Geometry and trigonometry
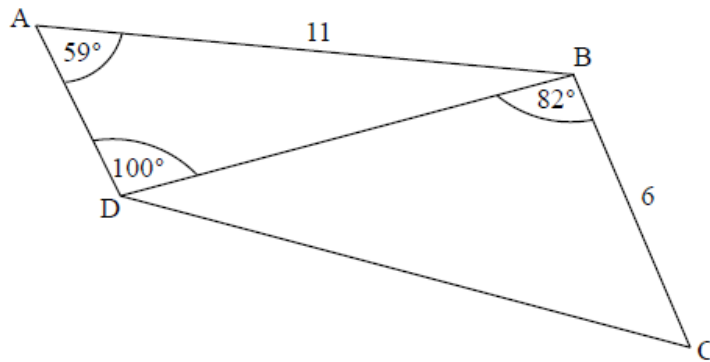## 28.11 *[78 marks]*

The following diagram shows quadrilateral ABCD.

diagram not to scale

$AB = 11\,cm,\ BC = 6\,cm,\ B\hat{A}D = 100°,\ and\ C\hat{B}D = 82°$
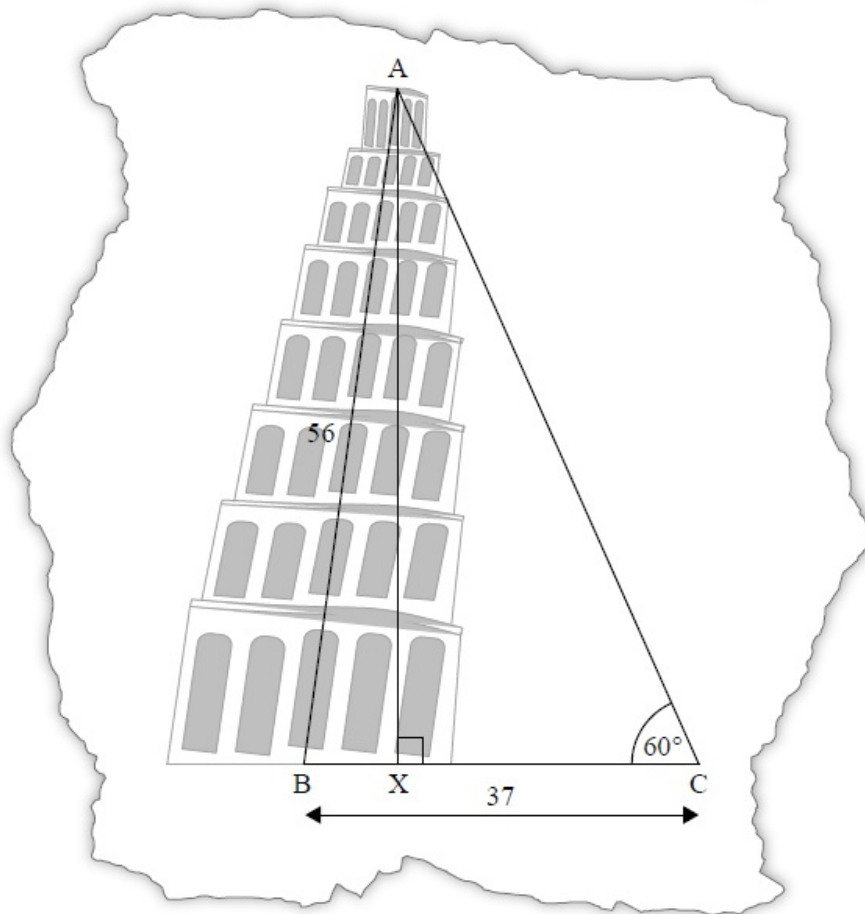
1a. Find DB. *[3 marks]*

1b. Find DC. *[3 marks]*

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni's diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale

2a. Use Giovanni's diagram to show that angle ABC, the angle at which the Tower is leaning relative to the
horizontal, is 85° to the nearest degree.   *[5 marks]*

