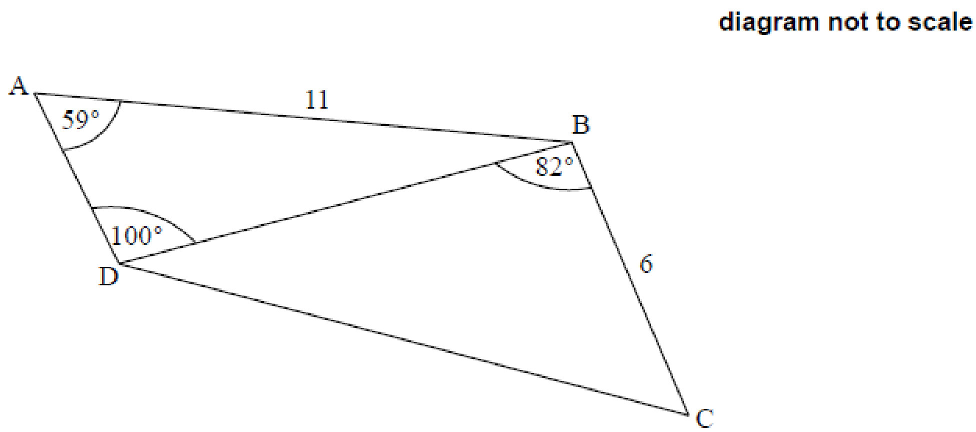
Geometry and trigonometry 28.11 *[78 marks]*

The following diagram shows quadrilateral ABCD.



$\mathrm{AB} = 11\,\mathrm{cm},\ \mathrm{BC} = 6\,\mathrm{cm},\ \mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 100°,\ \text{and}\ \mathrm{C}\hat{\mathrm{B}}\mathrm{D} = 82°$

1a. Find DB. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

evidence of choosing sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

correct substitution ***(A1)***

eg $\frac{\mathrm{DB}}{\sin 59°} = \frac{11}{\sin 100°}$

9.57429

DB = 9.57 (cm) ***A1 N2***

[3 marks]

1b. Find DC. *[3 marks]*

Markscheme

evidence of choosing cosine rule ***(M1)***

eg

$a^2 = b^2 + c^2 - 2bc\ \cos\ (A)\,,\ \ \mathrm{DC}^2 =\ \mathrm{DB}^2 +\ \mathrm{BC}^2\ -\ 2\mathrm{DB} \times\ \mathrm{BC} \times\ \cos\ \Big(\mathrm{D}\hat{\mathrm{B}}\mathrm{C}$

correct substitution into RHS ***(A1)***

eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times\ \cos\ 82^\circ,\ \ 111.677$

