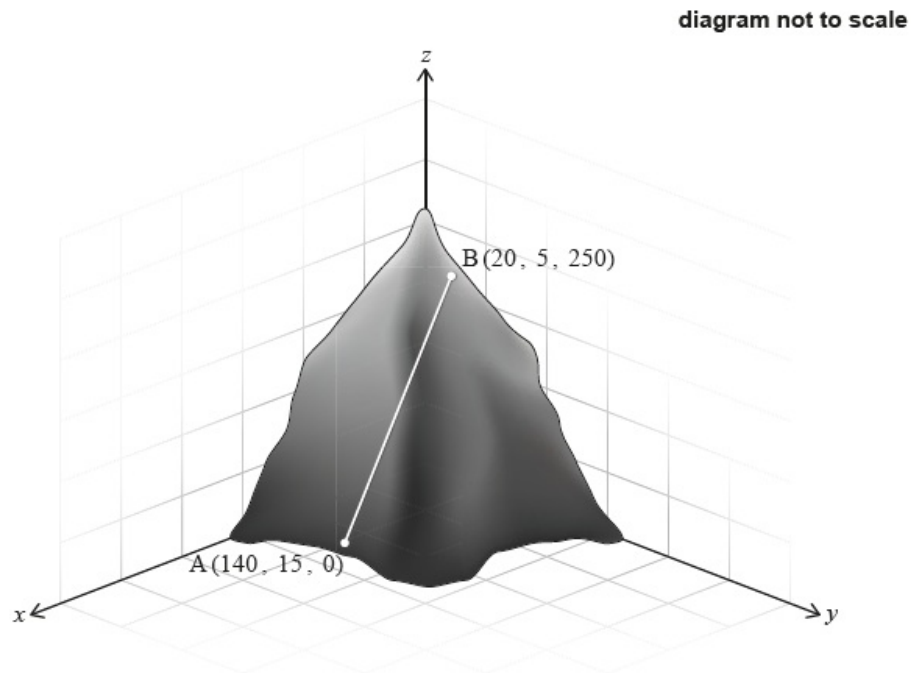


Geometry and trigonometry

29.11 [47 marks]

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.



The locations of the stations on the railway can be described by coordinates in reference to x , y , and z -axes, where the x and y axes are in the horizontal plane and the z -axis is vertical.

The ground level station A has coordinates $(140, 15, 0)$ and station B , located near the top of the hill, has coordinates $(20, 5, 250)$. All coordinates are given in metres.

1a. Find the distance between stations A and B . [2 marks]

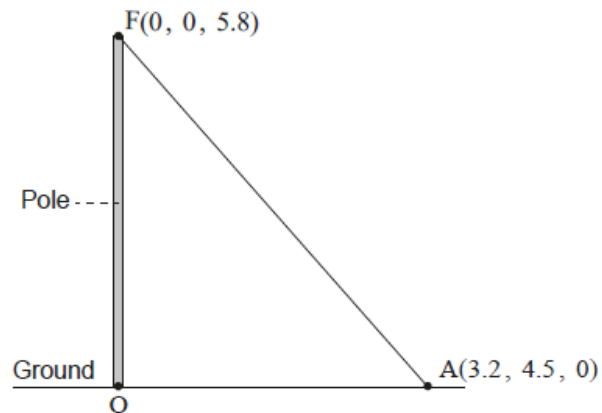
Station M is to be built halfway between stations A and B .

1b. Find the coordinates of station M . [2 marks]

1c. Write down the height of station M , in metres, above the ground. [1 mark]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O , of a coordinate system in which the top, F , of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F .

One of the ropes is attached to the ground at a point A with coordinates $(3.2, 4.5, 0)$. The rope forms a straight line from A to F .

2a. Find the length of the rope connecting A to F .

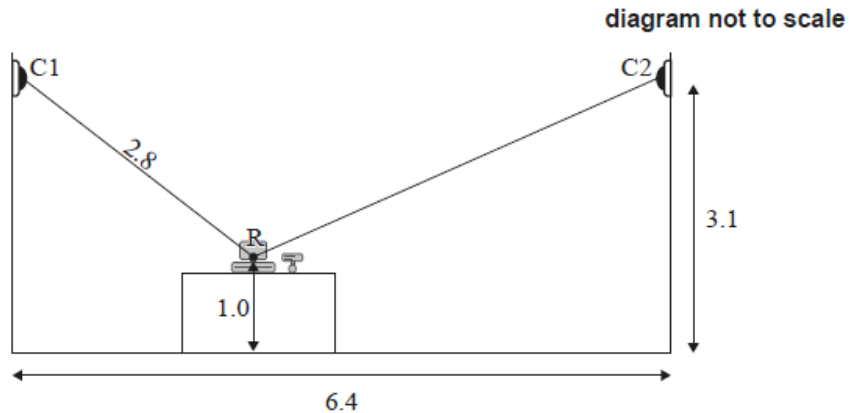
[2 marks]

2b. Find \hat{FAO} , the angle the rope makes with the ground.

[2 marks]

The owner of a convenience store installs two security cameras, represented by points C_1 and C_2 . Both cameras point towards the centre of the store's cash register, represented by the point R .