2b. Use Giovanni's diagram to calculate the length of AX.   *[2 marks]*

2c. Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower.   *[2 marks]*
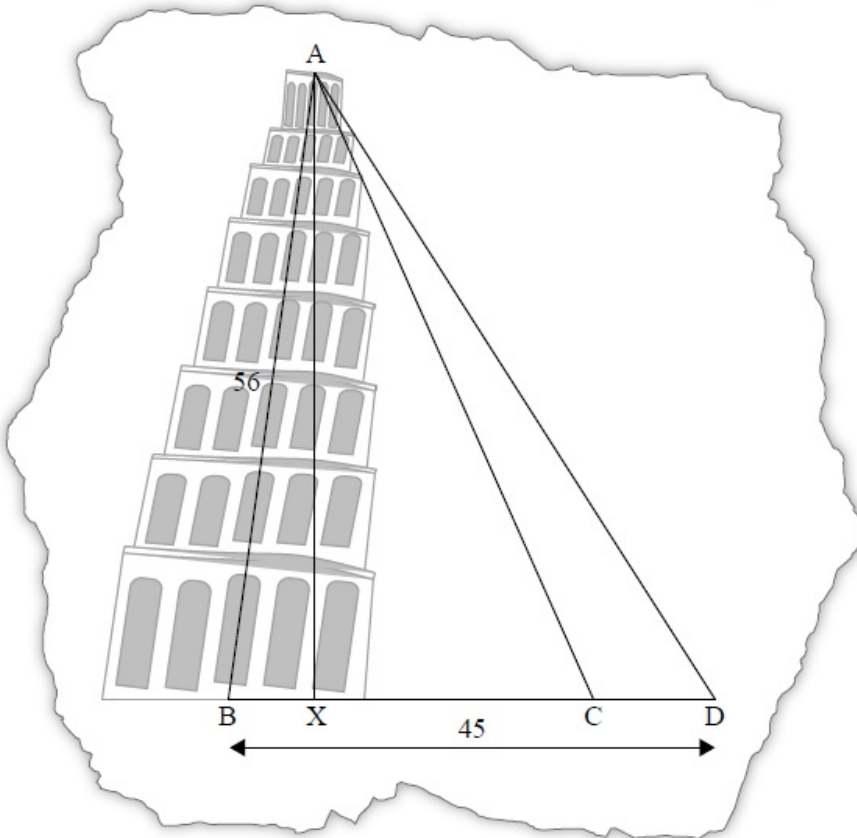
Giovanni's tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

2d. Find the percentage error on Giovanni's diagram.   *[2 marks]*

2e. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed.
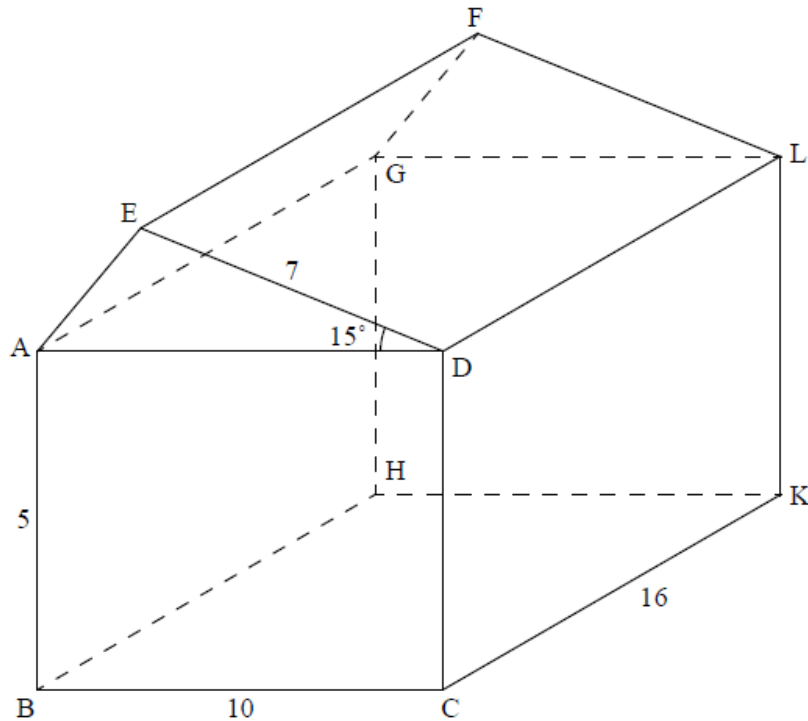
[3 marks]

diagram not to scale

Find the angle of elevation of A from D.

Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

diagram not to scale

The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
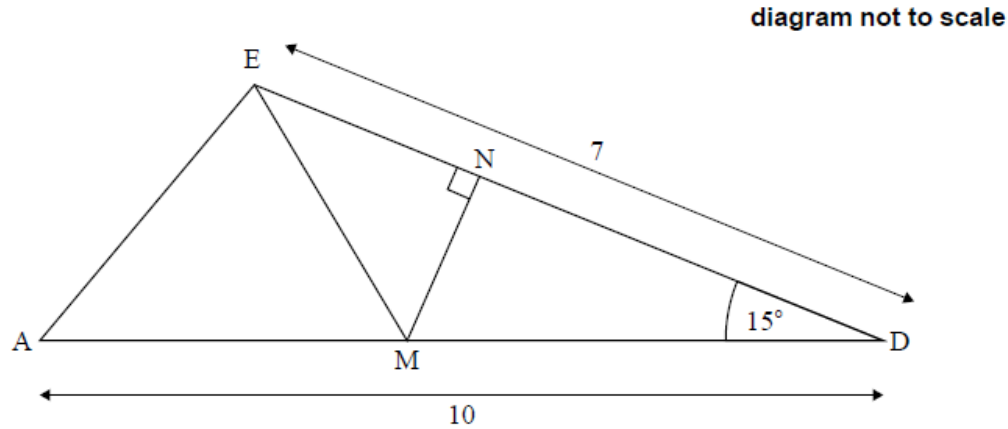The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

3a. Calculate the area of triangle EAD.   *[3 marks]*

3b. Calculate the **total** volume of the barn.   *[3 marks]*

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.

diagram not to scale

ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

3c. Calculate the length of MN.   *[2 marks]*

3d. Calculate the length of AE.   *[3 marks]*

Farmer Brown believes that N is the midpoint of ED.

3e. Show that Farmer Brown is incorrect.   *[3 marks]*

3f. Calculate the **total** length of metal required for one support.   *[4 marks]*

Emily's kite ABCD is hanging in a tree. The plane ABCDE is vertical.

Emily stands at point E at some distance from the tree, such that EAD is a straight line and angle BED = 7°. Emily knows BD = 1.2 metres and angle BDA = 53°, as shown in the diagram
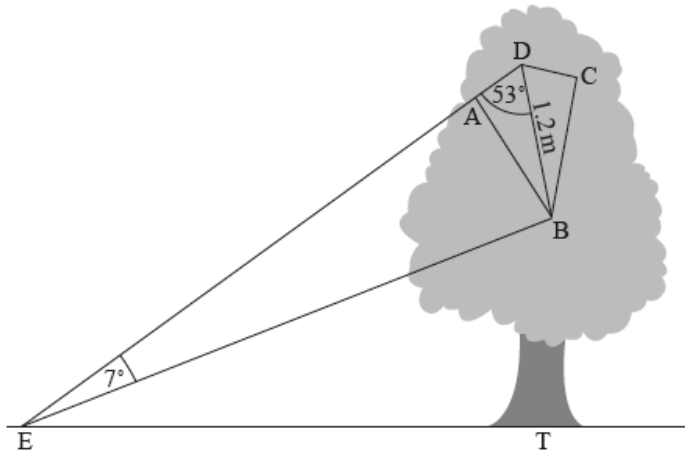
4a. Find the length of EB.   [3 marks]

T is a point at the base of the tree. ET is a horizontal line. The angle of elevation of A from E is 41°.

4b. Write down the angle of elevation of B from E.   [1 mark]
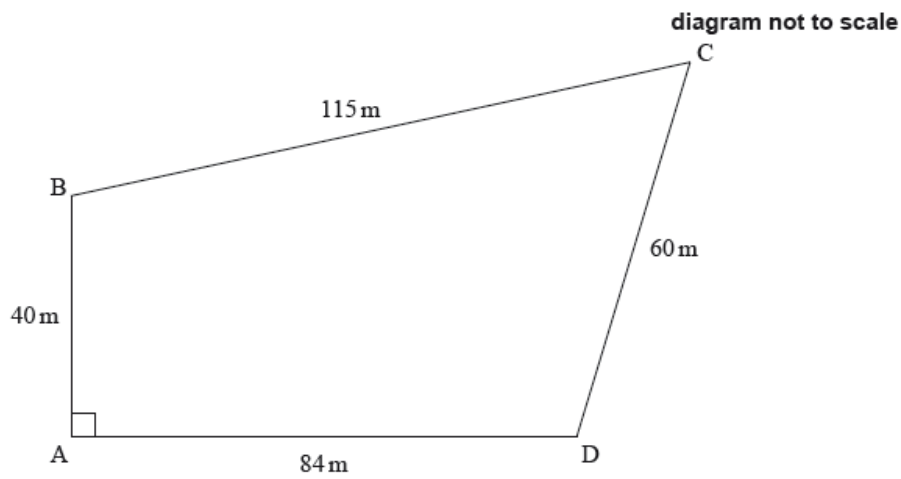
4c. Find the vertical height of B above the ground.   [2 marks]

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD.