10.5677

DC = 10.6 (cm) ***A1 N2***

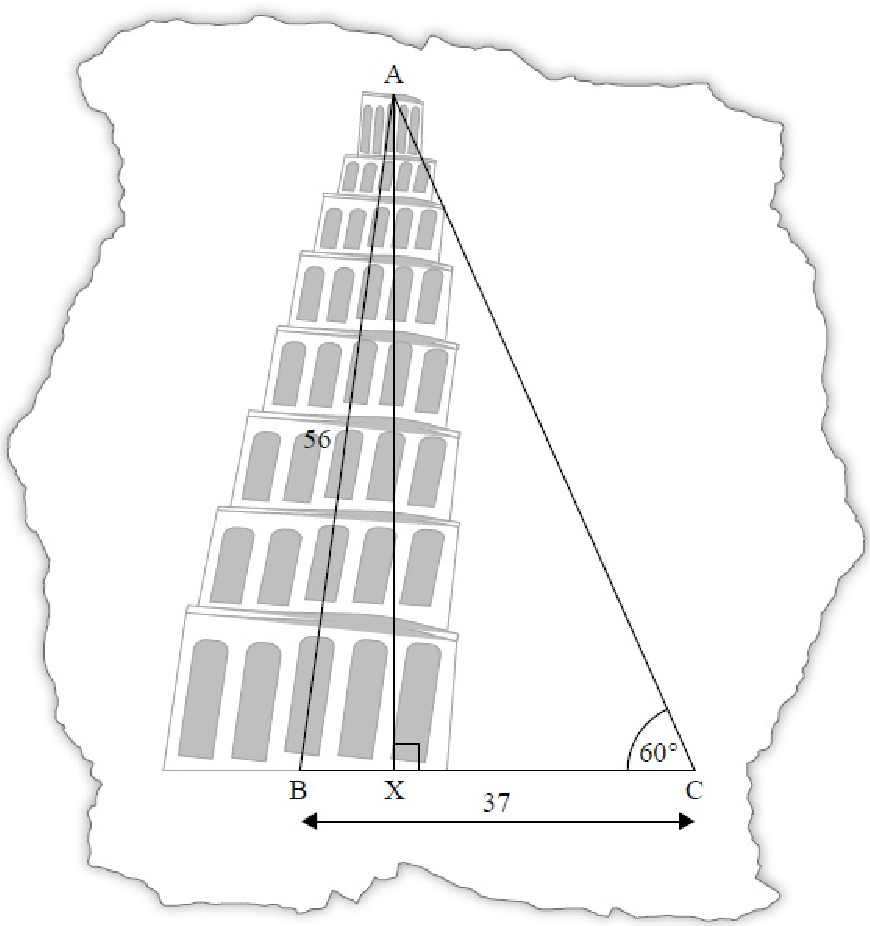
[3 marks]

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



2a. Use Giovanni's diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. *[5 marks]*

Markscheme

$\frac{\sin \text{BAC}}{37} = \frac{\sin 60}{56}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting the sine rule formula, ***(A1)*** for correct substitution.

angle $\text{B}\overset{\wedge}{\text{A}}\text{C}$ = 34.9034...° ***(A1)***

Note: Award ***(A0)*** if unrounded answer does not round to 35. Award ***(G2)*** if 34.9034... seen without working.

angle $\text{A}\overset{\wedge}{\text{B}}\text{C}$ = 180 − (34.9034... + 60) ***(M1)***

Note: Award ***(M1)*** for subtracting their angle BAC + 60 from 180.

85.0965...° ***(A1)***

85° ***(AG)***

Note: Both the unrounded and rounded value must be seen for the final ***(A1)*** to be awarded. If the candidate rounds 34.9034...° to 35° while substituting to find angle $\text{A}\overset{\wedge}{\text{B}}\text{C}$, the final ***(A1)*** can be awarded but **only** if both 34.9034...° and 35° are seen.
If 85 is used as part of the workings, award at most ***(M1)(A0)(A0)(M0)(A0)(AG)***. This is the reverse process and not accepted.

2b. Use Giovanni's diagram to calculate the length of AX. *[2 marks]*

Markscheme

sin 85... × 56 ***(M1)***

= 55.8 (55.7869...) (m) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

2c. Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. *[2 marks]*

Markscheme

$\sqrt{56^2 - 55.7869\ldots^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the Pythagoras theorem formula. Follow through from part (a)(ii).

OR

cos(85) × 56 ***(M1)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

= 4.88 (4.88072...) (m) ***(A1)*****(ft)*****(G2)***

Note: Accept 4.73 (4.72863...) (m) from using their 3 s.f answer. Accept equivalent methods.

[2 marks]

