Revision - kinematics + crv *[264 marks]*

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \leq t \leq 5$.

1a. Find the maximum distance of the particle from O. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

1b. Find the acceleration of the particle at the instant it first changes direction. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

2a. Find P($24.15 < X < 25$). *[2 marks]*

2b. Find σ, the standard deviation of X. *[3 marks]*

2c. Hence, find the probability that a seashell selected at random has a length greater than 26 mm. *[2 marks]*

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

2d. Find E(Y). *[3 marks]*

2e. Find the probability that exactly three of these seashells have a length greater than 26 mm. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2f. A seashell selected at random has a length less than 26 mm. *[3 marks]*

Find the probability that its length is between 24.15 mm and 25 mm.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

In a city, the number of passengers, X, who ride in a taxi has the following probability distribution.

x	1	2	3	4	5
$P(X=x)$	0.60	0.30	0.03	0.05	0.02

After the opening of a new highway that charges a toll, a taxi company introduces a charge for passengers who use the highway. The charge is $ 2.40 per taxi plus $ 1.20 per passenger. Let T represent the amount, in dollars, that is charged by the taxi company per ride.

3a. Find E(T). *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

3b. Given that Var(X) = 0.8419, find Var(T). *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

A particle moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by

$v = 4t^2 - 6t + 9 - 2\sin(4t), 0 \leq t \leq 1.$

The particle's acceleration is zero at $t = T$.

4a. Find the value of T. *[2 marks]*

.....

.....

.....

.....

.....

.....

4b. Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$. *[3 marks]*

Show that $s_2 > s_1$.

.....

.....

.....

.....

.....

.....

.....

.....

.....

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

5a. By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. *[4 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Use $\mu = 32.29$ in the remainder of the question.

5b. Find the 86th percentile time to complete the jigsaw puzzle. *[2 marks]*

5c. Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. *[2 marks]*

Six randomly chosen people complete the jigsaw puzzle.

5d. Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5e. Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle. *[4 marks]*

Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The continuous random variable X has probability density function

$$f(x)=\begin{cases} \frac{k}{\sqrt{4-3x^2}}, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

6a. Find the value of k. *[4 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

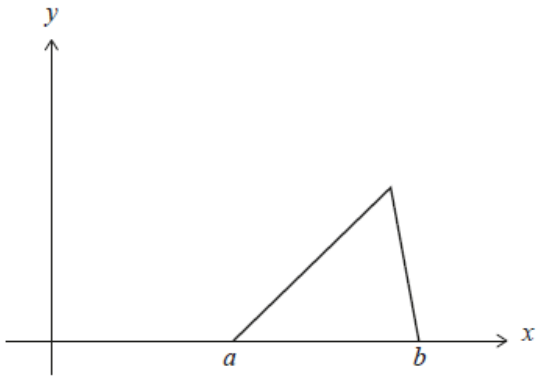
.....

6b. Find $\mathrm{E}(X)$. *[4 marks]*

7. A continuous random variable X has the probability density function *[6 marks]*

$$f(x)=\begin{cases} \frac{2}{(b-a)(c-a)}(x-a), & a \leq x \leq c \\ \frac{2}{(b-a)(b-c)}(b-x), & c < x \leq b \\ 0, & \text{otherwise} \end{cases}.$$

The following diagram shows the graph of $y = f(x)$ for $a \leq x \leq b$.



Given that $c \geq \frac{a+b}{2}$, find an expression for the median of X in terms of a, b and c.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A particle moves along a straight line so that its velocity, $v\ \mathrm{m\,s^{-1}}$, after t seconds is given by $v(t) = \mathrm{e}^{\sin t} + 4\sin t$ for $0 \leq t \leq 6$.

