revision Sequences+Financial Maths+Rounding *[168 marks]*

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

1a. Write down the value of the common difference, d *[1 mark]*

Markscheme

$(d =) -250$ ***A1***

[1 mark]

1b. Calculate the price of a ticket in the 16th row. *[2 marks]*

Markscheme

$(u_{16} =)\, 6800 + (16 - 1)\,(-250)$ ***M1***

(¥)3050 ***A1***

[2 marks]

1c. Find the total cost of buying 2 tickets in each of the first 16 rows. *[3 marks]*

Markscheme

$(S_{16} =)\left(\frac{16}{2}\right)(2 \times 6800 + (16 - 1)\,(-\,250)) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula.
Award ***M1*** for multiplication by 2 seen.

OR

$(S_{16} =)\left(\frac{16}{2}\right)(6800 + 3050) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula.
Award ***M1*** for multiplication by 2 seen.

(¥)158 000 (157 600) ***A1***

[3 marks]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

2a. Find the repayment made each quarter. *[3 marks]*

Markscheme

N = 24
I % = 14
PV = −14000
FV = 0
P/Y = 4
C/Y = 4 ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept PV = 14000.

(€)871.82 ***A1***

[3 marks]

2b. Find the total amount paid for the car. *[2 marks]*

Markscheme

$4 \times 6 \times 871.82$ ***(M1)***

(€) 20923.68 ***A1***

[2 marks]

2c. Find the interest paid on the loan. *[2 marks]*

Markscheme

$20923.68 - 14000$ ***(M1)***

(€) 6923.68 ***A1***

[2 marks]

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**. Terms of the loan require a 10 % deposit and monthly repayments of €250.

2d. Find the amount to be borrowed for this option. *[2 marks]*

Markscheme

0.9×14000 $(= 14000 - 0.10 \times 14000)$ ***M1***

(€) 12600.00 ***A1***

[2 marks]

2e. Find the annual interest rate, r. *[3 marks]*

Markscheme

N = 72

PV = 12600

PMT = −250

FV = 0

P/Y = 12

C/Y = 12 ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept PV = −12600 provided PMT = 250.

12.56(%) ***A1***

[3 marks]

2f. State which option Bryan should choose. Justify your answer. *[2 marks]*

Markscheme

EITHER

Bryan should choose Option A ***A1***

no deposit is required ***R1***

Note: Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1***.

OR

Bryan should choose Option B ***A1***

cost of Option A (6923.69) > cost of Option B (72 × 250 − 12600 = 5400) ***R1***

Note: Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

[2 marks]

2g. Bryan's car depreciates at an annual rate of 25 % per year. *[3 marks]*

Find the value of Bryan's car six years after it is purchased.

Markscheme

$14\,000\left(1-\frac{25}{100}\right)^6$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula.
Award ***A1*** for correct substitutions.

= (€)2491.70 ***A1***

OR

N = 6

I% = −25

PV = ±14 000

P/Y = 1

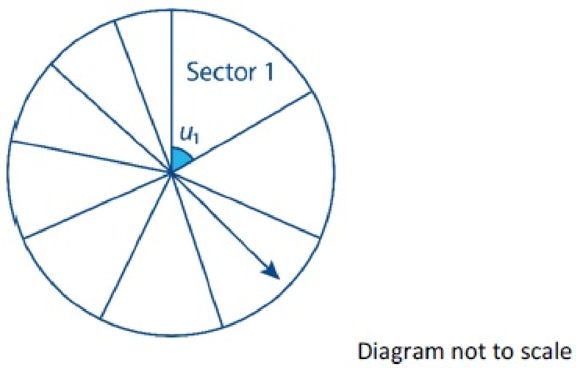
C/Y = 1 ***(A1)(M1)***

Note: Award ***A1*** for PV = ±14 000, ***M1*** for other entries correct.

(€)2491.70 ***A1***

[3 marks]

A disc is divided into 9 sectors, number 1 to 9. The angles at the centre of each of the sectors u_n form an arithmetic sequence, with u_1 being the largest angle.



3a. Write down the value of $\sum_{i=1}^{9} u_i$. *[1 mark]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

360° **A1**

[1 mark]

It is given that $u_9 = \frac{1}{3}u_1$.

