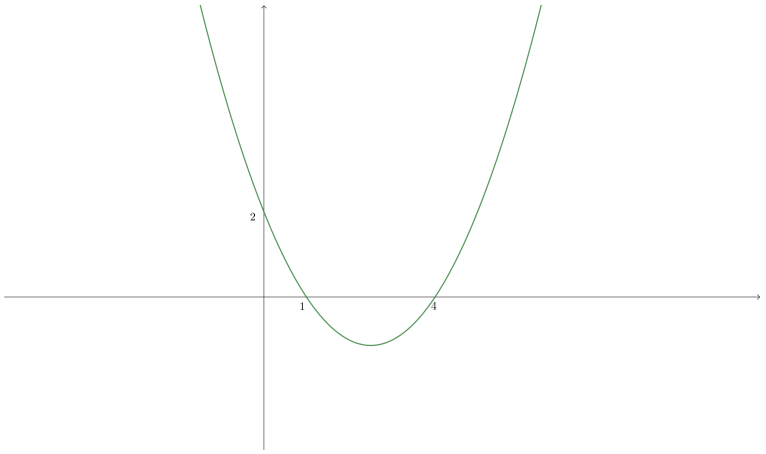
1. The following diagram shows part of a graph of a quadratic function:



a) Find the equation of the function.

b) The graph has been reflected in y-axis and then translated by the vector $\binom{1}{2}$. Find the equation of the resulting graph in the standard form.

2. Let $f(x) = x^2 - 2x - 3$ and $g(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 1$.

a) On the same set of axes sketch the graphs of $y = f(x)$ and $y = g(x)$. Clearly indicate all axes intercepts, vertices and points of intersections.

b) Find a sequence of transformations that maps the graph of $f(x)$ onto the graph of $g(x)$.

3. Let α and β be the roots of equation

$$3x^2 - x - 4 = 0$$

Find an equation with integer coefficients whose roots are:

i) α^3 and β^3.

ii) $\dfrac{1}{\alpha^2}$ and $\dfrac{1}{\beta^2}$.

4. Find equations of lines that are tangent to the parabola $y = x^2 + 2x$ and which pass through the point $(2, 7)$. Find the coordinates of the points of tangency of both of these lines with the parabola.

5. Find the set of all possible values of parameter m such that the equation:

$$(m-5)x^2 - 4mx + m - 2 = 0$$

has two solutions x_1 and x_2 such that $x_1 < 1 < x_2$.

6. Solve the following equations and inequalities:

a) $(x^2 + 2x - 4)^2 - 3(x^2 + 2x - 4) = 4$

b) $x - 3\sqrt{x-2} = 12$

c) $\dfrac{2x+1}{x-3} \geqslant 9$

7. Find the set of all possible values of parameter m for which the equation:

$$(m-2)x^4 - 2(m+3)x^2 + m + 1 = 0$$

has four distinct solutions.

8. A piece of wire of length 1 metre is used to make

a) a rectangle with ratio of sides 2:1 and a square,

b) a right isosceles triangle and a circle,

calculate how the wire should be divided between the two figures so that their total area is minimal.

9. A shop sells apples for 3 zł. per kg. At that price the shop sells 400 kg of apples per day. It was observed that each reduction in price of 10 gr. corresponds to an increase of sales by 100 kg. Given that the shop pays 1.40 zł per kg for purchasing the apples and other associated costs (storage etc.), find the optimal price the shopkeeper should set.