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of 3.1 m, and the horizontal distance between the cameras is 6.4 m. The cash register is sitting on a counter so that its centre, R , is 1.0 m above the floor.

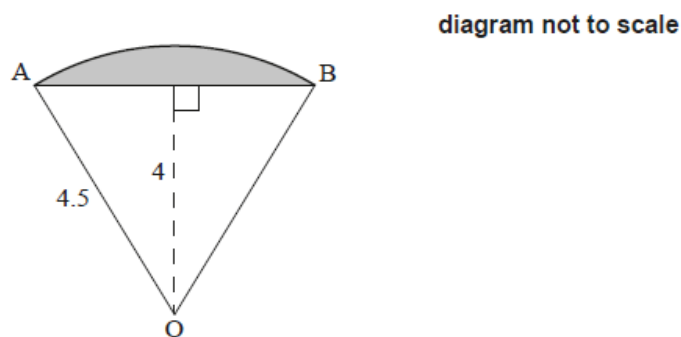
The distance from Camera 1 to the centre of the cash register is 2.8 m.

3a. Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2 marks]

3b. Calculate the distance from Camera 2 to the centre of the cash register. [4 marks]

3c. Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2 marks]

A sector of a circle, centre O and radius 4.5 m, is shown in the following diagram.

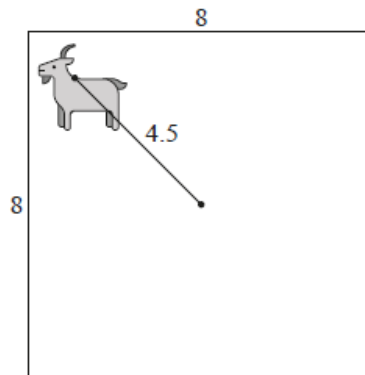


4a. Find the angle $\hat{A}OB$. [3 marks]

4b. Find the area of the shaded segment. [5 marks]

A square field with side 8 m has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to 4.5 m from the post.

diagram not to scale



[Source: mynamepong, n.d. Goat [image online] Available at: <https://thenounproject.com/term/goat/1761571/> This file is licensed under the Creative Commons Attribution-ShareAlike 3.0 Unported (CC BY-SA 3.0) <https://creativecommons.org/licenses/by-sa/3.0/deed.en> [Accessed 22 April 2010] Source adapted.]

4c. Find the area of a circle with radius 4.5 m. [2 marks]

4d. Find the area of the field that can be reached by the goat. [3 marks]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

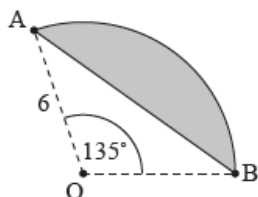
The goat eats grass at the rate of $\frac{dV}{dt} = 0.3 te^{-t}$.

4e. Find the value of t at which the goat is eating grass at the greatest rate. [2 marks]

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius 6 m, such that $\angle AOB = 135^\circ$. The straight border of the lawn is defined by chord $[AB]$.

The lawn is shown as the shaded region in the following diagram.

diagram not to scale

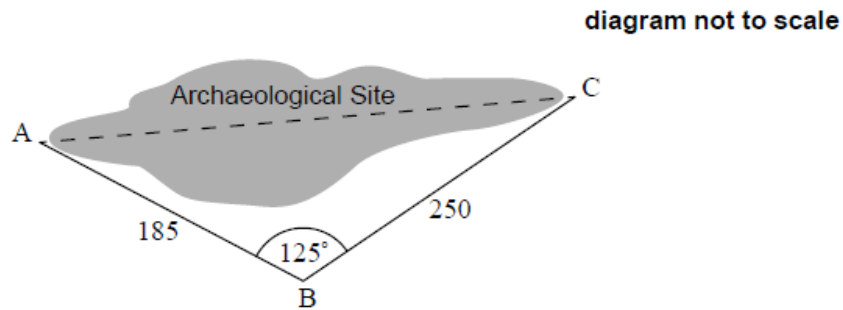


5a. A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3 marks]

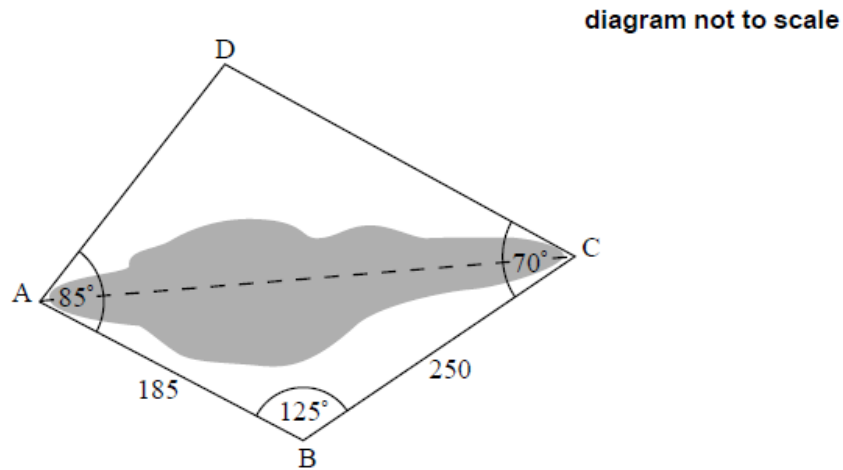
5b. Find the area of the lawn.