Giovanni’s tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

2d. Find the percentage error on Giovanni’s diagram. *[2 marks]*

Markscheme

$\left|\frac{4.88-3.9}{3.9}\right| \times 100$ ***(M1)***

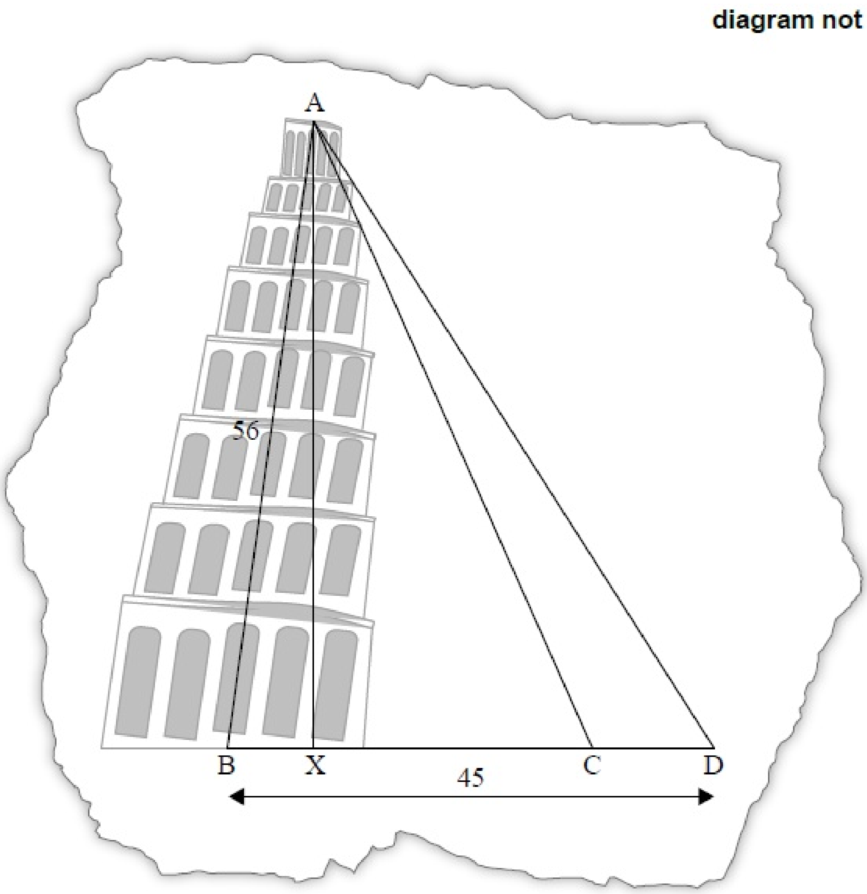
Note: Award ***(M1)*** for correct substitution into the percentage error formula.

= 25.1 (25.1282) (%) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a)(iii).

[2 marks]

2e. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. *[3 marks]*



Find the angle of elevation of A from D.

Markscheme

$\tan^{-1}\left(\frac{55.7869\ldots}{40.11927\ldots}\right)$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for their 40.11927… seen. Award ***(M1)*** for correct substitution into trigonometric ratio.

OR

$(37 - 4.88072\ldots)^2 + 55.7869\ldots^2$

(AC =) 64.3725…

$64.3726\ldots^2 + 8^2 - 2 \times 8 \times 64.3726\ldots \times \cos 120$

(AD =) 68.7226…

$\frac{\sin 120}{68.7226\ldots} = \frac{\sin \mathrm{A\hat{D}C}}{64.3725\ldots}$ ***(A1)*****(ft)*****(M1)***

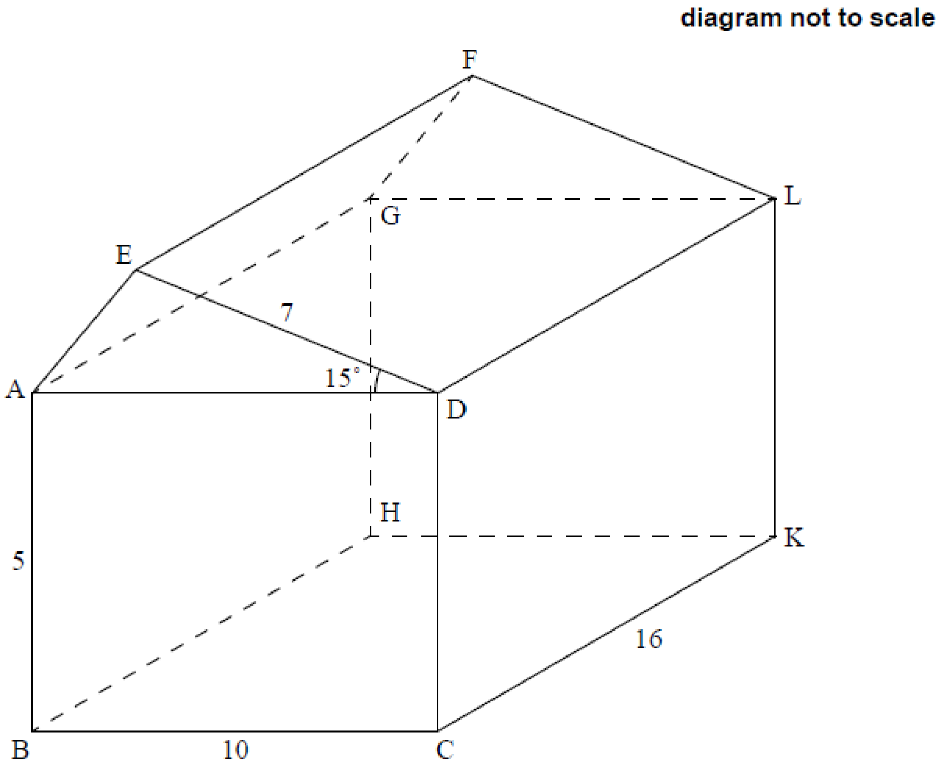
Note: Award ***(A1)*****(ft)** for their correct values seen, ***(M1)*** for correct substitution into the sine formula.

