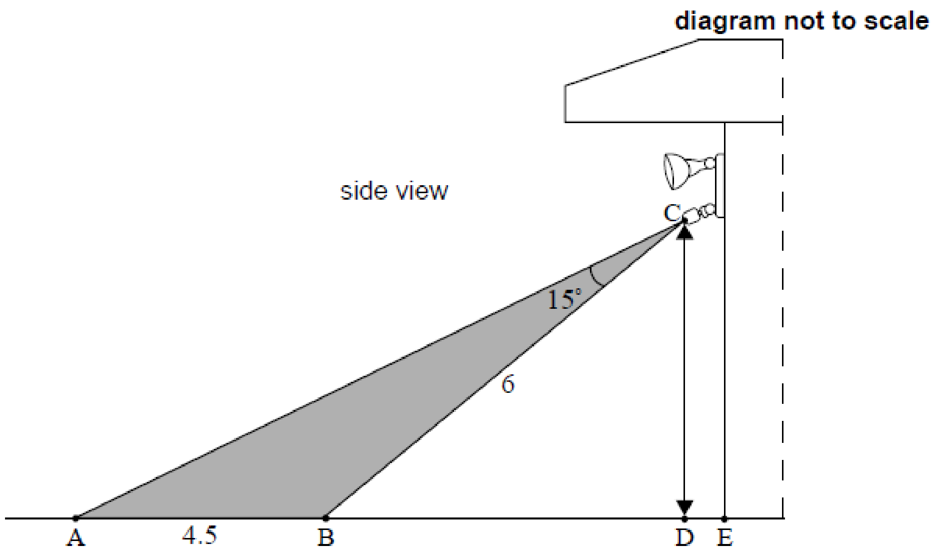
Geometry and trigonometry

[70 marks]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle AĈB is 15°.



1a. Find CÂB. *[3 marks]*

Markscheme

$$\frac{\sin C\hat{A}B}{6} = \frac{\sin 15^\circ}{4.5}$$ ***(M1)(A1)***

CÂB = 20.2º (20.187415…) ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

1b. Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B. *[5 marks]*

Find the distance Ollie is **from the entrance to his house** when he first activates the sensor.

Markscheme

$\mathrm{C\hat{B}D} = 20.2 + 15 = 35.2°$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots° = \frac{1.8}{\mathrm{BX}}$ ***(M1)***

Note: Award ***A1*** for their correct angle $\mathrm{C\hat{B}D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\mathrm{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

= 2.45 (m) (2.44714...) ***A1***

[5 marks]

A farmer owns a triangular field ABC. The length of side $[\mathrm{AB}]$ is $85\ \mathrm{m}$ and side $[\mathrm{AC}]$ is $110\ \mathrm{m}$. The angle between these two sides is 55°.

2a. Find the area of the field. *[3 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

Area $= \frac{1}{2} \times 110 \times 85 \times \sin 55^\circ$ **(M1)(A1)**

$= 3830(3829.53\ldots)\mathrm{m}^2$ **A1**

Note: units must be given for the final **A1** to be awarded.

[3 marks]

2b. The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[\mathrm{BC}]$. *[6 marks]*

Find BD. Fully justify any assumptions you make.

Markscheme

$\mathrm{BC}^2 = 110^2 + 85^2 - 2 \times 110 \times 85 \times \cos 55^\circ$ **(M1)A1**

$\mathrm{BC} = 92.7(92.7314\ldots)(\mathrm{m})$ **A1**

METHOD 1

Because the height and area of each triangle are equal they must have the same length base **R1**

D must be placed half-way along BC **A1**

$\mathrm{BD} = \frac{92.731\ldots}{2} \approx 46.4(\mathrm{m})$ **A1**

Note: the final two marks are dependent on the **R1** being awarded.