3b. Find the value of u_1. *[4 marks]*

Markscheme

EITHER

$360 = \frac{9}{2}(u_1 + u_9)$ **M1**

$360 = \frac{9}{2}\left(u_1 + \frac{1}{3}u_1\right) = 6u_1$ **M1A1**

OR

$360 = \frac{9}{2}(2u_1 + 8d)$ **M1**

$u_9 = \frac{1}{3}u_1 = u_1 + 8d \Rightarrow u_1 = -12d$ **M1**

Substitute this value $360 = \frac{9}{2}\left(2u_1 - 8 \times \frac{u_1}{12}\right)\left(= \frac{9}{2} \times \frac{4}{3}u_1 = 6u_1\right)$ **A1**

THEN

$u_1 = 60^\circ$ **A1**

[4 marks]

Sophia pays $\$200$ into a bank account at the end of each month. The annual interest paid on money in the account is 3.1% which is compounded monthly.

4a. Find the value of her investment after a period of 5 years. *[3 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Number of time periods $12 \times 5 = 60$ **(A1)**

N = 60
I% = 3.1
PV = 0
PMT = 200
P/Y = 12
C/Y = 12
Value $(\$)12,961.91$ **(M1)A1**

[3 marks]

The average rate of inflation per year over the 5 years was 2%.

4b. Find an approximation for the real interest rate for the money invested in the account. *[2 marks]*

Markscheme

METHOD 1

Real interest rate $= 3.1 - 2.0 = 1.1\%$ **(M1)A1**

METHOD 2

$\frac{1+0.031}{1+0.02} = 1.01078\ldots$ **(M1)**

1.08% (accept 1.1%) **A1**

[2 marks]

4c. Hence find the real value of Sophia’s investment at the end of 5 years. *[2 marks]*

Markscheme

N = 60
I% = 1.1
PV = 0
PMT = 200
P/Y = 12
C/Y = 12
$(\$)12,300(12,330.33\ldots)$ **(M1)A1**

Note: Award **A1** for $\$12,300$ only.

[2 marks]

Give your answers to this question correct to two decimal places.

Gen invests $2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

5a. Calculate the value of her savings after 10 years. *[2 marks]*

Markscheme

$2400(1.04)^{10} = \$3552.59$ ***M1A1***

[2 marks]

5b. The rate of inflation during this 10 year period is 1.5% per year. *[3 marks]*

Calculate the real value of her savings after 10 years.

Markscheme

real interest rate = $4 - 1.5 = 2.5\%$ ***A1***

$2400(1.025)^{10} = \$3072.20$ ***M1A1***

[3 marks]

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of $4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

6a. Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. *[3 marks]*

Markscheme

Use of finance solver ***M1***

N = 360, I = 5%, Pmt = 4000, FV = 0, PpY = 12, CpY = 1 ***A1***

$755000 (correct to 3 s.f.) ***A1***

[3 marks]

6b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. *[3 marks]*

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

Markscheme

$N = 384$, $I = 5\%$, PV = 0, FV = 754638, PpY = 12, CpY = 1 ***M1A1***

$817 per month (correct to 3 s.f.) ***A1***

[3 marks]

Paul wants to buy a car. He needs to take out a loan for $7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay $200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

7a. the number of months it will take for Paul to repay the loan. *[3 marks]*

Markscheme

evidence of using Finance solver on GDC ***M1***

$N = 39.8$ ***A1***

It will take 40 months ***A1***

[3 marks]

7b. the total amount that Paul has to pay. *[2 marks]*

Markscheme

$40 \times 200 = \$8000$ ***M1A1***

[2 marks]

Use option 2 to calculate

7c. the amount Paul pays each month. *[2 marks]*

Markscheme

Monthly payment = $316 ($315.70) ***M1A1***

[2 marks]

7d. the total amount that Paul has to pay. *[2 marks]*

Markscheme

$24 \times 315.7 = \$7580\ (\$7576.80)$ ***M1A1***

[2 marks]

Give a reason why Paul might choose

7e. option 1. *[1 mark]*

Markscheme

The monthly repayment is lower, he might not be able to afford $316 per month. ***R1***

[1 mark]

7f. option 2. *[1 mark]*

Markscheme

the total amount to repay is lower. ***R1***

[1 mark]

Sophie is planning to buy a house. She needs to take out a mortgage for $120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay $1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