= 54.3° (54.2781…°) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a). Accept equivalent methods.

[3 marks]

Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

3a. Calculate the area of triangle EAD. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(Area of EAD =) $\frac{1}{2} \times 10 \times 7 \times \sin 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into area of a triangle formula, **(A1)** for correct substitution. Award ***(M0)(A0)(A0)*** if EAD or AED is considered to be a right-angled triangle.

= 9.06 m^2 (9.05866... m^2) ***(A1) (G3)***

[3 marks]

3b. Calculate the **total** volume of the barn. *[3 marks]*

Markscheme

(10 × 5 × 16) + (9.05866… × 16) ***(M1)(M1)***

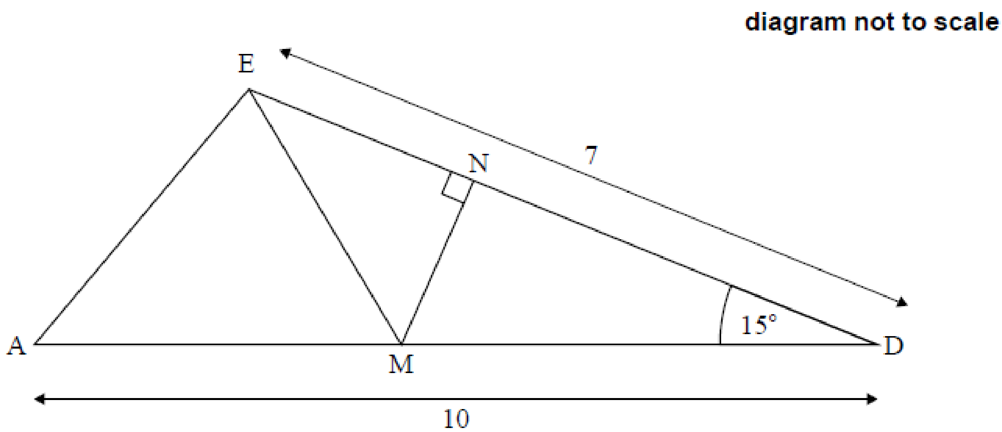
Note: Award ***(M1)*** for correct substitution into volume of a cuboid, ***(M1)*** for adding the correctly substituted volume of their triangular prism.

= 945 m^3 (944.938… m^3) ***(A1)*(ft)** ***(G3)***

Note: Follow through from part (a).

[3 marks]

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

3c. Calculate the length of MN. *[2 marks]*

Markscheme

$\frac{\text{MN}}{5} = \sin 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trigonometric equation.

(MN =) 1.29(m) (1.29409… (m)) ***(A1) (G2)***

[2 marks]

3d. Calculate the length of AE. *[3 marks]*

Markscheme

(AE^2 =) $10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, and ***(A1)*** for correct substitution.

(AE =) 3.71(m) (3.71084... (m)) ***(A1) (G2)***

[3 marks]

Farmer Brown believes that N is the midpoint of ED.