METHOD 2

Let $\mathrm{C\hat{B}A} = \theta^\circ$

$\frac{\sin\theta}{110} = \frac{\sin 55^\circ}{92.731\ldots}$ **M1**

$\Rightarrow \theta = 76.3^\circ(76.3354\ldots)$

Use of area formula

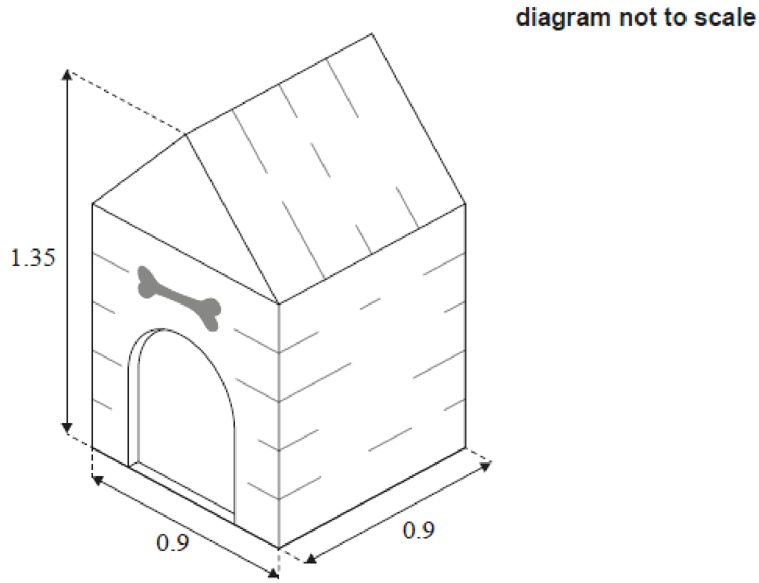
$\frac{1}{2} \times 85 \times \mathrm{BD} \times \sin(76.33\ldots^\circ) = \frac{3829.53\ldots}{2}$ **A1**

$\mathrm{BD} = 46.4(46.365\ldots)(\mathrm{m})$ **A1**

[6 marks]

3. The front view of a doghouse is made up of a square with an isosceles triangle on top. *[5 marks]*

The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted.

Markscheme

height of triangle at roof $= 1.35 - 0.9 = 0.45$ ***(A1)***

Note: Award ***A1*** for 0.45 (height of triangle) seen on the diagram.

$\text{slant height} = \sqrt{0.45^2 + 0.45^2}$ **OR** $\sin(45^\circ) = \frac{0.45}{\text{slant height}}$ ***(M1)***

$= \sqrt{0.405}$ $\left(0.636396\ldots,\ 0.45\sqrt{2}\right)$ ***A1***

Note: If using $\sin(45^\circ) = \frac{0.45}{\text{slant height}}$ then ***(A1)*** for angle of 45°, ***(M1)*** for a correct trig statement.

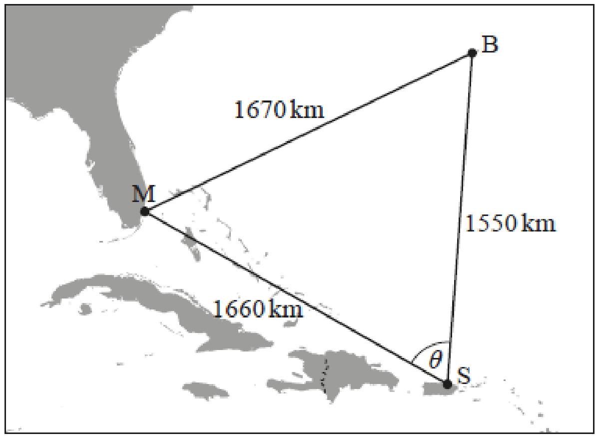
area of one rectangle on roof $= \sqrt{0.405} \times 0.9$ $(= 0.572756\ldots)$ ***M1***

area painted $= \left(2 \times \sqrt{0.405} \times 0.9\ = 2 \times 0.572756\ldots\right)$

1.15 m^2 $\left(1.14551\ldots\text{ m}^2,\ 0.81\sqrt{2}\text{ m}^2\right)$ ***A1***

[5 marks]

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

4a. Calculate the value of θ, the measure of angle $\mathrm{M\hat{S}B}$. *[3 marks]*