[4 marks]

An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle $\hat{A}B C$ is 125° .



The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle $\hat{B}A D$ is to be made equal to 85° and angle $\hat{B}C D$ is to be made equal to 70° as shown in the following diagram.



6a. Find the size of angle $\hat{C}A D$.

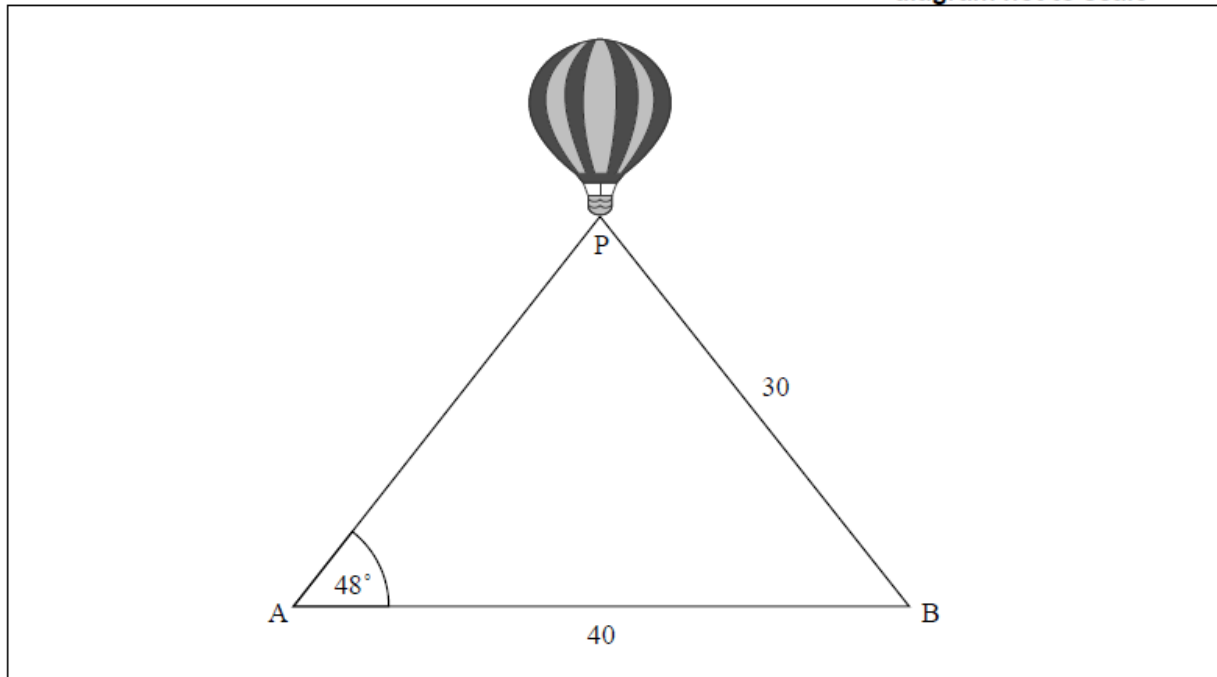
[1 mark]

6b. Find the size of angle $\hat{A}C D$.

[2 marks]

Two fixed points, A and B, are 40 m apart on horizontal ground. Two straight ropes, AP and BP, are attached to the same point, P, on the base of a hot air balloon which is vertically above the line AB. The length of BP is 30 m and angle BAP is 48° .

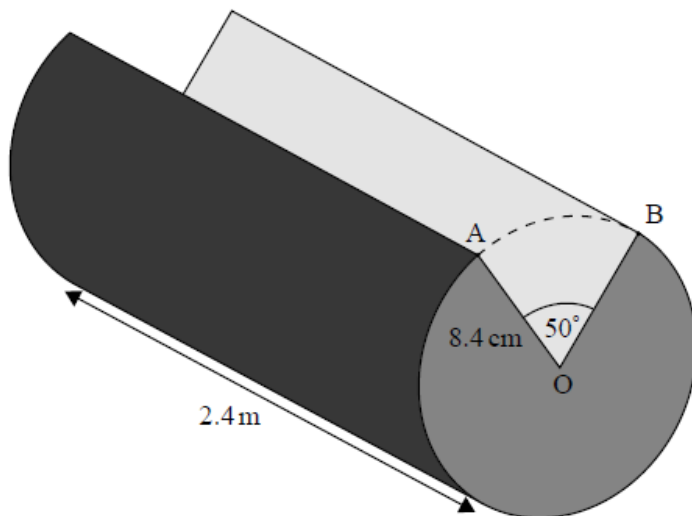
diagram not to scale



7. On the diagram, draw and label with an x the angle of depression of B from P. [1 mark]

8. Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram. [4 marks]

diagram not to scale



Find the volume of this log.

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