T<10.7)}{\mathrm{P}(T<11)}$ ***(M1)***

$\mathrm{P}\left(T < 10.7\right) = 0.1302\ldots$ ***(A1)***

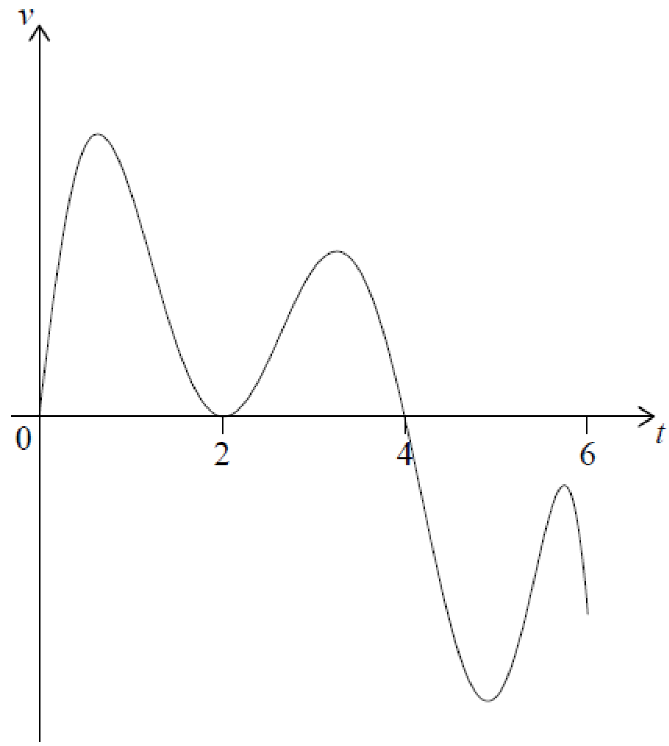
$\mathrm{P}\left(T < 11\right) = 0.2266\ldots$ ***(A1)***

$\mathrm{P}\left(T < 10.7 \mid T < 11\right) = 0.575$ ***A1***

Note: Accept only 0.575.

[6 marks]

A particle P starts from point O and moves along a straight line. The graph of its velocity, v ms^{-1} after t seconds, for $0 \le t \le 6$, is shown in the following diagram.



The graph of v has t-intercepts when t = 0, 2 and 4.

The function $s\left(t\right)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s\left(2\right) = s\left(5\right)$ and $\int_2^4 v\,\mathrm{d}t = 9$.

22a. Find the value of $s\left(4\right) - s\left(2\right)$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing relationship between v and s ***(M1)***

eg $\int v = s,\ s' = v$

$s\left(4\right) - s\left(2\right) = 9$ ***A1 N2***

[2 marks]

22b. Find the total distance travelled in the first 5 seconds. *[5 marks]*

Markscheme

correctly interpreting distance travelled in first 2 seconds (seen anywhere, including part (a) or the area of 15 indicated on diagram) ***(A1)***

eg $\int_0^2 v = 15,\ s(2) = 15$

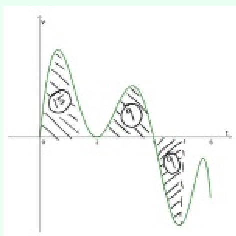
valid approach to find total distance travelled ***(M1)***

eg sum of 3 areas, $\int_0^4 v + \int_4^5 v,$ shaded areas in diagram between 0 and 5

Note: Award ***M0*** if only $\int_0^5 |v|$ is seen.

correct working towards finding distance travelled between 2 and 5 (seen anywhere including within total area expression or on diagram) ***(A1)***

eg $\int_2^4 v - \int_4^5 v,\ \int_2^4 v = \int_4^5 |v|,\ \int_4^5 v\,\mathrm{d}t = -9,\ s(4) - s(2) - [s(5) - s(4)],$