Markscheme

attempt at substituting the cosine rule formula ***(M1)***

$\cos\theta = \frac{1660^2+1550^2-1670^2}{2(1660)(1550)}$ ***(A1)***

$(\theta =)\ 62.6^\circ \quad (62.5873\ldots)$ (accept 1.09 rad $(1.09235\ldots)$)
A1

[3 marks]

4b. Find the area of the Bermuda Triangle. *[2 marks]*

Markscheme

correctly substituted area of triangle formula ***(M1)***

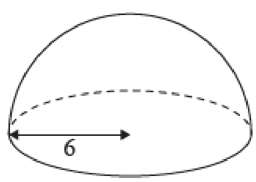
$A = \frac{1}{2}(1660)(1550)\sin(62.5873\ldots)$

$(A =)\ 1140\ 000\ \left(1.14 \times 10^6,\ 1142\ 043.327\ldots\right)\ \mathrm{km}^2$ ***A1***

Note: Accept $1150\ 000\ \left(1.15 \times 10^6,\ 1146\ 279.893\ldots\right)\ \mathrm{km}^2$ from use of 63°. Other angles and their corresponding sides may be used.

[2 marks]

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is $6\ \mathrm{mm}$.



5a. Calculate the **total** surface area of one piece of candy. *[4 marks]*

Markscheme

$\frac{1}{2} \times 4 \times \pi \times 6^2 + \pi \times 6^2$ **OR** $3 \times \pi \times 6^2$ ***(M1)(A1)(M1)***

Note: Award ***M1*** for use of surface area of a sphere formula (or curved surface area of a hemisphere), ***A1*** for substituting correct values into hemisphere formula, ***M1*** for adding the area of the circle.

$= 339\mathrm{mm}^2(108\pi, 339.292\ldots)$ ***A1***

[4 marks]

5b. The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of $240\ \mathrm{mm}^2$. *[2 marks]*

Calculate the weight of chocolate required to coat one piece of candy.

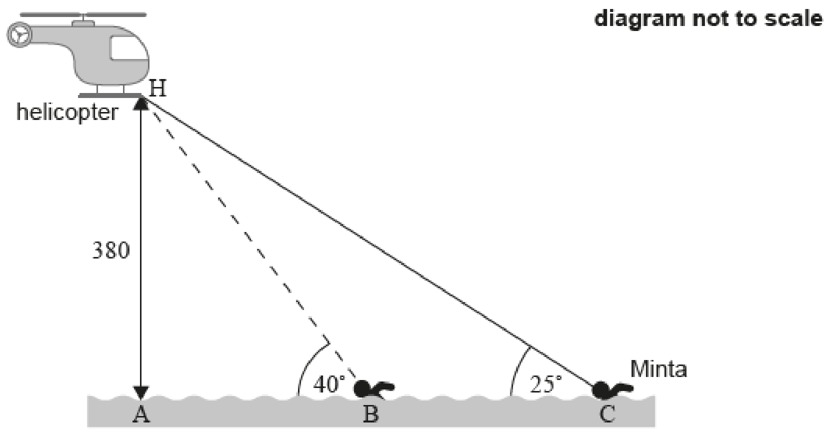
Markscheme

$\frac{339.292...}{240}$ ***(M1)***

$= 1.41(\text{g})\left(\frac{9\pi}{20}, 0.45\pi, 1.41371...\right)$ ***A1***

[2 marks]

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

6a. Write down the size of the angle of depression from H to C. *[1 mark]*

Markscheme

25° ***A1***

[1 mark]

6b. Find the distance from A to C. *[2 marks]*

Markscheme

$\mathrm{AC} = \frac{380}{\tan 25°}$ **OR** $\mathrm{AC} = \sqrt{\left(\frac{380}{\sin 25°}\right)^2 - 380^2}$ **OR** $\frac{380}{\sin 25°} = \frac{\mathrm{AC}}{\sin 65°}$ ***(M1)***

$\mathrm{AC} = 815\ \mathrm{m}(814.912\ldots)$ ***A1***

[2 marks]