8a. Find the value of t when the particle is at rest. *[2 marks]*

8b. Find the acceleration of the particle when it changes direction. *[3 marks]*

8c. Find the total distance travelled by the particle. *[2 marks]*

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

9a. Find the probability that a randomly selected chocolate muffin weighs less than 61 g. *[2 marks]*

9b. In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

9c. Find the probability that the randomly selected muffin weighs less than 61 g. *[4 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

9d. Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

9e. Find the value of σ. *[5 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

A particle moves in a straight line such that its velocity, $v\ \mathrm{m\,s^{-1}}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4},\ 0 \leq t \leq 3$.

10a. Determine when the particle changes its direction of motion. *[2 marks]*

10b. Find the times when the particle's acceleration is $-1.9\ \mathrm{m\,s^{-2}}$. *[3 marks]*

10c. Find the particle's acceleration when its speed is at its greatest. *[2 marks]*

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

11a. Find the value of σ. *[4 marks]*

11b. On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. *[2 marks]*

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

11c. Find the probability that she will be late to work at least one day next week. *[3 marks]*

11d. Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. *[5 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

11e. Find the probability that Suzi will receive a bonus. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

12. Rachel and Sophia are competing in a javelin-throwing competition. *[7 marks]*

The distances, R metres, thrown by Rachel can be modelled by a normal distribution with mean 56.5 and standard deviation 3.

The distances, S metres, thrown by Sophia can be modelled by a normal distribution with mean 57.5 and standard deviation 1.8.

In the first round of competition, each competitor must have five throws. To qualify for the next round of competition, a competitor must record at least one throw of 60 metres or greater in the first round.

Find the probability that only one of Rachel or Sophia qualifies for the next round of competition.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A particle P moves along the x-axis. The velocity of P is $v\ \mathrm{m\ s^{-1}}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0,\ P$ is at the origin O.

13a. Find the value of t when P reaches its maximum velocity. *[2 marks]*

.....

.....

.....

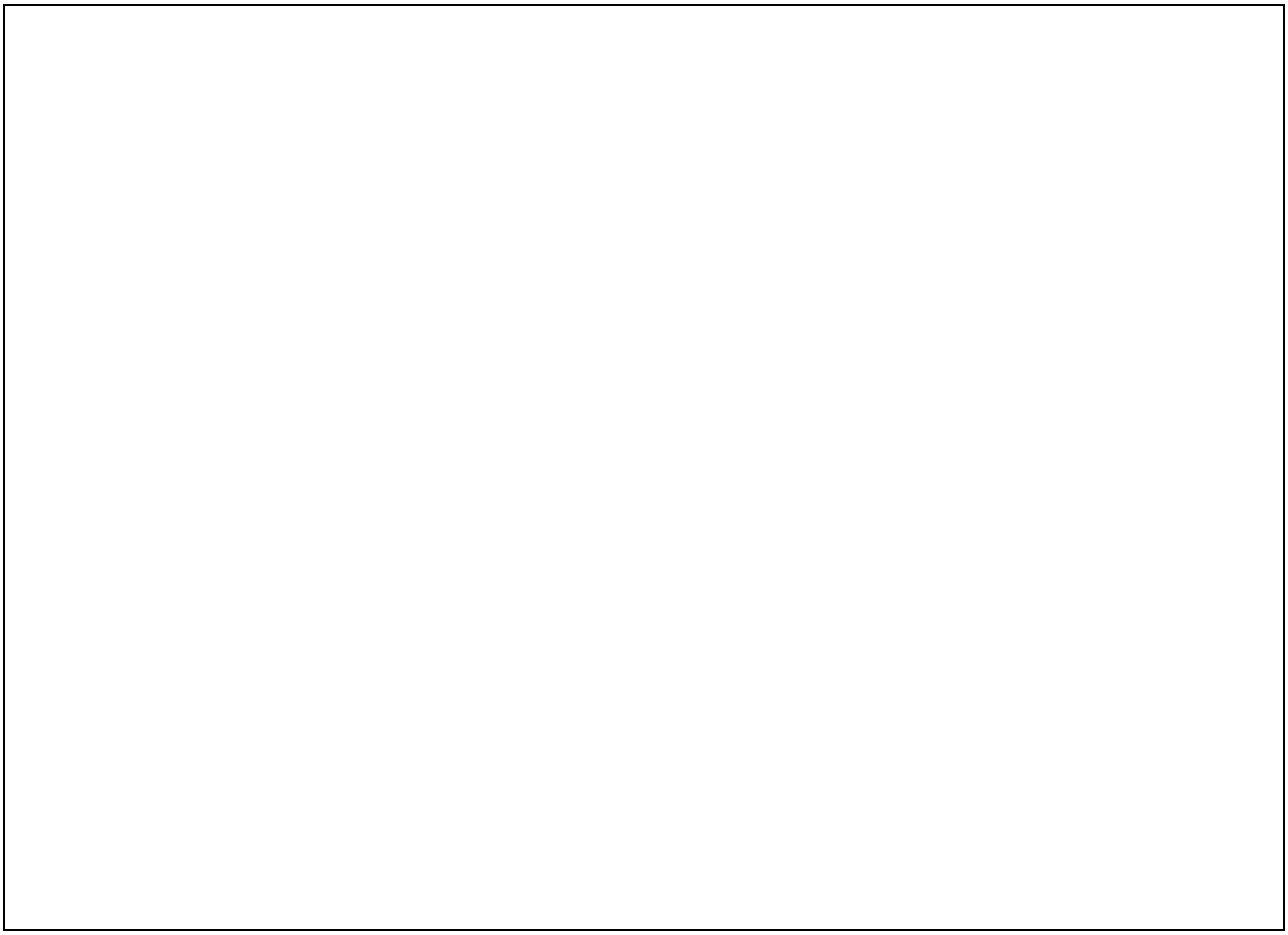
.....