8a. Calculate the monthly repayment using option 1. *[2 marks]*

Markscheme

evidence of using Finance solver on GDC ***M1***

Monthly payment = $785 ($784.60) ***A1***

[2 marks]

8b. Calculate the total amount Sophie would pay, using option 1. *[2 marks]*

Markscheme

$240 \times 785 = \$188000$ ***M1A1***

[2 marks]

8c. Calculate the number of months it will take to repay the mortgage using option 2. *[3 marks]*

Markscheme

$N = 180.7$ ***M1A1***

It will take 181 months ***A1***

[3 marks]

8d. Calculate the total amount Sophie would pay, using option 2. *[2 marks]*

Markscheme

$181 \times 1000 = \$\,181000$ ***M1A1***

[2 marks]

Give a reason why Sophie might choose

8e. option 1. *[1 mark]*

Markscheme

The monthly repayment is lower, she might not be able to afford $1000 per month. ***R1***

[1 mark]

8f. option 2. *[1 mark]*

Markscheme

the total amount to repay is lower. ***R1***

[1 mark]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

8g. Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years. *[2 marks]*

Markscheme

$74400 (accept $74300) ***M1A1***

[2 marks]

8h. Hence calculate her monthly repayment for the final 10 years. *[2 marks]*

Markscheme

Use of finance solver with *N* =120, *PV* = $74400, *I* = 7%   ***A1***

$855 (accept $854 − $856)   ***A1***

[2 marks]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

9a. Calculate the percentage increase in applications from the first year to the second year. *[2 marks]*

Markscheme

$\frac{12669-12300}{12300}\times 100$ ***(M1)***

3% ***A1***

[2 marks]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

9b. Write down the common ratio of the sequence. *[1 mark]*

Markscheme

1.03 ***A1***

Note: Follow through from part (a).

[1 mark]

9c. Find an expression for u_n. *[1 mark]*

Markscheme

$(u_n =)\ 12\,300 \times 1.03^{n-1}$ ***A1***

[1 mark]

9d. Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. *[2 marks]*

Markscheme

$(u_{11} =)\ 12\,300 \times 1.03^{10}$ ***(M1)***

16530 ***A1***

Note: Answer must be to the nearest integer. Do not accept 16500.

[2 marks]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

9e. Write down an expression for v_n . *[2 marks]*

Markscheme

$(v_n =)\ 10380 + 600(n-1)$ **OR** $600n + 9780$ ***M1A1***

Note: Award ***M1*** for substituting into arithmetic sequence formula, ***A1*** for correct substitution.

[2 marks]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

9f. Calculate the total amount of acceptance fees paid to the university in the first 10 years. *[3 marks]*

Markscheme

$80 \times \frac{10}{2}(2(10380)+9(600))$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying by 80 and ***(M1)*** for substitution into sum of arithmetic sequence formula.

$\$10\ 500\ 000 \quad (\$10\ 464\ 000)$ ***A1***

[3 marks]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

9g. Find k. *[3 marks]*

Markscheme

$12\,300 \times 1.03^{n-1} < 10\,380 + 600(n-1)$ or equivalent ***(M1)***

Note: Award ***(M1)*** for equating their expressions from parts (b) and (c).

EITHER

graph showing $y = 12\,300 \times 1.03^{n-1}$ and $y = 10\,380 + 600(n-1)$ ***(M1)***

OR

graph showing $y = 12\,300 \times 1.03^{n-1} - (10\,380 + 600(n-1))$ ***(M1)***

OR

list of values including, $(u_{n=})\ 17537$ and $(v_{n=})\ 17580$ ***(M1)***

OR

$12.4953\ldots$ from graphical method or solving numerical equality ***(M1)***

Note: Award ***(M1)*** for a valid attempt to solve.

THEN

$(k =)13$ ***A1***

[3 marks]

9h. State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. *[2 marks]*

Markscheme

this will not guarantee enough places. ***A1***

EITHER

A written statement that $u_n > v_n$, with range of n. ***R1***

Example: “when $n = 24$ (or greater), the number of applications will exceed the number of places again” (“$u_n > v_n,\ n \geq 24$”).

OR

exponential growth will always exceed linear growth ***R1***

Note: Accept an equivalent sketch. Do not award ***A1R0***.