3e. Show that Farmer Brown is incorrect. *[3 marks]*

Markscheme

$ND^2 = 5^2 - (1.29409\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$\frac{1.29409\ldots}{\mathrm{ND}} = \tan 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into tangent.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$\frac{\mathrm{ND}}{5} = \cos 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$ND^2 = 1.29409\ldots^2 + 5^2 - 2 \times 1.29409\ldots \times 5 \times \cos 75^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

$4.82962\ldots \neq 3.5$ (ND $\neq$ 3.5) ***(R1)*****(ft)**

OR

$4.82962\ldots \neq 2.17038\ldots$ (ND $\neq$ NE) ***(R1)*****(ft)**

(hence Farmer Brown is incorrect)

Note: Do not award ***(M0)(A0)(R1)*****(ft)**. Award ***(M0)(A0)(R0)*** for a correct conclusion without any working seen.

[3 marks]

3f. Calculate the **total** length of metal required for one support. *[4 marks]*

Markscheme

(EM^2 =) 1.29409...^{2} + (7 − 4.82962...)^2 ***(M1)***

Note: Award ***(M1)*** for their correct substitution into Pythagoras theorem.

OR

(EM^2 =) $5^2 + 7^2 - 2 \times 5 \times 7 \times \cos 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule formula.

(EM =) 2.53(m) (2.52689...(m)) ***(A1)*(ft)** ***(G2)*(ft)**

Note: Follow through from parts (c), (d) and (e).

(Total length =) 2.52689... + 3.71084... + 1.29409... +10 + 7 ***(M1)***

Note: Award ***(M1)*** for adding their EM, their parts (c) and (d), and 10 and 7.

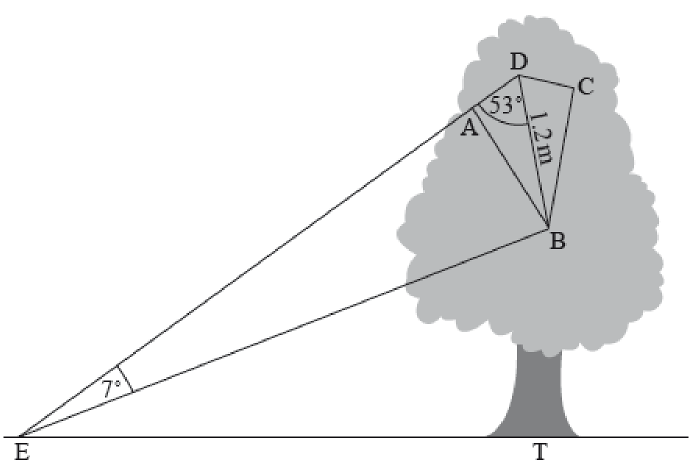
= 24.5 (m) (24.5318... (m)) ***(A1)*(ft)** ***(G4)***

Note: Follow through from parts (c) and (d).

[4 marks]

Emily's kite ABCD is hanging in a tree. The plane ABCDE is vertical.

Emily stands at point E at some distance from the tree, such that EAD is a straight line and angle BED = 7°. Emily knows BD = 1.2 metres and angle BDA = 53°, as shown in the diagram



4a. Find the length of EB. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

Units are required in parts (a) and (c).

$\frac{\mathrm{EB}}{\sin 53^\circ} = \frac{1.2}{\sin 7^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into sine formula, ***(A1)*** for correct substitution.

$(\mathrm{EB} =)\ 7.86\ \mathrm{m}$ **OR** $786\ \mathrm{cm}\ (7.86385\ldots\ \mathrm{m}$ **OR** $786.385\ldots\ \mathrm{cm})$ ***(A1)*** ***(C3)***

[3 marks]

T is a point at the base of the tree. ET is a horizontal line. The angle of elevation of A from E is 41°.

4b. Write down the angle of elevation of B from E. *[1 mark]*

Markscheme

34° ***(A1)*** ***(C1)***

[1 mark]

4c. Find the vertical height of B above the ground. *[2 marks]*

Markscheme

Units are required in parts (a) and (c).