The lengths of the sides are $AB = 40$ m, $BC = 115$ m, $CD = 60$ m, $AD = 84$ m and angle $B\hat{A}D = 90°$.

This information is shown on the diagram.

diagram not to scale

5a. Show that $BD = 93$ m correct to the nearest metre. *[2 marks]*

5b. Calculate angle $B\hat{C}D$. *[3 marks]*

5c. Find the area of ABCD. *[4 marks]*

The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$$\text{area} = \frac{(AB+CD)(AD+BC)}{4}.$$

Abdallah uses this formula to estimate the area of his plot of land.

5d. Calculate Abdallah's estimate for the area. *[2 marks]*

5e. Find the percentage error in Abdallah's estimate. *[2 marks]*

A ship is sailing north from a point A towards point D. Point C is 175 km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.

diagram not to scale
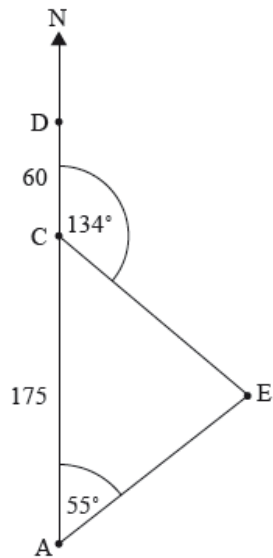
6. When the ship reaches D, it changes direction and travels directly to the island at 50 km per hour. At the same time as the ship changes direction, a boat starts travelling to the island from a point B. This point B lies on (AC), between A and C, and is the closest point to the island. The ship and the boat arrive at the island at the same time. Find the speed of the boat. *[5 marks]*

A farmer owns a plot of land in the shape of a quadrilateral ABCD.

$AB = 105m, BC = 95m, CD = 40m, DA = 70m$ and angle $DCB = 90°$.

diagram not to scale
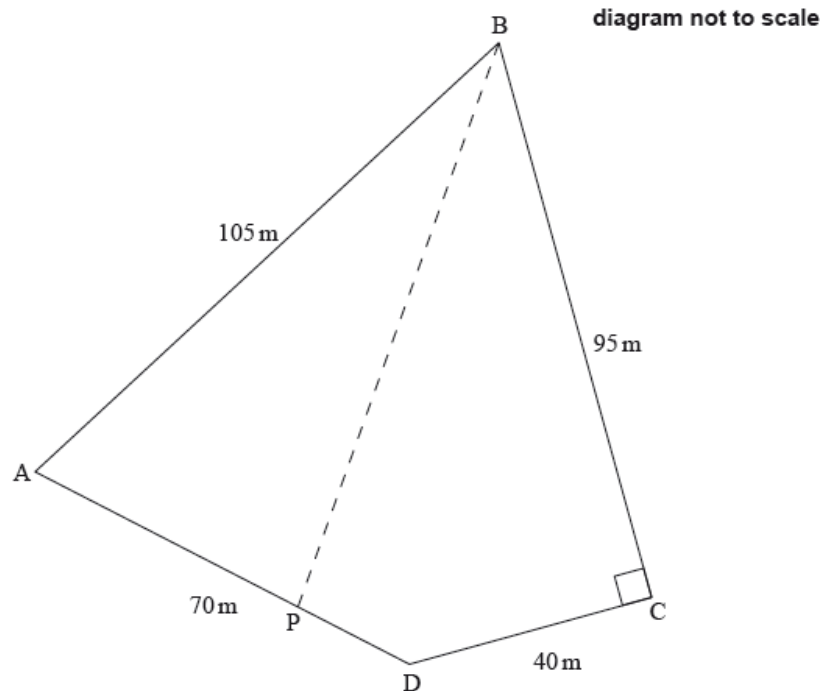
The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

7a. the length of BD;   *[2 marks]*

7b. the size of angle DAB;   *[3 marks]*

7c. the area of triangle ABD;   *[3 marks]*

7d. the area of quadrilateral ABCD;   *[2 marks]*

7e. the length of AP;   *[3 marks]*

7f. the length of the fence, BP.   *[3 marks]*