[2 marks]

Katya approximates π, correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6+\frac{13}{16}}$$

10a. Calculate Katya’s approximation of π, correct to four decimal places. *[2 marks]*

Markscheme

$\pi \approx 3 + \frac{1}{6+\frac{13}{16}}$

$= 3.14678\ldots \left(\frac{343}{109}, 3\frac{16}{109}\right)$ ***(A1)***

$= 3.1468$ ***A1***

Note: Award ***A1*** for correct rounding to 4 decimal places. Follow through within this part.

[2 marks]

10b. Calculate the percentage error in using Katya’s four decimal place approximation of π, compared to the exact value of π in your calculator. *[2 marks]*

Markscheme

$\left|\frac{3.1468-\pi}{\pi}\right| \times 100$ ***(M1)***

Note: Award ***M1*** for substitution of their final answer in part (a) into the percentage error formula. Candidates should use the exact value of π from their GDC.

$= 0.166\,(\%)(0.165754\ldots)$ ***A1***

[2 marks]

Charlie and Daniella each began a fitness programme. On day one, they both ran 500 m. On each subsequent day, Charlie ran 100 m more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

Calculate how far

11a. Charlie ran on day 20 of his fitness programme. *[2 marks]*

Markscheme

attempt to find u_{20} using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ \ldots$

(Charlie ran) 2400 m ***A1***

[2 marks]

11b. Daniella ran on day 20 of her fitness programme. *[3 marks]*

Markscheme

$(r =) 1.02$ ***(A1)***

attempt to find u_{20} using a geometric sequence ***(M1)***

e.g. $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** $500, 510, 520.2, \ldots$

(Daniella ran) $728\ \mathrm{m} (728.405\ldots)$ ***A1***

[3 marks]

11c. On day n of the fitness programmes Daniella runs more than Charlie for the first time. *[3 marks]*

Find the value of n.

Markscheme

$500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215\ldots$

$n = 185$ ***A1***

[3 marks]

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

12a. Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places. *[3 marks]*

Markscheme

METHOD 1

$N = 5$ **OR** $N = 10$
$I\% = 2.75$ $I\% = 2.75$
$PV = -1500$ $PV = -1500$
$PMT = 0$ $PMT = 0$
$P/Y = 1$ $P/Y = 2$
$C/Y = 2$ $C/Y = 2$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, ***A1*** for all entries correct.

METHOD 2

$1500\left(1 + \frac{2.75}{2 \times 100}\right)^{2 \times 5}$ ***(M1)(A1)***

1719.49 euro ***A1***

[3 marks]

12b. Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5 years. *[3 marks]*

Assume the account pays a nominal annual interest of $r\%$ **compounded quarterly**.

Determine the value of r.

Markscheme

METHOD 1

$N = 5$ **OR** $N = 20$
$PV = \pm 1500$ $PV = \pm 1500$
$FV = \mp 2250$ $FV = \mp 2250$
$PMT = 0$ $PMT = 0$
$P/Y = 1$ $P/Y = 4$
$C/Y = 4$ $C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, ***A1*** for all entries correct. PV and FV must have opposite signs.

METHOD 2

$1500\left(1 + \frac{r}{4 \times 100}\right)^{4 \times 5} = 2250$ **OR** $\left(1 + \frac{r}{4 \times 100}\right)^{4 \times 5} = 1.5$ ***(M1)(A1)***

Note: Award ***M1*** for substitution in compound interest formula, ***A1*** for correct substitution and for equating to 2250 (if using LHS equation) or to 1.5 (if using RHS equation).

$r = 8.19 (8.19206\ldots)$ ***A1***

Note: Accept $r = 8.19\%$.

Accept a trial and error method which leads to $r = 8.19$.

[3 marks]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested $37\ 000$ Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

13a. Calculate the value of Daisy's investment after 2 years. *[3 marks]*

Markscheme

EITHER

$N = 2$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$N = 8$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$FV = 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times 2}$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

$= 42\,010\,\mathrm{AUD}$ ***A1***

Note: Award ***(M1)(A1)A0*** for unsupported 42009.87.

[3 marks]

After m months, the amount of money in the fixed deposit account has appreciated to more than $50\,000\,\mathrm{AUD}$.

13b. Find the minimum value of m, where $m \in \mathbb{N}$. *[4 marks]*

Markscheme

EITHER

$PV = -37\ 000$
$FV = 50\ 000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. The final mark can still be awarded for the correct number of months (multiple of 3).