$\sin 34^\circ = \frac{\text{height}}{7.86385\ldots}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into a trigonometric ratio.

$(\text{height} =)\ 4.40\ \text{m}$ **OR** $440\ \text{cm}\ (4.39741\ldots\ \text{m}$ **OR** $439.741\ldots\ \text{cm})$ ***(A1)*****(ft)** ***(C2)***

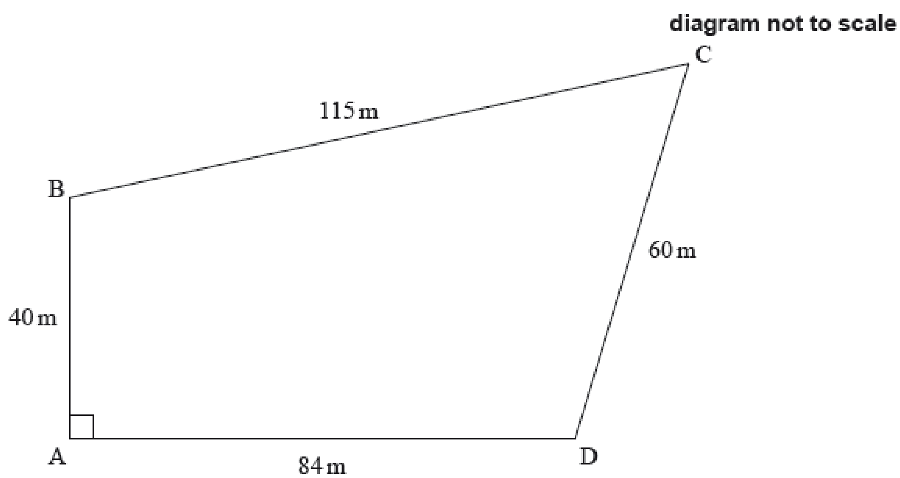
Note: Accept "BT" used for height. Follow through from parts (a) and (b). Use of 7.86 gives an answer of 4.39525….

[2 marks]

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD.

The lengths of the sides are $\text{AB} = 40\ \text{m}$, $\text{BC} = 115\ \text{m}$, $\text{CD} = 60\ \text{m}$, $\text{AD} = 84\ \text{m}$ and angle $\text{B}\hat{\text{A}}\text{D} = 90^\circ$.

This information is shown on the diagram.



5a. Show that $\text{BD} = 93\ \text{m}$ correct to the nearest metre. *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$\mathrm{BD}^2 = 40^2 + 84^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras.

Accept correct substitution into cosine rule.

$\mathrm{BD} = 93.0376\ldots$ ***(A1)***

$= 93$ ***(AG)***

Note: Both the rounded and unrounded value must be seen for the ***(A1)*** to be awarded.

[2 marks]

5b. Calculate angle $\mathrm{B\hat{C}D}$. *[3 marks]*

Markscheme

$\cos C = \frac{115^2+60^2-93^2}{2\times115\times60}$ $\left(93^2 = 115^2 + 60^2 - 2\times115\times60\times\cos C\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine formula, ***(A1)*** for correct substitutions.

$= 53.7^\circ$ $(53.6679\ldots^\circ)$ ***(A1)(G2)***

[3 marks]

5c. Find the area of ABCD. *[4 marks]*

Markscheme

$\frac{1}{2}(40)(84)+\frac{1}{2}(115)(60)\sin(53.6679\ldots)$ ***(M1)(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for correct substitution into right-angle triangle area. Award ***(M1)*** for substitution into area of triangle formula and ***(A1)*****(ft)** for correct substitution.

$=4460\text{ m}^2\ (4459.30\ldots\text{ m}^2)$ ***(A1)*****(ft)*****(G3)***

Notes: Follow through from part (b).

[4 marks]

The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$\text{area}=\frac{(\text{AB}+\text{CD})(\text{AD}+\text{BC})}{4}$.

Abdallah uses this formula to estimate the area of his plot of land.

5d. Calculate Abdallah’s estimate for the area. *[2 marks]*

Markscheme

$\frac{(40+60)(84+115)}{4}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the area formula used by ‘Ancient Egyptians’.