equal areas 

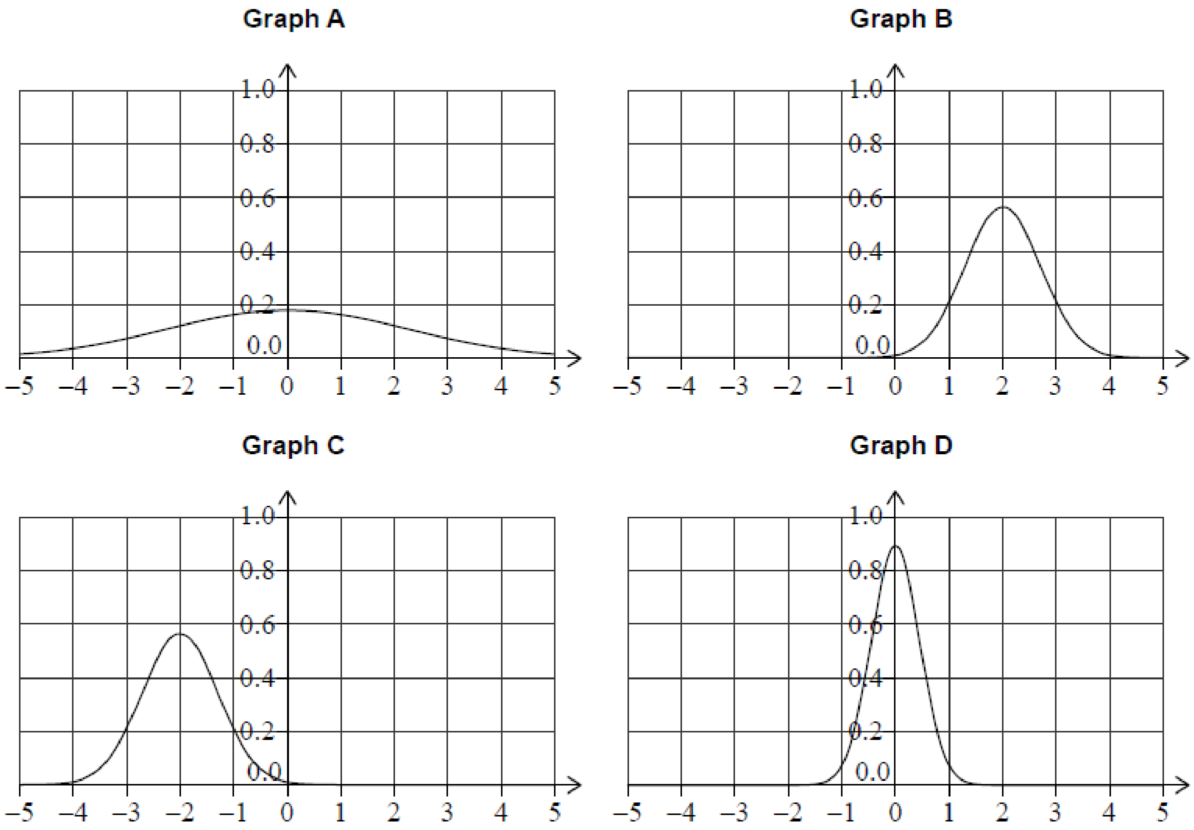
correct working using $s(5) = s(2)$ ***(A1)***

eg $15 + 9 - (-9),\ 15 + 2[s(4) - s(2)],\ 15 + 2(9),\ 2 \times s(4) - s(2),$ $48 - 15$

total distance travelled = 33 (m) ***A1 N2***

[5 marks]

Consider the following graphs of normal distributions.



23a. In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation. *[2 marks]*

Mean and standard deviation	Graph
Mean $= -2$; standard deviation $= 0.707$	
Mean $= 0$; standard deviation $= 0.447$	

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Mean and standard deviation	Graph
Mean $= -2$; standard deviation $= 0.707$	C
Mean $= 0$; standard deviation $= 0.447$	D

(A1)(A1)

(C2)

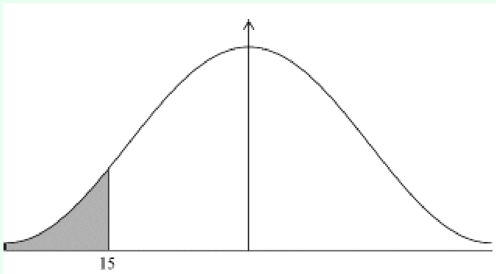
Note: Award ***(A1)*** for each correct entry.

[2 marks]

At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

23b. Find the probability that a suitcase weighs less than 15 kg. *[2 marks]*

Markscheme



(M1)

Note: Award ***(M1)*** for sketch with 15 labelled and left tail shaded **OR** for a correct probability statement, $P(X < 15)$.

0.0766 (0.0765637…, 7.66%) ***(A1) (C2)***

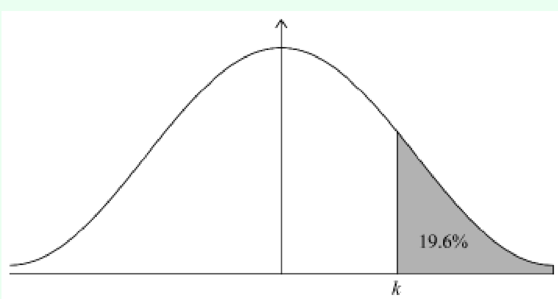
[2 marks]

23c. Any suitcase that weighs more than k kg is identified as excess baggage. *[2 marks]*

19.6 % of the suitcases at this airport are identified as excess baggage.

Find the value of k.

Markscheme



(M1)

Note: Award ***(M1)*** for a sketch showing correctly shaded region to the right of the mean with 19.6% labelled (accept shading of the complement with 80.4% labelled) **OR** for a correct probability statement, $P(X > k) = 0.196$ or $P(X \leq k) = 0.804$.