OR

$PV = -37\ 000$
$FV = 50\ 000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$50\ 000 < 37\ 000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times n}$ **OR** $50\ 000 < 37\ 000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{n}$ ***(M1)(A1)***

Note: Award ***M1*** for the correct inequality, $50\ 000$ and substituted compound interest formula. Allow an equation. Award ***A1*** for correct substitution.

THEN

$N = 4.74(\text{years})(4.74230\ldots)$ **OR** $N = 18.9692\ldots(\text{quarters})$ ***(A1)***

$m = 57$ months ***A1***

Note: Award ***A1*** for rounding their m to the correct number of months. The final answer must be a multiple of 3. Follow through within this part.

[4 marks]

Daisy is saving to purchase a new apartment. The price of the apartment is $200\ 000\mathrm{AUD}$.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

13c. Write down the amount of the loan. *[1 mark]*

Markscheme

$150\ 000\mathrm{AUD}$ ***A1***

[1 mark]

The loan is for 10 years, compounded monthly, with equal monthly payments of $1700\mathrm{AUD}$ made by Daisy at the end of each month.

For this loan, find

13d. the amount of interest paid by Daisy. *[2 marks]*

Markscheme

$120 \times 1700 - 150\ 000$ ***(M1)***

$= 54\ 000\mathrm{AUD}$ ***A1***

[2 marks]

13e. the annual interest rate of the loan. *[3 marks]*

Markscheme

$N = 120$
$PV = -150\ 000$
$PMT = 1700$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula or $FV = 0$ seen. If a compound interest formula is equated to zero, award ***M1***, otherwise award ***M0*** for a substituted compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in annuity formula, but award ***A0*** for a substituted compound interest formula.
Follow through marks in part (d)(ii) are contingent on working seen.

$r = 6.46(\%)(6.45779\ldots)$ ***A1***

[3 marks]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

13f. Find the amount of Daisy’s final payment. *[3 marks]*

Markscheme

$N = 60$
$I = 6.46(6.45779\ldots)$
$PV = -150\ 000$
$PMT = 1700$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula. Award ***(M0)*** for a substituted compound interest formula. Award ***A1*** for all entries correct. Follow through marks in part (e) are contingent on working seen.

$FV = 86973\text{AUD}$ ***A1***

[3 marks]

13g. Find how much money Daisy saved by making one final payment after 5 years. *[3 marks]*

Markscheme

$204\ 000 - (60 \times 1700 + 86973)$ **OR** $204\ 000 - 188\ 973$ ***(M1)(A1)***

Note: Award ***M1*** for 60×1700. Award ***M1*** for subtracting their $(60 \times 1700 + 86973)$ from their $(204\ 000)$. Award at most ***M1M0*** for their $204\ 000 - (60 \times 1700)$ or ***M0M0*** for their $204\ 000 - (86973)$. Follow through from parts (d)(i) and (e). Follow through marks in part (f) are contingent on working seen.

$= 15\ 027 \mathrm{AUD}$ ***A1***

[3 marks]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

Find:

14a. the number of seats in the last row. *[3 marks]*

Markscheme

recognition of arithmetic sequence with common difference 2 ***(M1)***

use of arithmetic sequence formula ***(M1)***

$14 + 2(20 - 1)$

52 ***A1***

[3 marks]

14b. the total number of seats in the concert hall. *[2 marks]*

Markscheme

use of arithmetic series formula ***(M1)***

$\frac{14+52}{2} \times 20$

660 ***A1***

[2 marks]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

14c. Find the average number of visitors per concert in 2020. *[2 marks]*

Markscheme

$584 + (584 \times 0.012)$ **OR** $584 \times (1.012)^1$ ***(M1)***

$591 (591.008)$ ***A1***

Note: Award ***M0A0*** if incorrect r used in part (b), and ***FT*** with their r in parts (c) and (d).

[2 marks]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

14d. Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. *[5 marks]*

Markscheme

recognition of geometric sequence ***(M1)***

equating their nth geometric sequence term to their 660 ***(M1)***

Note: Accept inequality.