$=4980\text{ m}^2\ (4975\text{ m}^2)$ ***(A1)(G2)***

[2 marks]

5e. Find the percentage error in Abdallah’s estimate. *[2 marks]*

Markscheme

$\left|\frac{4975-4459.30\ldots}{4459.30\ldots}\right| \times 100$ ***(M1)***

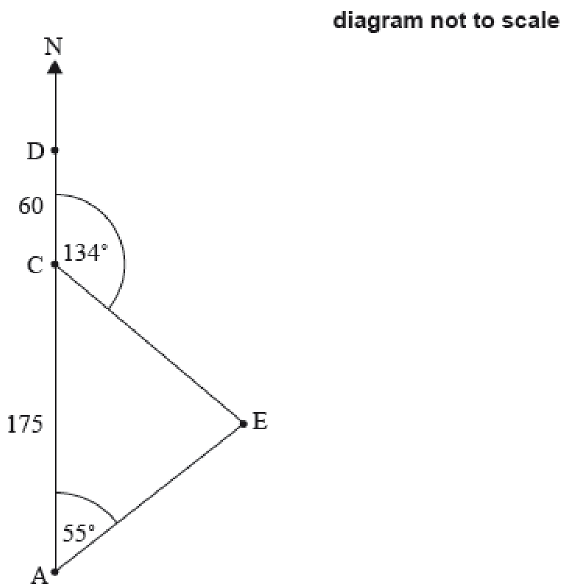
Notes: Award ***(M1)*** for correct substitution into percentage error formula.

$= 11.6\ (\%)\ (11.5645\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Follow through from parts (c) and (d)(i).

[2 marks]

A ship is sailing north from a point A towards point D. Point C is 175 km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.



6. When the ship reaches D, it changes direction and travels directly to the island at 50 km per hour. At the same time as the ship changes direction, a boat starts travelling to the island from a point B. This point B lies on (AC), between A and C, and is the closest point to the island. The ship and the boat arrive at the island at the same time. Find the speed of the boat. *[5 marks]*

Markscheme

valid approach for locating B ***(M1)***

eg BE is perpendicular to ship's path, angle $\mathrm{B} = 90$

correct working for BE ***(A1)***

eg $\sin 46^\circ = \frac{\mathrm{BE}}{146.034},\ \mathrm{BE} = 146.034 \sin 46^\circ,\ 105.048$

valid approach for expressing time ***(M1)***

eg $t = \frac{d}{s},\ t = \frac{d}{r},\ t = \frac{192.612}{50}$

correct working equating time ***(A1)***

eg $\frac{146.034 \sin 46^\circ}{r} = \frac{192.612}{50},\ \frac{s}{105.048} = \frac{50}{192.612}$

