M10/5/MATHL/HP1/ENG/TZ1/XX

22107203

International Baccalaureate®
Baccalauréat International
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MATHEMATICS
HIGHER LEVEL
PAPER 1

Wednesday 5 May 2010 (afternoon)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Given that $Ax^3 + Bx^2 + x + 6$ is exactly divisible by $(x+1)(x-2)$, find the value of A and the value of B.

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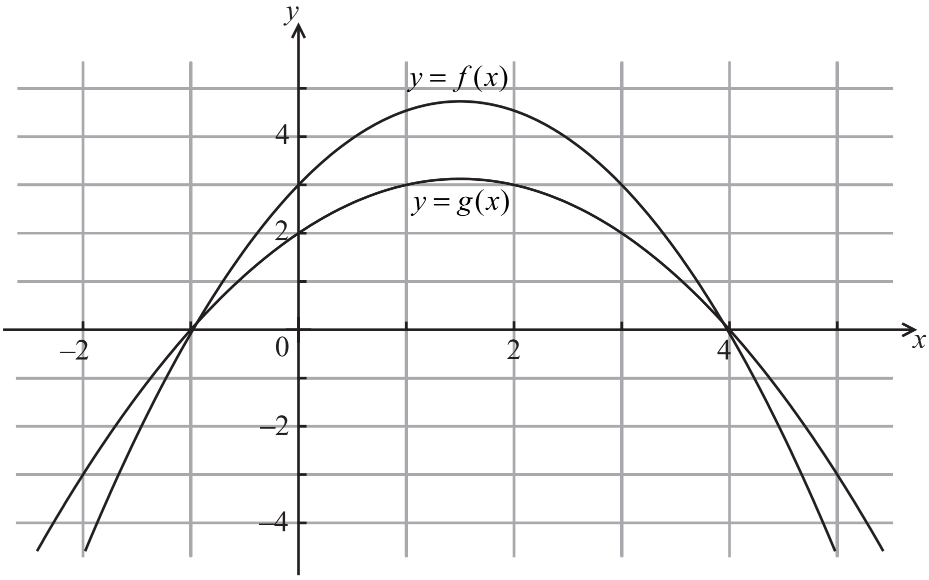
2. *[Maximum mark: 4]*

Shown below are the graphs of $y = f(x)$ and $y = g(x)$.



If $(f \circ g)(x) = 3$, find all possible values of x.

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3. *[Maximum mark: 7]*

(a) Show that the two planes

$$\pi_1 : x + 2y - z = 1$$
$$\pi_2 : x + z = -2$$

are perpendicular. *[3 marks]*

(b) Find the equation of the plane π_3 that passes through the origin and is perpendicular to both π_1 and π_2. *[4 marks]*

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4. *[Maximum mark: 5]*

Solve the equation $4^{x-1} = 2^x + 8$.