.....

.....

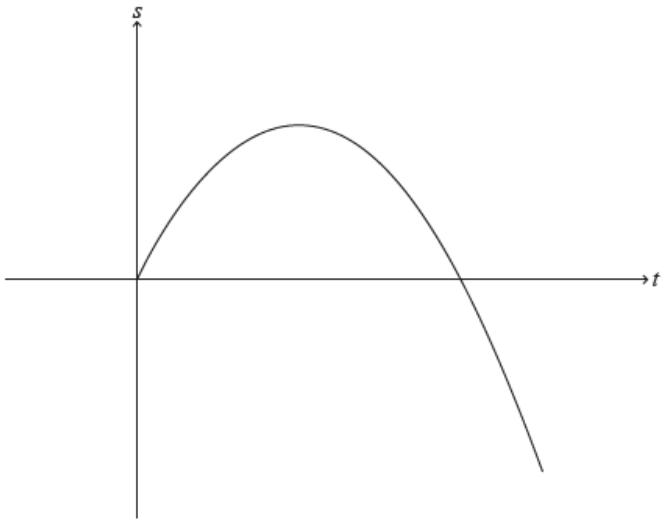
13b. Show that the distance of P from O at this time is $\frac{88}{27}$ metres. *[5 marks]*

13c. Sketch a graph of v against t, clearly showing any points of intersection with the axes. *[4 marks]*



13d. Find the total distance travelled by P. *[5 marks]*

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

14a. Find the value of p. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Particle A changes direction when $t = q$.

14b. Find the value of q. *[2 marks]*

14c. Find the displacement of particle A from the origin when $t = q$. *[2 marks]*

14d. Find the distance of particle A from the origin when $t = 10$. *[2 marks]*

The total distance travelled by particle A is given by d.

14e. Find the value of d. *[2 marks]*

14f. A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$. *[4 marks]*

When $t = k$, the distance travelled by particle B is equal to d.

Find the value of k.