23.0 (kg) (22.9959… (kg)) ***(A1) (C2)***

[2 marks]

Let X be a random variable which follows a normal distribution with mean μ. Given that $\mathrm{P}\left(X < \mu - 5\right) = 0.2$, find

24a. $\mathrm{P}\left(X > \mu + 5\right)$. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of symmetry *eg* diagram ***(M1)***

$\mathrm{P}\left(X > \mu + 5\right) = 0.2$ ***A1***

[2 marks]

24b. $\mathrm{P}\left(X < \mu + 5 \mid X > \mu - 5\right)$. *[5 marks]*

Markscheme

EITHER

$\mathrm{P}\left(X < \mu + 5 \mid X > \mu - 5\right) = \frac{\mathrm{P}(X > \mu - 5 \cap X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(M1)***

$= \frac{\mathrm{P}(\mu - 5 < X < \mu + 5)}{\mathrm{P}(X > \mu - 5)}$ ***(A1)***

$= \frac{0.6}{0.8}$ ***A1A1***

Note: ***A1*** for denominator is independent of the previous ***A*** marks.

OR

use of diagram ***(M1)***

Note: Only award ***(M1)*** if the region $\mu - 5 < X < \mu + 5$ is indicated and used.

$\mathrm{P}\left(X > \mu - 5\right) = 0.8$ $\mathrm{P}\left(\mu - 5 < X < \mu + 5\right) = 0.6$ ***(A1)***

Note: Probabilities can be shown on the diagram.

$= \frac{0.6}{0.8}$ ***M1A1***

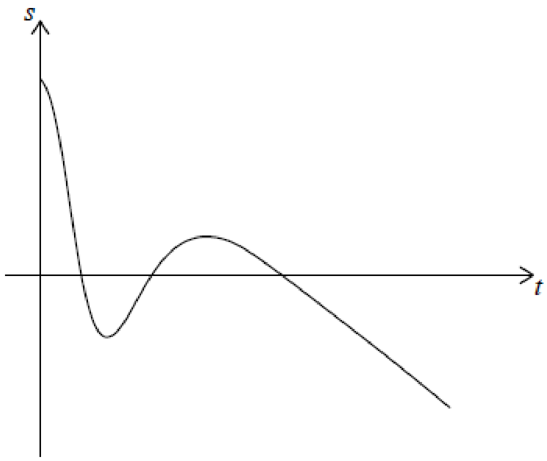
THEN

$= \frac{3}{4} = (0.75)$ ***A1***

[5 marks]

In this question distance is in centimetres and time is in seconds.

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_{\text{A}} = 15 - t - 6t^3\text{e}^{-0.8t}$, 0 ≤ t ≤ 25. This is shown in the following diagram.



25a. Find the initial displacement of particle A from point P. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach ***(M1)***

eg $s_{\text{A}}(0)$, $s(0)$, $t = 0$

15 (cm) ***A1 N2***

[2 marks]

25b. Find the value of t when particle A first reaches point P. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $s_{\text{A}} = 0,\ s = 0,\ 6.79321,\ 14.8651$

2.46941

t = 2.47 (seconds) ***A1 N2***

[2 marks]

25c. Find the value of t when particle A first changes direction. *[2 marks]*

Markscheme

recognizing when change in direction occurs ***(M1)***

eg slope of s changes sign, $s' = 0$, minimum point, 10.0144, (4.08, −4.66)

4.07702

t = 4.08 (seconds) ***A1 N2***

[2 marks]

25d. Find the total distance travelled by particle A in the first 3 seconds. *[3 marks]*

Markscheme

METHOD 1 (using displacement)

correct displacement or distance from P at $t = 3$ (seen anywhere) ***(A1)***

eg −2.69630, 2.69630

valid approach ***(M1)***

eg 15 + 2.69630, $s(3) - s(0)$, −17.6963

17.6963

17.7 (cm) ***A1 N2***

METHOD 2 (using velocity)

attempt to substitute either limits or the velocity function into distance formula involving $|v|$ ***(M1)***

eg $\int_0^3 |v|\,\mathrm{d}t$, $\int \left|-1 - 18t^2\mathrm{e}^{-0.8t} + 4.8t^3\mathrm{e}^{-0.8t}\right|$

17.6963

17.7 (cm) ***A1 N2***

[3 marks]

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_{\mathrm{B}} = 8 - 2t$, 0 ≤ t ≤ 25.

25e. Given that particles A and B start at the same point, find the displacement function s_{B} for particle B. *[5 marks]*

Markscheme

recognize the need to integrate velocity ***(M1)***

eg $\int v(t)$

$8t - \frac{2t^2}{2} + c$ (accept x instead of t and missing c) ***(A2)***

substituting initial condition into their integrated expression (must have c) ***(M1)***

eg $15 = 8(0) - \frac{2(0)^2}{2} + c, \quad c = 15$

$s_{\mathrm{B}}(t) = 8t - t^2 + 15$ ***A1 N3***

[5 marks]

25f. Find the other value of t when particles A and B meet. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $s_{\mathrm{A}} = s_{\mathrm{B}}$, sketch, (9.30404, 2.86710)

9.30404

$t = 9.30$ (seconds) ***A1 N2***

Note: If candidates obtain $s_{\mathrm{B}}(t) = 8t - t^2$ in part (e)(i), there are 2 solutions for part (e)(ii), 1.32463 and 7.79009. Award the last ***A1*** in part (e)(ii) only if both solutions are given.

[2 marks]