Geometry and trigonometry

[70 marks]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle AĈB is 15°.

1a. Find CÂB.   *[3 marks]*

1b. Point B on the ground is 5 m from point E at the entrance to Ollie's   *[5 marks]*
    house. He is 1.8 m tall and is standing at point D, below the sensor. He
    walks towards point B.

Find the distance Ollie is **from the entrance to his house** when he first
activates the sensor.

A farmer owns a triangular field ABC. The length of side $[AB]$ is 85 m and side $[AC]$ is 110 m. The angle between these two sides is $55°$.
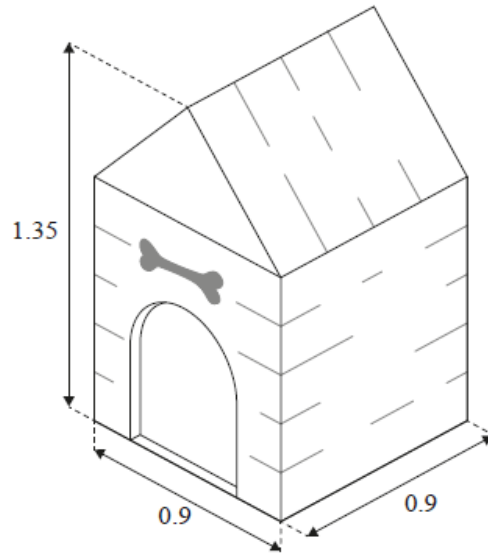
2a. Find the area of the field. *[3 marks]*

2b. The farmer would like to divide the field into two equal parts by constructing a straight fence from A to a point D on $[BC]$. *[6 marks]*

Find BD. Fully justify any assumptions you make.

3. The front view of a doghouse is made up of a square with an isosceles triangle on top. *[5 marks]*

The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.

1.35

0.9

0.9

The top of the rectangular surfaces of the roof of the doghouse are to be painted. Find the area to be painted.

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale
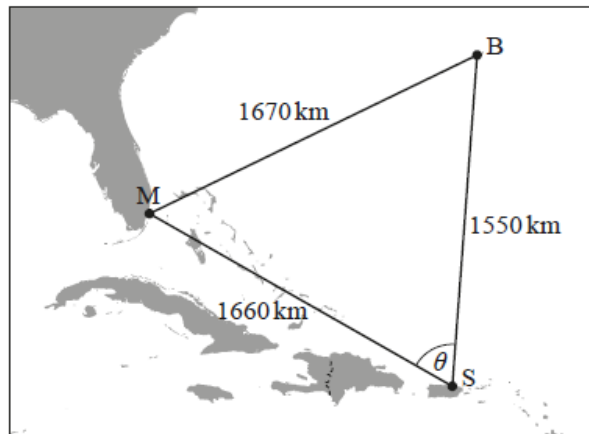
The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

4a. Calculate the value of θ, the measure of angle $M\hat{S}B$.   *[3 marks]*

4b. Find the area of the Bermuda Triangle. *[2 marks]*

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is 6 mm.

5a. Calculate the **total** surface area of one piece of candy. *[4 marks]*

5b. The total surface of the candy is coated in chocolate. It is known that 1   *[2 marks]*
gram of the chocolate covers an area of 240 mm^2.

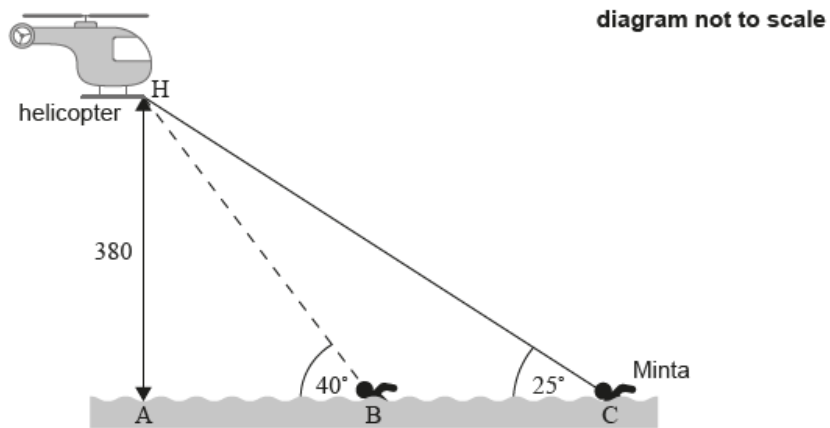Calculate the weight of chocolate required to coat one piece of candy.

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.

Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of $25°$. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of $40°$.

6a. Write down the size of the angle of depression from H to C.   *[1 mark]*

6b. Find the distance from A to C.   *[2 marks]*

6c. Find the distance from B to C.   *[3 marks]*

6d. Find Minta's speed, in metres per hour.   *[1 mark]*

A farmer owns a field in the shape of a triangle ABC such that $AB = 650$ m, $AC = 1005$ m and $BC = 1225$ m.

diagram not to scale

7a. Find the size of $A\hat{C}B$. *[3 marks]*

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210$ m and $\hat{CED} = 100°$, as shown in the diagram below.

diagram not to scale

highway

F

A

650

210

D

100°

C

E

B

7b. Find DE. [3 marks]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

7c. Find the area of triangle DCE. *[5 marks]*

7d. Estimate DF. You may assume the highway has a width of zero. *[4 marks]*

Using geometry software, Pedro draws a quadrilateral $ABCD$. $AB = 8$ cm and $CD = 9$ cm. Angle $BAD = 51.5°$ and angle $ADB = 52.5°$. This information is shown in the diagram.

diagram not to scale

8a. Calculate the length of BD.   [3 marks]

$\mathrm{CE} = 7 \text{ cm}$, where point E is the midpoint of BD.

8b. Show that angle $\mathrm{EDC} = 48.0°$, correct to three significant figures. *[4 marks]*

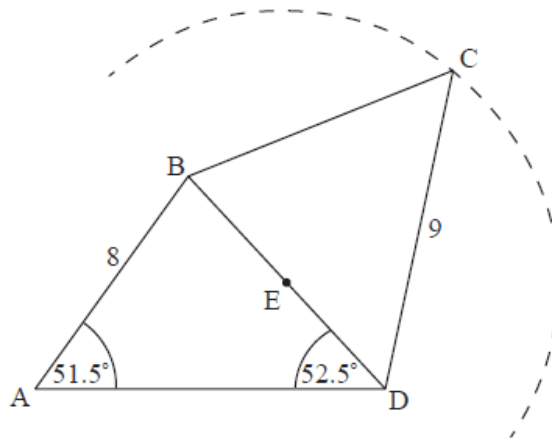8c. Calculate the area of triangle BDC. *[3 marks]*

8d. Pedro draws a circle, with centre at point E, passing through point C.   *[5 marks]*
Part of the circle is shown in the diagram.

diagram not to scale

Show that point A lies outside this circle. Justify your reasoning.

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