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

15a. Find the probability that a bag selected at random is rejected. *[2 marks]*

15b. Estimate the number of bags which will be rejected from a random sample of 100 bags. *[1 mark]*

15c. Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. *[3 marks]*

A particle P moves in a straight line such that after time t seconds, its velocity, v in $\mathrm{m\,s^{-1}}$, is given by $v = \mathrm{e}^{-3t} \sin 6\,t$, where $0 < t < \frac{\pi}{2}$.

16a. Find the times when P comes to instantaneous rest. *[2 marks]*

At time t, P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

16b. Find an expression for s in terms of t. *[7 marks]*

16c. Find the maximum displacement of P, in metres, from its initial position. *[2 marks]*

16d. Find the total distance travelled by P in the first 1.5 seconds of its motion. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

At successive times when the acceleration of P is$0\ \mathrm{m\,s^{-2}}$, the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1, t_2, t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1, v_2, v_3.

16e. Show that, at these times, $\tan 6t = 2$. *[2 marks]*

.....

.....

.....

.....

.....

.....

16f. Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -\mathrm{e}^{-\frac{\pi}{2}}$. *[5 marks]*

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean 102 g and standard deviation 8 g.

17a. Find the probability that a randomly selected packet has a weight less than 100 g. *[2 marks]*

17b. The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. *[2 marks]*

17c. A packet is randomly selected. Given that the packet has a weight greater than 105 g, find the probability that it has a weight greater than 110 g. *[3 marks]*

17d. From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. *[3 marks]*

17e. Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than 95 g. *[4 marks]*

Let X and Y be normally distributed with $X \sim \mathrm{N}\left(14, a^2\right)$ and $Y \sim \mathrm{N}\left(22, a^2\right)$, $a > 0$.

18a. Find b so that $\mathrm{P}(X > b) = \mathrm{P}(Y < b)$. *[2 marks]*

18b. It is given that $\mathrm{P}(X > 20) = 0.112$. *[4 marks]*

Find $\mathrm{P}(16 < Y < 28)$.

A rocket is travelling in a straight line, with an initial velocity of $140\,\text{m s}^{-1}$. It accelerates to a new velocity of $500\,\text{m s}^{-1}$ in two stages.

During the first stage its acceleration, $a\,\text{m s}^{-2}$, after t seconds is given by $a(t) = 240\sin(2t)$, where $0 \leqslant t \leqslant k$.

19a. Find an expression for the velocity, $v\,\text{m s}^{-1}$, of the rocket during the first stage. *[4 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

The first stage continues for k seconds until the velocity of the rocket reaches 375 $\mathrm{m\,s^{-1}}$.

19b. Find the distance that the rocket travels during the first stage. *[4 marks]*

19c. During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage. *[6 marks]*

Find the total time taken for the two stages.

SpeedWay airline flies from city A to city B. The flight time is normally distributed with a mean of 260 minutes and a standard deviation of 15 minutes.

A flight is considered late if it takes longer than 275 minutes.

20a. Calculate the probability a flight is **not** late. *[2 marks]*

The flight is considered to be **on time** if it takes between m and 275 minutes. The probability that a flight is on time is 0.830.

20b. Find the value of m. *[3 marks]*

During a week, SpeedWay has 12 flights from city A to city B. The time taken for any flight is independent of the time taken by any other flight.

20c. Calculate the probability that at least 7 of these flights are **on time**. *[3 marks]*

20d. Given that at least 7 of these flights are on time, find the probability that exactly 10 flights are on time. *[4 marks]*

20e. SpeedWay increases the number of flights from city A to city B to 20 *[3 marks]*

flights each week, and improves their efficiency so that more flights are on time. The probability that at least 19 flights are on time is 0.788.

A flight is chosen at random. Calculate the probability that it is on time.