METHOD 1

EITHER

$600 = 584 \times (1.012)^{x-1}$ ***A1***

$(x - 1 =)10.3(10.2559\ldots)$

$x = 11.3(11.2559\ldots)$ ***A1***

2030 ***A1***

OR

$600 = 584 \times (1.012)^{x}$ ***A1***

$x = 10.3(10.2559\ldots)$ ***A1***

2030 ***A1***

METHOD 2

11^{th} term $658(657.987\ldots)$ ***(M1)A1***

12^{th} term $666(666.883\ldots)$ ***(M1)A1***

2030 ***A1***

Note: The last mark can be awarded if both their 11^{th} and 12^{th} correct terms are seen.

[5 marks]

14e. It is assumed that the concert hall will host 50 concerts each year. *[4 marks]*

Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025.

Markscheme

7 seen ***(A1)***

EITHER

$584\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)***

multiplying their sum by 50 ***(M1)***

OR

sum of the number of visitors for their r and their seven years ***(M1)***

multiplying their sum by 50 ***(M1)***

OR

$29\ 200\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)(M1)***

THEN

$212000(211907.3\ldots)$ ***A1***

Note: Follow though from their r from part (b).

[4 marks]

Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of $30\ \mathrm{cm}^3$. The fifth smallest slice has a volume of $240\ \mathrm{cm}^3$.

15a. Find the common ratio of the sequence. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$u_1 r = 30$ **and** $u_1 r^4 = 240,$ ***(M1)***

Note: Award ***(M1)*** for both the given terms expressed in the formula for u_n.

OR

$30r^3 = 240 \quad \left(r^3 = 8\right)$ ***(M1)***

Note: Award ***(M1)*** for a correct equation seen.

$(r =)\ 2$ ***(A1)*** ***(C2)***

[2 marks]

15b. Find the volume of the smallest slice of pie. *[2 marks]*

Markscheme

$u_1 \times 2 = 30$ **OR** $u_1 \times 2^4 = 240$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in geometric sequence formula.

$(u_1 =)\ 15$ ***(A1)*(ft)** ***(C2)***

Note: Follow through from part (a).

[2 marks]

15c. The apple pie has a volume of $61\ 425\ \mathrm{cm}^3$. *[2 marks]*

Find the total number of slices Mia can cut from this pie.

Markscheme

$\frac{15(2^n-1)}{2-1} = 61425$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted geometric series formula equated to 61425.

$(n =)\ 12$ (slices) ***(A1)*(ft)*** ***(C2)***

Note: Follow through from parts (a) and (b).

[2 marks]

A geometric sequence has a first term of $\frac{8}{3}$ and a fourth term of 9.

16a. Find the common ratio. *[2 marks]*

Markscheme

$9 = \left(\frac{8}{3}\right) r^3$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted geometric sequence formula equated to 9.

$(r =)\ 1.5\ \ \left(\frac{3}{2}\right)$ ***(A1) (C2)***

[2 marks]

16b. Write down the second term of this sequence. *[1 mark]*

Markscheme

4 ***(A1)*(ft)** ***(C1)***

Note: Follow through from part (a).

[1 mark]

16c. The sum of the first k terms is greater than 2500. *[3 marks]*

Find the smallest possible value of k.

Markscheme

$2500 < \frac{\left(\frac{8}{3}\right)\left((1.5)^k - 1\right)}{1.5-1}$ ***(M1)***

Note: Award ***(M1)*** for *their* correctly substituted geometric series formula compared to 2500.

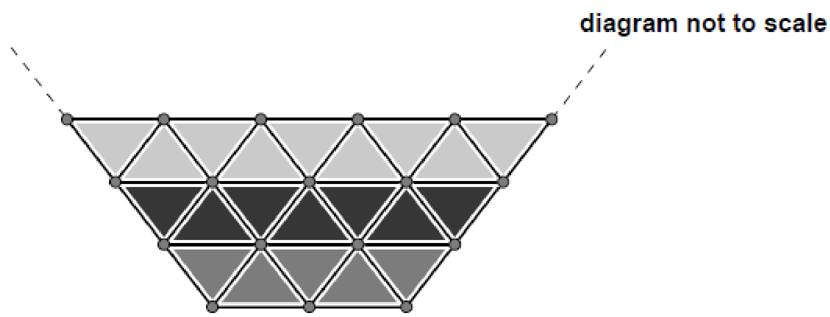
$k = 15.2 \ \ (15.17319\ldots)$ ***(A1)*****(ft)**

$(k =)\ 16$ ***(A1)*****(ft)** ***(C3)***

Note: Answer must be an integer for the final ***(A1)*****(ft)** to be awarded. Follow through from part (a).