6c. Find the distance from B to C. *[3 marks]*

Markscheme

METHOD 1

attempt to find AB ***(M1)***

$\mathrm{AB} = \frac{380}{\tan 40°}$

$= 453\ \mathrm{m}(452.866\ldots)$ ***(A1)***

$\mathrm{BC} = 814.912\ldots - 452.866\ldots$

$= 362\ \mathrm{m}(362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$\mathrm{HB} = \frac{380}{\sin 40°}$

$591\ \mathrm{m}(= 591.175\ldots)$ ***(A1)***

$\mathrm{BC} = \frac{591.175\ldots \times \sin 15°}{\sin 25°}$

$= 362\ \mathrm{m}(362.046\ldots)$ ***A1***

[3 marks]

6d. Find Minta’s speed, in metres per hour. *[1 mark]*

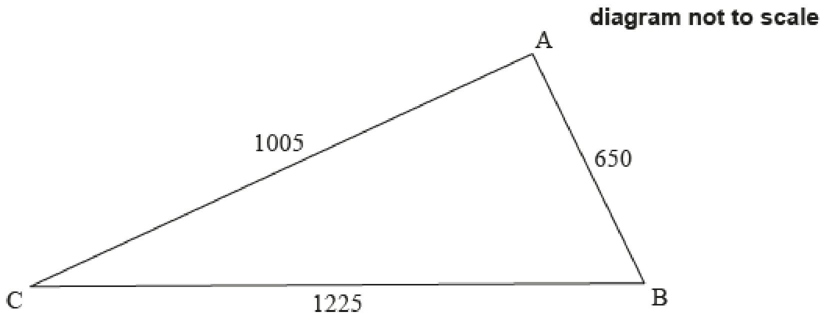
Markscheme

$362.046\ldots \times 4$

$= 1450\ \mathrm{m\,h^{-1}} (1448.18\ldots)$ ***A1***

[1 mark]

A farmer owns a field in the shape of a triangle ABC such that $\mathrm{AB} = 650\ \mathrm{m}$, $\mathrm{AC} = 1005\ \mathrm{m}$ and $\mathrm{BC} = 1225\ \mathrm{m}$.



7a. Find the size of $\mathrm{A\hat{C}B}$. *[3 marks]*

Markscheme

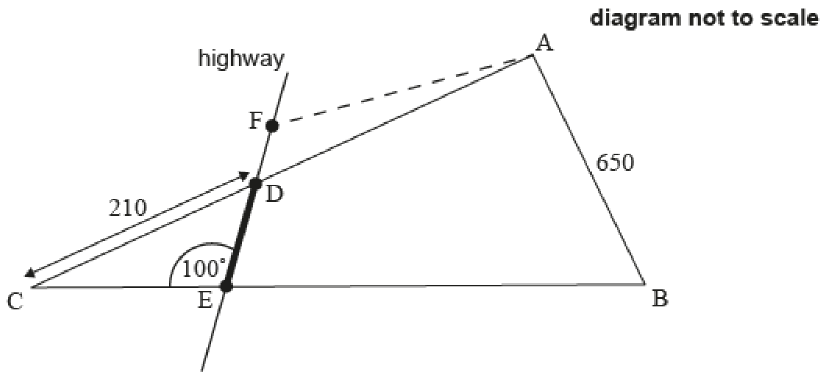
use of cosine rule ***(M1)***

$\mathrm{A\hat{C}B} = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right)$ ***(A1)***

$= 32^\circ (31.9980\ldots)$ ***A1***

[3 marks]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $\mathrm{DC} = 210\ \mathrm{m}$ and $\mathrm{C\hat{E}D} = 100^\circ$, as shown in the diagram below.



