SL 1.2 Percentage error

You have to be able to calculate the percentage error of an approximated or estimated answer.

We will go through a few examples. Calculating the percentage error is just a matter of substituting numbers into a formula. The only part that may cause problems is figuring out which value is the exact value and which is the approximated value.

Error

Error (ϵ) is the absolute difference between the approximated value and the exact value:

$$\epsilon = |v_A - v_E|$$

Example: Basket contains 36 apples. I estimate the number of apples in the basket to be 30. In this case the exact value (v_E) is 36, the approximated value is (v_A) is 30.
The error is

$$\epsilon = |v_A - v_E| = 6$$

Note that the absolute value (the $| \ |$ brackets) guarantees that the answer is always positive.

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Two examples

Let's say we have two experiments. In the first one the exact answer should be 1.23, but the approximated answers comes out to be 0.61. In the other experiment the exact answers should be 321 and the approximated answer is 323.

If we calculate the error in both approximations we see that in the first case $\epsilon_1 = 0.62$ and in the second case we get $\epsilon_2 = 2$. The error in the second experiment is larger, but one could (and should) argue that in the second scenario the error is almost insignificant and in the first we got less than a half of the right answer. This is why, we often want to express the error in relation to the exact answer.

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Percentage error

The formula for percentage error is as follows:

$$\epsilon_{\%} = \left|\frac{v_A - v_E}{v_E}\right| \cdot 100\%$$

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Percentage error

Let's go back to the two examples. In the first scenario we had:

$$v_E = 1.23$$
$$v_A = 0.61$$

So the percentage error is:

$$\epsilon_\% = \left|\frac{v_A - v_E}{v_E}\right| \cdot 100\% = \left|\frac{0.61 - 1.23}{1.23}\right| \cdot 100\% = 50.4\%$$

In the second scenario we had:

$$v_E = 321$$
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These answers tell us much more about the relative size of the error. In the first case the error was very large compared to the quantities involved in the problem. In the second case it was very small.

On the next slides we will go through the exercises 1 to 4 from the page I've uploaded on the site.

Exercise 1

We first want to calculate the exact answer:

$$v_E = 3a + b^3 = 119.423$$

Because we want the **exact** answer, we should never round this answer!

The estimate (approximated answer) is 140, i.e. $v_A = 140$. We can now plug in those values into the percentage error formula:

$$\epsilon_{\%} = \left|\frac{v_A - v_E}{v_E}\right| \cdot 100\% = \left|\frac{140 - 119.423}{119.423}\right| \cdot 100\% = 17.2\%$$

The percentage error should be given correct to 3 s.f. (or exactly).

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Exercise 2

We first need to calculate the final grade, which will be our exact answer:

$$v_E = \frac{8.3 + 6.8 + 9.4}{3} = 8\frac{1}{6}$$

Again this is the exact answer, so no rounding here! Now the approximated answers is calculated by first rounding the marks to the nearest unit. So the rounded marks are: 8, 7 and 9. The approximated answer is then:

$$v_A = \frac{8 + 7 + 9}{3} = 8$$

The percentage error is:

$$\epsilon_{\%} = \left|\frac{v_A - v_E}{v_E}\right| \cdot 100\% = \left|\frac{8 - 8\frac{1}{6}}{8\frac{1}{6}}\right| \cdot 100\% = 2.04\%$$

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Exercise 3

First the exact area:

$$v_E = 5.34m \cdot 3.48m = 18.5832m^2$$

The dimensions correct to 1 d.p. are $5.3m$ and $3.5m$. So the approximated area is:

$$v_A = 5.3m \cdot 3.5m = 18.55m^2$$

This gives the percentage error of the approximation as:

$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{18.55 - 18.5832}{18.5832}| \cdot 100\% = 0.179\%$$

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Exercise 4

The area of a circle is πr^2, so we want to solve $\pi r^2 = 89$, which gives $r = \sqrt{\frac{89}{\pi}} = 5.323$ (correct to 3 decimal places).

The perimeter of a circle is given by $2\pi r$, so the perimeter is $2\pi \cdot \sqrt{\frac{89}{\pi}} = 33.4$ (3 s.f.).

We have $v_E = 33.4$ and $v_A = 30$, so the percentage error is:

$$\epsilon_{\%} = |\frac{v_A - v_E}{v_E}| \cdot 100\% = |\frac{30 - 33.4}{33.4}| \cdot 100\% = 10\%$$

The final answer is given to 2 s.f. (as the question required).

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Exercise 4

Note that in general you should never round exact answers. Exercise 4 may seem different in this regard, but it's not. For part (b) we are not asked to calculate the exact answer, the precision is not specified, so we calculate correct to 3 s.f. In part (c) we are told to use our answer to part (b) as the exact answer. This is why, we can use the rounded answer.

In case of any questions you can email me at T.J.Lechowski@gmail.com.