21. Runners in an athletics club have season's best times for the 100 m, which can be modelled by a normal distribution with mean 11.6 seconds and standard deviation 0.8 seconds. To qualify for a particular competition a runner must have a season's best time of under 11 seconds. A runner from this club who has qualified for the competition is selected at random. Find the probability that he has a season's best time of under 10.7 seconds. *[6 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

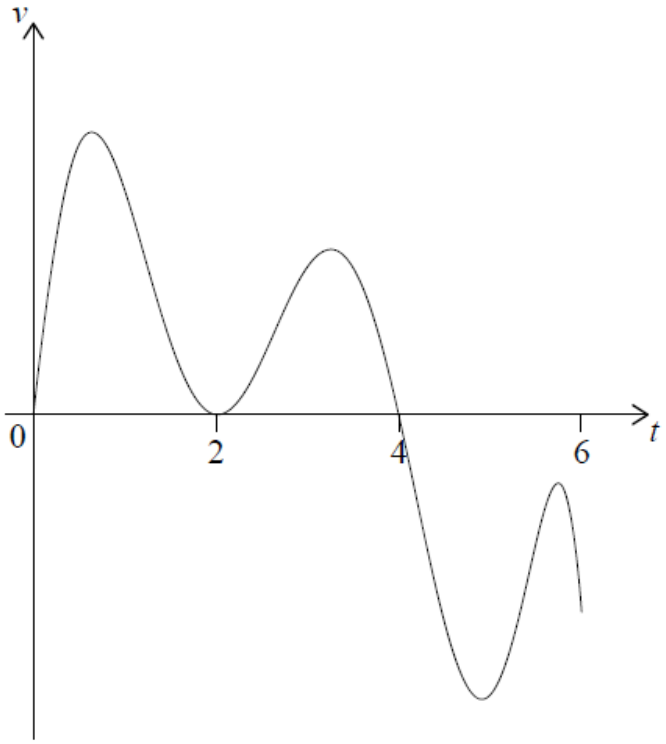
.....

.....

.....

.....

A particle P starts from point O and moves along a straight line. The graph of its velocity, v ms^{-1} after t seconds, for $0 \leq t \leq 6$, is shown in the following diagram.



The graph of v has t-intercepts when t = 0, 2 and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v \, \mathrm{d}t = 9$.

22a. Find the value of $s(4) - s(2)$. *[2 marks]*

.....

.....

.....

.....

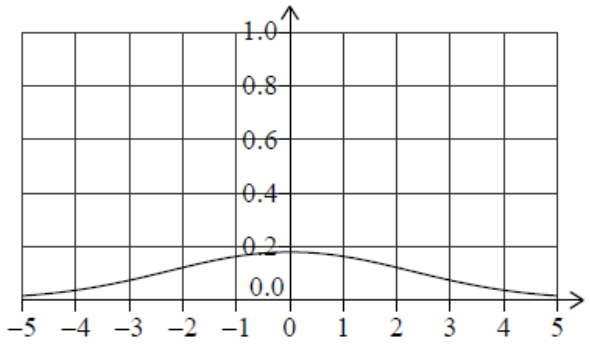
.....

.....

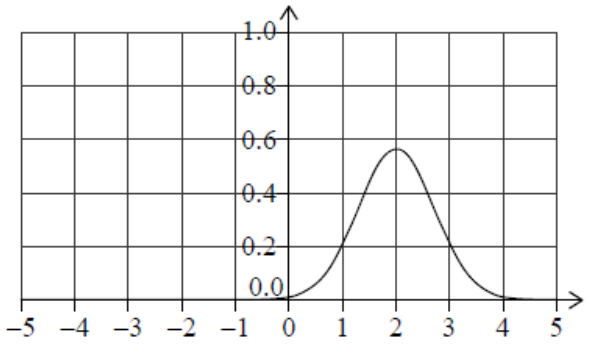
22b. Find the total distance travelled in the first 5 seconds. *[5 marks]*

Consider the following graphs of normal distributions.