7b. Find DE. *[3 marks]*

Markscheme

use of sine rule ***(M1)***

$\frac{\mathrm{DE}}{\sin 31.9980\ldots^\circ} = \frac{210}{\sin 100^\circ}$ ***(A1)***

$(\mathrm{DE} =) 113\ \mathrm{m} (112.9937\ldots)$ ***A1***

[3 marks]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

7c. Find the area of triangle DCE. *[5 marks]*

Markscheme

METHOD 1

$180^\circ - (100^\circ + \text{their part}(a))$ ***(M1)***

$= 48.0019\ldots^\circ$ **OR** $0.837791\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.9937\ldots \times 210 \times \sin 48.002^\circ$ ***(A1)***

$8820\ \text{m}^2 (8817.18\ldots)$ ***A1***

METHOD 2

$\frac{\text{CE}}{\sin(180-100-\text{their part}(a))} = \frac{210}{\sin 100}$ ***(M1)***

$(\text{CE} =) 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

EITHER

$\frac{1}{2} \times 112.993\ldots \times 158.472\ldots \times \sin 100$ ***(A1)***

OR

$\frac{1}{2} \times 210\ldots \times 158.472\ldots \times \sin(\text{their part}(a))$ ***(A1)***

THEN

$8820\ \text{m}^2 (8817.18\ldots)$ ***A1***

METHOD 3

$\text{CE}^2 = 210^2 + 112.993\ldots^2 - (2 \times 210 \times 112.993\ldots \times \cos(180 - 100 - \text{their pa}$
(M1)

$(\text{CE} =) 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.993\ldots \times 158.472\ldots \times \sin 100$ ***(A1)***

$8820\ \text{m}^2 (8817.18\ldots)$ ***A1***

[5 marks]

7d. Estimate DF. You may assume the highway has a width of zero. *[4 marks]*

Markscheme

$1005 - 210$ **OR** 795 ***(A1)***

equating answer to part (c) to area of a triangle formula ***(M1)***

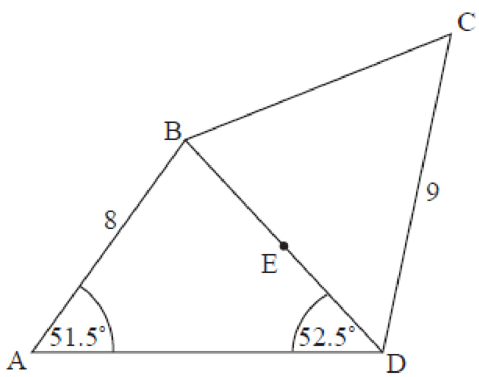
$8817.18\ldots = \frac{1}{2} \times \text{DF} \times (1005 - 210) \times \sin 48.002\ldots^\circ$ ***(A1)***

$(\text{DF}=) 29.8 \text{ m} (29.8473\ldots)$ ***A1***

[4 marks]

Using geometry software, Pedro draws a quadrilateral ABCD. $\text{AB} = 8 \text{ cm}$ and $\text{CD} = 9 \text{ cm}$. Angle $\text{BAD} = 51.5^\circ$ and angle $\text{ADB} = 52.5^\circ$. This information is shown in the diagram.

diagram not to scale



8a. Calculate the length of BD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\frac{\text{BD}}{\sin 51.5^\circ} = \frac{8}{\sin 52.5^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$(\text{BD} =)\ 7.89\ (\text{cm})\ \ (7.89164\ldots)$ ***(A1)(G2)***

Note: If radians are used the answer is $9.58723\ldots$ award at most ***(M1)(A1)(A0)***.

[3 marks]

$\mathrm{CE} = 7\ \mathrm{cm}$, where point E is the midpoint of BD.

8b. Show that angle $\mathrm{EDC} = 48.0^\circ$, correct to three significant figures. *[4 marks]*

Markscheme

$\cos \mathrm{EDC} = \frac{9^2 + 3.94582\ldots^2 - 7^2}{2 \times 9 \times 3.94582\ldots}$ ***(A1)*****(ft)*****(M1)(A1)*****(ft)**

Note: Award ***(A1)*** for $3.94582\ldots$ or $\frac{7.89164\ldots}{2}$ seen, ***(M1)*** for substituted cosine rule, ***(A1)*****(ft)** for correct substitutions.