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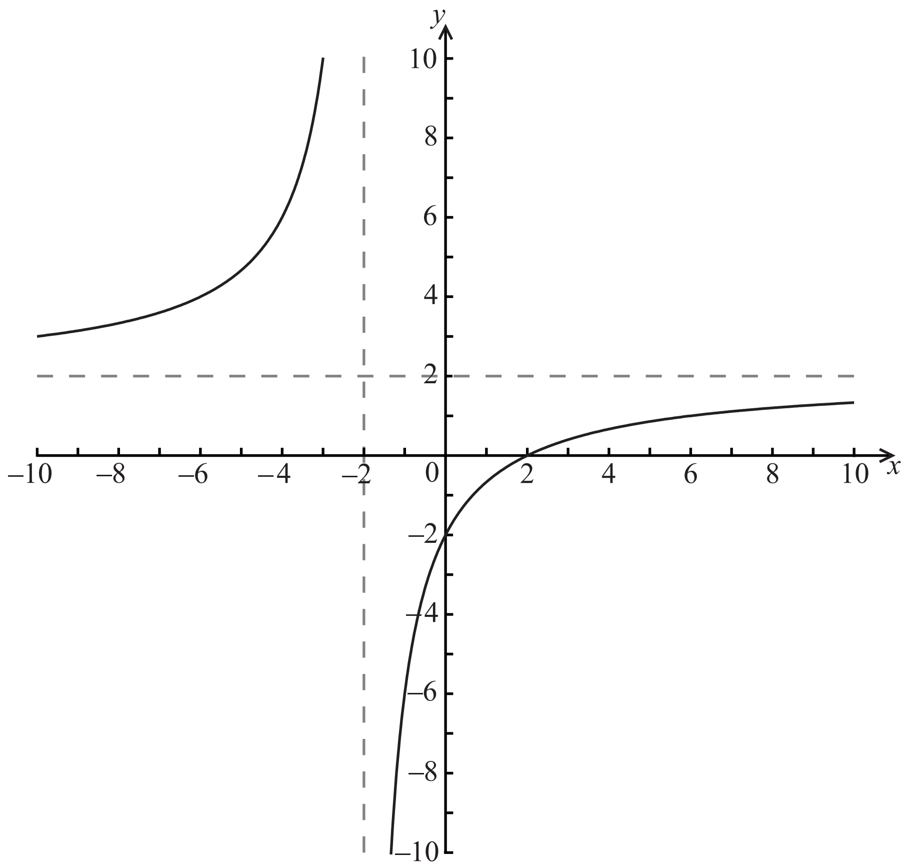
5. *[Maximum mark: 8]*

The graph of $y = \dfrac{a+x}{b+cx}$ is drawn below.



(a) Find the value of a, the value of b and the value of c. *[4 marks]*

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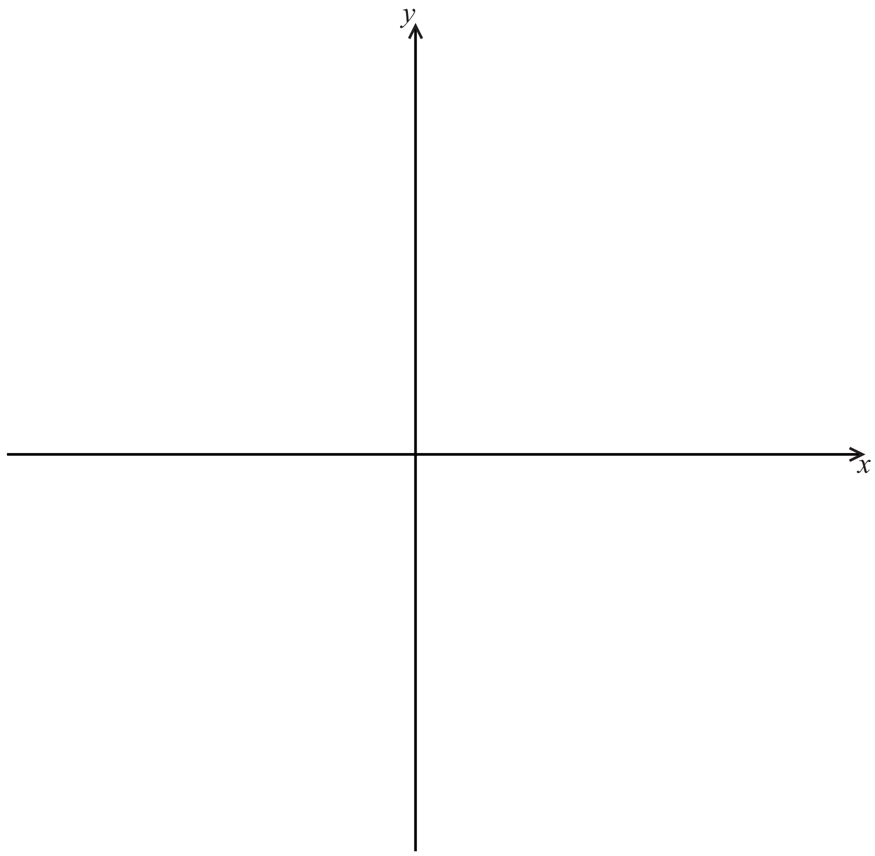
(This question continues on the following page)