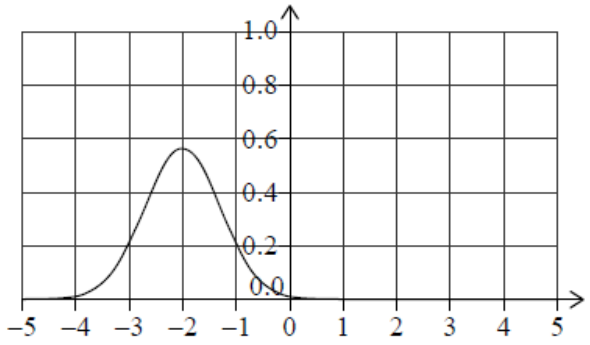
Graph A



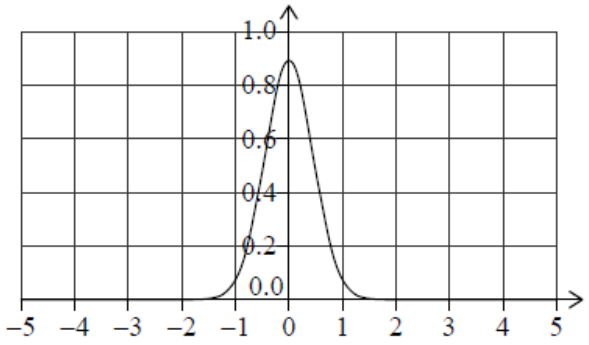
Graph B



Graph C



Graph D



23a. In the following table, write down the letter of the corresponding graph next to the given mean and standard deviation. *[2 marks]*

Mean and standard deviation	Graph
Mean = −2; standard deviation = 0.707	
Mean = 0; standard deviation = 0.447	

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

At an airport, the weights of suitcases (in kg) were measured. The weights are normally distributed with a mean of 20 kg and standard deviation of 3.5 kg.

23b. Find the probability that a suitcase weighs less than 15 kg. *[2 marks]*

23c. Any suitcase that weighs more than k kg is identified as excess baggage. *[2 marks]*
19.6 % of the suitcases at this airport are identified as excess baggage.

Find the value of k.

Let X be a random variable which follows a normal distribution with mean μ. Given that $\mathrm{P}(X < \mu - 5) = 0.2$, find

24a. $\mathrm{P}(X > \mu + 5)$. *[2 marks]*

.....

.....

.....

.....

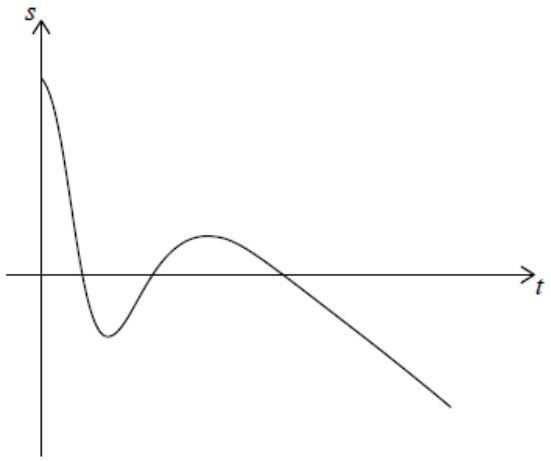
.....

.....

24b. $\mathrm{P}\left(X < \mu + 5 \mid X > \mu - 5\right)$. *[5 marks]*

In this question distance is in centimetres and time is in seconds.

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_{\mathrm{A}} = 15 - t - 6t^3\mathrm{e}^{-0.8t}$, $0 \leq t \leq 25$. This is shown in the following diagram.



25a. Find the initial displacement of particle A from point P. *[2 marks]*

25b. Find the value of t when particle A first reaches point P. *[2 marks]*

25c. Find the value of t when particle A first changes direction. *[2 marks]*

25d. Find the total distance travelled by particle A in the first 3 seconds. *[3 marks]*

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_{\mathrm{B}} = 8 - 2t$, $0 \leq t \leq 25$.

25e. Given that particles A and B start at the same point, find the displacement function s_{B} for particle B. *[5 marks]*

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

25f. Find the other value of t when particles A and B meet. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .