revision Sequences+Financial Maths+Rounding *[168 marks]*

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

1a. Write down the value of the common difference, d *[1 mark]*

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1b. Calculate the price of a ticket in the 16th row. *[2 marks]*

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1c. Find the total cost of buying 2 tickets in each of the first 16 rows. *[3 marks]*

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In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14 % **compounded quarterly**. No deposit required and repayments are made each quarter.

2a. Find the repayment made each quarter. *[3 marks]*

2b. Find the total amount paid for the car. *[2 marks]*

2c. Find the interest paid on the loan. *[2 marks]*

Finance option B:

A 6 year loan at a nominal annual interest rate of r % **compounded monthly**.
Terms of the loan require a 10 % deposit and monthly repayments of €250.

2d. Find the amount to be borrowed for this option. *[2 marks]*

2e. Find the annual interest rate, r. *[3 marks]*

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2f. State which option Bryan should choose. Justify your answer. *[2 marks]*

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2g. Bryan’s car depreciates at an annual rate of 25 % per year. *[3 marks]*

Find the value of Bryan’s car six years after it is purchased.

A disc is divided into 9 sectors, number 1 to 9. The angles at the centre of each of the sectors u_n form an arithmetic sequence, with u_1 being the largest angle.

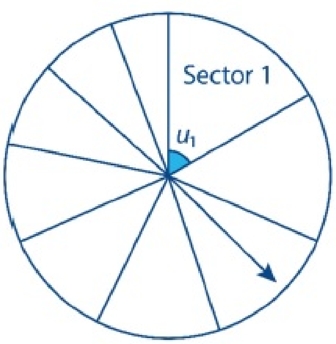

Diagram not to scale

3a. *[1 mark]*

Write down the value of $\sum_{i=1}^{9} u_i$.

It is given that $u_9 = \frac{1}{3}u_1$.

3b. Find the value of u_1. *[4 marks]*

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Sophia pays $\$200$ into a bank account at the end of each month. The annual interest paid on money in the account is 3.1% which is compounded monthly.

4a. Find the value of her investment after a period of 5 years. *[3 marks]*

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The average rate of inflation per year over the 5 years was 2%.

4b. Find an approximation for the real interest rate for the money invested in the account. *[2 marks]*

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4c. Hence find the real value of Sophia’s investment at the end of 5 years. *[2 marks]*

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Give your answers to this question correct to two decimal places.

Gen invests \$2400 in a savings account that pays interest at a rate of 4% per year, compounded annually. She leaves the money in her account for 10 years, and she does not invest or withdraw any money during this time.

5a. Calculate the value of her savings after 10 years. *[2 marks]*

5b. The rate of inflation during this 10 year period is 1.5% per year. *[3 marks]*

Calculate the real value of her savings after 10 years.

Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of $4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

6a. Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. *[3 marks]*

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6b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. *[3 marks]*

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

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Paul wants to buy a car. He needs to take out a loan for $7000. The car salesman offers him a loan with an interest rate of 8%, compounded annually. Paul considers two options to repay the loan.

Option 1: Pay $200 each month, until the loan is fully repaid

Option 2: Make 24 equal monthly payments.

Use option 1 to calculate

7a. the number of months it will take for Paul to repay the loan. *[3 marks]*

7b. the total amount that Paul has to pay. *[2 marks]*

Use option 2 to calculate

7c. the amount Paul pays each month. *[2 marks]*

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7d. the total amount that Paul has to pay. *[2 marks]*

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Give a reason why Paul might choose

7e. option 1. *[1 mark]*

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7f. option 2. *[1 mark]*

Sophie is planning to buy a house. She needs to take out a mortgage for $120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay $1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

8a. Calculate the monthly repayment using option 1. *[2 marks]*

8b. Calculate the total amount Sophie would pay, using option 1. *[2 marks]*

8c. Calculate the number of months it will take to repay the mortgage using option 2. *[3 marks]*

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8d. Calculate the total amount Sophie would pay, using option 2. *[2 marks]*

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Give a reason why Sophie might choose

8e. option 1. *[1 mark]*

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8f. option 2. *[1 mark]*

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

8g. Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years. *[2 marks]*

8h. Hence calculate her monthly repayment for the final 10 years. *[2 marks]*

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

9a. Calculate the percentage increase in applications from the first year to the second year. *[2 marks]*

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

9b. Write down the common ratio of the sequence. *[1 mark]*

9c. Find an expression for u_n. *[1 mark]*

9d. Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. *[2 marks]*

In the first year there were $10\ 380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

9e. Write down an expression for v_n . *[2 marks]*

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

9f. Calculate the total amount of acceptance fees paid to the university in the first 10 years. *[3 marks]*

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When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

9g. Find k. *[3 marks]*

9h. State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. *[2 marks]*

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Katya approximates π, correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6 + \frac{13}{16}}$$

10a. Calculate Katya's approximation of π, correct to four decimal places. *[2 marks]*

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10b. Calculate the percentage error in using Katya's four decimal place approximation of π, compared to the exact value of π in your calculator. *[2 marks]*

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Charlie and Daniella each began a fitness programme. On day one, they both ran $500\ \mathrm{m}$. On each subsequent day, Charlie ran $100\ \mathrm{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

Calculate how far

11a. Charlie ran on day 20 of his fitness programme. *[2 marks]*

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11b. Daniella ran on day 20 of her fitness programme. *[3 marks]*

11c. On day n of the fitness programmes Daniella runs more than Charlie for the first time. *[3 marks]*

Find the value of n.

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

12a. Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places. *[3 marks]*

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12b. Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5 years. *[3 marks]*

Assume the account pays a nominal annual interest of $r\%$ **compounded quarterly**.

Determine the value of r.

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Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested $37\,000$ Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

13a. Calculate the value of Daisy's investment after 2 years. *[3 marks]*

After m months, the amount of money in the fixed deposit account has appreciated to more than $50\ 000 \mathrm{AUD}$.

13b. Find the minimum value of m, where $m \in \mathbb{N}$. *[4 marks]*

Daisy is saving to purchase a new apartment. The price of the apartment is $200\ 000 \mathrm{AUD}$.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

13c. Write down the amount of the loan. *[1 mark]*

The loan is for 10 years, compounded monthly, with equal monthly payments of $1700 \mathrm{AUD}$ made by Daisy at the end of each month.

For this loan, find

13d. the amount of interest paid by Daisy. *[2 marks]*

13e. the annual interest rate of the loan. *[3 marks]*

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

13f. Find the amount of Daisy’s final payment. *[3 marks]*

13g. Find how much money Daisy saved by making one final payment after 5 years. *[3 marks]*

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

Find:

14a. the number of seats in the last row. *[3 marks]*

14b. the total number of seats in the concert hall. *[2 marks]*

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

14c. Find the average number of visitors per concert in 2020. *[2 marks]*

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The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

14d. Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. *[5 marks]*

14e. It is assumed that the concert hall will host 50 concerts each year. *[4 marks]*

Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025.

Mia baked a very large apple pie that she cuts into slices to share with her friends. The smallest slice is cut first. The volume of each successive slice of pie forms a geometric sequence.

The second smallest slice has a volume of $30\ \mathrm{cm}^3$. The fifth smallest slice has a volume of $240\ \mathrm{cm}^3$.

15a. Find the common ratio of the sequence. *[2 marks]*

15b. Find the volume of the smallest slice of pie. *[2 marks]*

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15c. The apple pie has a volume of $61\ 425\ \mathrm{cm}^3$. *[2 marks]*

Find the total number of slices Mia can cut from this pie.

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A geometric sequence has a first term of $\frac{8}{3}$ and a fourth term of 9.

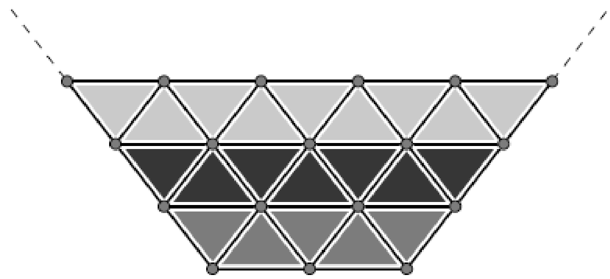
16a. Find the common ratio. *[2 marks]*

16b. Write down the second term of this sequence. *[1 mark]*

16c. The sum of the first k terms is greater than 2500. *[3 marks]*

Find the smallest possible value of k.

Maegan designs a decorative glass face for a new Fine Arts Centre. The glass face is made up of small triangular panes. The **first** three levels of the glass face are illustrated in the following diagram.



The 1st level, at the bottom of the glass face, has 5 triangular panes. The 2nd level has 7 triangular panes, and the 3rd level has 9 triangular panes. Each additional level has 2 more triangular panes than the level below it.

17a. Find the number of triangular panes in the 12th level. *[3 marks]*

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17b. Show that the total number of triangular panes, S_n, in the first n levels is given by: *[3 marks]*

$$S_n = n^2 + 4n.$$

17c. **Hence**, find the total number of panes in a glass face with 18 levels. *[2 marks]*

Maegan has 1000 triangular panes to build the decorative glass face and does not want it to have any incomplete levels.

17d. Find the maximum number of **complete** levels that Maegan can build. *[3 marks]*

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17e. Each triangular pane has an area of $1.84\ \mathrm{m}^2$. *[4 marks]*

Find the **total** area of the decorative glass face, if the maximum number of complete levels were built. Express your area to the nearest m^2.

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