(Question 5 continued)

(b) Using the values of a, b and c found in part (a), sketch the graph of $y = \left| \dfrac{b + cx}{a + x} \right|$ on the axes below, showing clearly all intercepts and asymptotes. *[4 marks]*



6. *[Maximum mark: 4]*

Consider the vectors $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{a} + \boldsymbol{b}$. Show that if $|\boldsymbol{a}| = |\boldsymbol{b}|$ then $(\boldsymbol{a} + \boldsymbol{b}) \bullet (\boldsymbol{a} - \boldsymbol{b}) = 0$. Comment on what this tells us about the parallelogram OACB.

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7. *[Maximum mark: 7]*

Two players, A and B, alternately throw a fair six-sided dice, with A starting, until one of them obtains a six. Find the probability that A obtains the first six.

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8. *[Maximum mark: 7]*

The region enclosed between the curves $y = \sqrt{x}\,e^x$ and $y = e\sqrt{x}$ is rotated through 2π about the x-axis. Find the volume of the solid obtained.

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9. *[Maximum mark: 7]*

(a) Given that $\alpha > 1$, use the substitution $u = \frac{1}{x}$ to show that

$$\int_1^\alpha \frac{1}{1+x^2}\,\mathrm{d}x = \int_{\frac{1}{\alpha}}^1 \frac{1}{1+u^2}\,\mathrm{d}u\ .$$ *[5 marks]*

(b) **Hence** show that $\arctan\alpha + \arctan\frac{1}{\alpha} = \frac{\pi}{2}$. *[2 marks]*

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10. *[Maximum mark: 6]*

The ten numbers $x_1, x_2, ..., x_{10}$ have a mean of 10 and a standard deviation of 3.

Find the value of $\sum_{i=1}^{10}(x_i - 12)^2$.

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 20]*

Consider $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 5x + 4}$.

(a) Find the equations of all asymptotes of the graph of f. *[4 marks]*

(b) Find the coordinates of the points where the graph of f meets the x and y axes. *[2 marks]*

(c) Find the coordinates of

(i) the maximum point and justify your answer;

(ii) the minimum point and justify your answer. *[10 marks]*

(d) Sketch the graph of f, clearly showing all the features found above. *[3 marks]*

(e) **Hence**, write down the number of points of inflexion of the graph of f. *[1 mark]*

12. *[Maximum mark: 20]*

A continuous random variable X has probability density function

$$f(x) = \begin{cases} 0, & x < 0 \\ a\mathrm{e}^{-ax}, & x \geq 0. \end{cases}$$

It is known that $\mathrm{P}(X < 1) = 1 - \dfrac{1}{\sqrt{2}}$.

(a) Show that $a = \dfrac{1}{2}\ln 2$. *[6 marks]*

(b) Find the median of X. *[5 marks]*

(c) Calculate the probability that $X < 3$ given that $X > 1$. *[9 marks]*

13. *[Maximum mark: 20]*

(a) Show that $\sin 2nx = \sin((2n+1)x)\cos x - \cos((2n+1)x)\sin x$. *[2 marks]*

(b) **Hence** prove, by induction, that

$$\cos x + \cos 3x + \cos 5x + \ldots + \cos((2n-1)x) = \frac{\sin 2nx}{2\sin x},$$

for all $n \in \mathbb{Z}^+$, $\sin x \neq 0$. *[12 marks]*

(c) Solve the equation $\cos x + \cos 3x = \frac{1}{2}$, $0 < x < \pi$. *[6 marks]*