[3 marks]

Maegan designs a decorative glass face for a new Fine Arts Centre. The glass face is made up of small triangular panes. The **first** three levels of the glass face are illustrated in the following diagram.



The 1st level, at the bottom of the glass face, has 5 triangular panes. The 2nd level has 7 triangular panes, and the 3rd level has 9 triangular panes. Each additional level has 2 more triangular panes than the level below it.

17a. Find the number of triangular panes in the 12th level. *[3 marks]*

Markscheme

$u_{12} = 5 + (12 - 1) \times (2)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitutions.

27 ***(A1)(G3)***

[3 marks]

17b. Show that the total number of triangular panes, S_n, in the first n levels is given by: *[3 marks]*

$S_n = n^2 + 4n$.

Markscheme

$S_n = \frac{n}{2}(2 \times 5 + (n-1)(2))$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted arithmetic sequence formula, ***(A1)*** for correct substitutions.

$S_n = \frac{n}{2}(8 + 2n)$ **OR** $S_n = n(5 + n - 1)$ ***(M1)***

Note: Award ***(M1)*** for evidence of expansion and simplification, or division by 2 leading to the final answer.

$S_n = n^2 + 4n$ ***(AG)***

Note: The final line must be seen, with no incorrect working, for the final ***(M1)*** to be awarded.

[3 marks]

17c. **Hence**, find the total number of panes in a glass face with 18 levels. *[2 marks]*

Markscheme

$(S_{18} =) 18^2 + 4 \times 18$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted formula for S_n.

$(S_{18} =)\ 396$ ***(A1)***

Note: The use of “hence” in the question paper means that the S_n formula (from part (b)) must be used.

[2 marks]

Maegan has 1000 triangular panes to build the decorative glass face and does not want it to have any incomplete levels.

17d. Find the maximum number of **complete** levels that Maegan can build. *[3 marks]*

Markscheme

$1000 = n^2 + 4n$ **OR** $1000 = \frac{n}{2}(10 + (n-1)2)$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for equating S_n to 1000 or for equating the correctly substituted sum of arithmetic sequence formula to 1000.

OR

a sketch of the graphs $S_n = n^2 + 4n$ and $S_n = 1000$ intersecting ***(M1)***

Note: Award ***(M1)*** for a sketch of a quadratic and a horizontal line with at least one point of intersection.

OR

a sketch of $n^2 + 4n - 1000$ intersecting the x-axis ***(M1)***

Note: Award ***(M1)*** for a sketch of $n^2 + 4n - 1000$ with at least one x-intercept.

$(n =)\ 29.6859\ldots$ **OR** $-2 + 2\sqrt{251}$ ***(A1)***

Note: Award ***(A1)*** for $29.6859\ldots$ or $-2 + 2\sqrt{251}$ seen. Can be implied by a correct final answer.

$(n =)\ 29$ ***(A1)*(ft)*(G2)***

Note: Do not accept 30. Award a maximum of ***(M1)(A1)(A0)*** if two final answers are given. Follow though from their unrounded answer.

OR

$S_{30} = 1020$ and $S_{29} = 957$ ***(A2)***

Note: Award ***(A2)*** for both “crossover” values seen. Do not split this ***(A2)*** mark.

$(n =)\ 29$ ***(A1)(G2)***

[3 marks]

17e. Each triangular pane has an area of $1.84\ \mathrm{m}^2$. *[4 marks]*

Find the **total** area of the decorative glass face, if the maximum number of complete levels were built. Express your area to the nearest m^2.

Markscheme

$(A =)\ (29^2 + 4 \times 29) \times (1.84)$ ***(M1)(M1)***

Note: Award ***(M1)*** for their correct substitution to find the total number of triangular panes. Award ***(M1)*** for multiplying their number of panes by 1.84.

OR

$(A =)\ 957 \times 1.84$ ***(A1)*(ft)*(M1)***

Note: Award ***(A1)*(ft)** for their 957 seen. Award ***(M1)*** for multiplying their number of panes by 1.84. Follow through from part (d).

$(A =)\ 1760.88\ (\mathrm{m}^2)$ ***(A1)*(ft)*(G2)***

$(A =)\ 1761\ (\mathrm{m}^2)$ ***(A1)*(ft)*(G3)***

[4 marks]