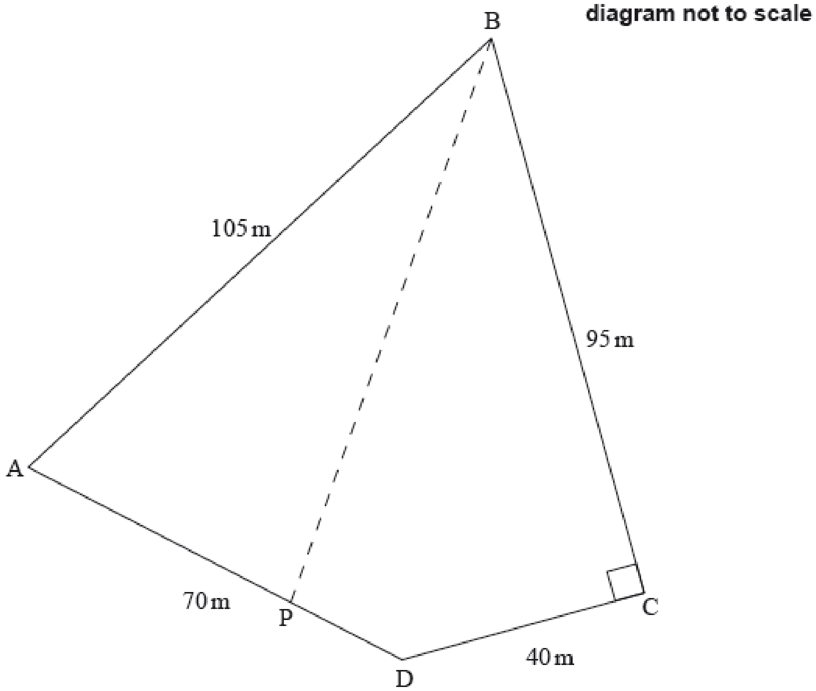
27.2694

27.3 (km per hour) ***A1*** ***N3***

[5 marks]

A farmer owns a plot of land in the shape of a quadrilateral ABCD.

$\mathrm{AB} = 105\mathrm{m}$, $\mathrm{BC} = 95\mathrm{m}$, $\mathrm{CD} = 40\mathrm{m}$, $\mathrm{DA} = 70\mathrm{m}$ and angle $\mathrm{DCB} = 90°$.



The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

7a. the length of BD; *[2 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

$(\mathrm{BD} =)\ \sqrt{95^2 + 40^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' theorem.

$= 103\ (\mathrm{m})\ \left(103.077\ldots,\ 25\sqrt{17}\right)$ ***(A1)(G2)***

[2 marks]

7b. the size of angle DAB; *[3 marks]*

Markscheme

$\cos \mathrm{B\hat{A}D} = \frac{105^2+70^2-(103.077\ldots)^2}{2\times105\times70}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into cosine rule, ***(A1)*****(ft)** for their correct substitutions. Follow through from part (a).

$(\mathrm{B\hat{A}D}) = 68.9^\circ\ (68.8663\ldots)$ ***(A1)*****(ft)*****(G2)***

Note: If their 103 used, the answer is $68.7995\ldots$

[3 marks]

7c. the area of triangle ABD; *[3 marks]*

Markscheme

$(\text{Area of ABD} =)\frac{1}{2} \times 105 \times 70 \times \sin(68.8663\ldots)$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into the trig form of the area of a triangle formula.

Award ***(A1)*****(ft)** for their correct substitutions.

Follow through from part (b).

If 68.8° is used the area $= 3426.28\ldots\ \mathrm{m}^2$.

$= 3430\ \mathrm{m}^2\ (3427.82\ldots)$ ***(A1)*****(ft)*****(G2)***

[3 marks]

7d. the area of quadrilateral ABCD; *[2 marks]*

Markscheme

$\text{area of ABCD} = \frac{1}{2} \times 40 \times 95 + 3427.82\ldots$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted area of triangle formula **added** to their answer to part (c).

$= 5330\ \text{m}^2\ (5327.83\ldots)$ ***(A1)*(ft)*(G2)***

[2 marks]

7e. the length of AP; *[3 marks]*

Markscheme

$\frac{1}{2} \times 105 \times \text{AP} \times \sin(68.8663\ldots) = 0.5 \times 5327.82\ldots$ ***(M1)(M1)***

Notes: Award ***(M1)*** for the correct substitution into triangle formula.

Award ***(M1)*** for equating their triangle area to half their part (d).

$(\text{AP} =)\ 54.4\ (\text{m})\ (54.4000\ldots)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from parts (b) and (d).

[3 marks]

7f. the length of the fence, BP. *[3 marks]*

Markscheme

$\mathrm{BP}^2 = 105^2 + (54.4000\ldots)^2 - 2 \times 105 \times (54.4000\ldots) \times \cos(68.8663\ldots)$

(M1)(A1)**(ft)**

Notes: Award ***(M1)*** for substituted cosine rule formula.

Award ***(A1)*****(ft)** for their correct substitutions. Accept the exact fraction $\frac{53}{147}$ in place of $\cos(68.8663\ldots)$.

Follow through from parts (b) and (e).

$(\mathrm{BP} =)\ 99.3\ (\mathrm{m})\ (99.3252\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: If 54.4 and $\cos(68.9)$ are used the answer is $99.3567\ldots$

[3 marks]