$(\mathrm{EDC} =)\ 47.9515\ldots^\circ$ ***(A1)***

48.0° (3 sig figures) ***(AG)***

Note: Both an unrounded answer that rounds to the given answer and the rounded value must be seen for the final ***(M1)*** to be awarded.
Award at most ***(A1)*****(ft)*****(M1)(A1)*****(ft)*****(A0)*** if the known angle 48.0° is used to validate the result. Follow through from their BD in part (a).

[4 marks]

8c. Calculate the area of triangle BDC. *[3 marks]*

Markscheme

Units are required in this question.

$(\text{area} =)\ \frac{1}{2} \times 7.89164\ldots \times 9 \times \sin 48.0^\circ$ ***(M1)(A1)*****(ft)**

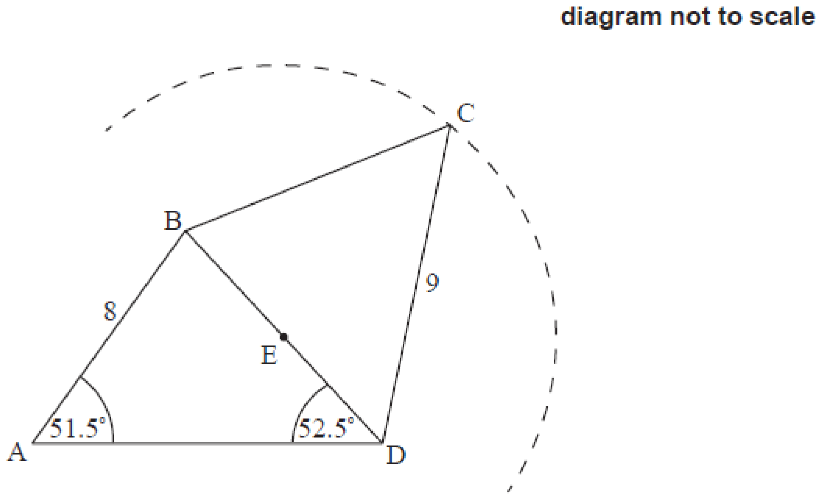
Note: Award ***(M1)*** for substituted area formula. Award ***(A1)*** for correct substitution.

$(\text{area} =)\ 26.4\ \mathrm{cm}^2\ \ (26.3908\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

[3 marks]

8d. Pedro draws a circle, with centre at point E, passing through point C. *[5 marks]*
Part of the circle is shown in the diagram.



Show that point A lies outside this circle. Justify your reasoning.

Markscheme

$\text{AE}^2 = 8^2 + (3.94582\ldots)^2 - 2 \times 8 \times 3.94582\ldots \cos(76^\circ)$ ***(A1)(M1)(A1)*(ft)**

Note: Award ***(A1)*** for 76° seen. Award ***(M1)*** for substituted cosine rule to find AE, ***(A1)*(ft)** for correct substitutions.

$(\text{AE} =)\ 8.02\,(\text{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

OR

$\text{AE}^2 = 9.78424\ldots^2 + (3.94582\ldots)^2 - 2 \times 9.78424\ldots \times 3.94582\ldots \cos(52.5^\circ$
***(A1)(M1)(A1)*(ft)**

Note: Award ***(A1)*** for AD $(9.78424\ldots)$ or 76° seen. Award ***(M1)*** for substituted cosine rule to find AE (do not award ***(M1)*** for cosine or sine rule to find AD), ***(A1)*(ft)** for correct substitutions.

$(\text{AE} =)\ 8.02\,(\text{cm})\ \ (8.01849\ldots)$ ***(A1)(ft)(G3)***

Note: Follow through from part (a).

$8.02 > 7.$ ***(A1)(ft)***

point A is outside the circle. ***(AG)***

Note: Award ***(A1)*** for a numerical comparison of AE and CE. Follow through for the final ***(A1)(ft)*** within the part for their 8.02. The final ***(A1)(ft)*** is contingent on a valid method to find the value of AE.
Do not award the final ***(A1)(ft)*** if the ***(AG)*** line is not stated.
Do not award the final ***(A1)(ft)*** if their point A is inside the circle.

[5 marks]