Monday 28.11 *[32 marks]*

1. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3 \left(4x^3\right)$, where $x > 0$. *[5 marks]*

Markscheme

attempt to use change the base ***(M1)***

$\log_3 \sqrt{x} = \frac{\log_3 2}{2} + \log_3 \left(4x^3\right)$

attempt to use the power rule ***(M1)***

$\log_3 \sqrt{x} = \log_3 \sqrt{2} + \log_3 \left(4x^3\right)$

attempt to use product or quotient rule for logs, $\ln a + \ln b = \ln ab$ ***(M1)***

$\log_3 \sqrt{x} = \log_3 \left(4\sqrt{2}x^3\right)$

Note: The ***M*** marks are for attempting to use the relevant log rule and may be applied in any order and at any time during the attempt seen.

$\sqrt{x} = 4\sqrt{2}x^3$

$x = 32x^6$

$x^5 = \frac{1}{32}$ ***(A1)***

$x = \frac{1}{2}$ ***A1***

[5 marks]

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

2a. Write down the equation of the vertical asymptote. *[1 mark]*

Markscheme

$x = -1$ ***A1***

[1 mark]

2b. Write down the equation of the horizontal asymptote. *[1 mark]*

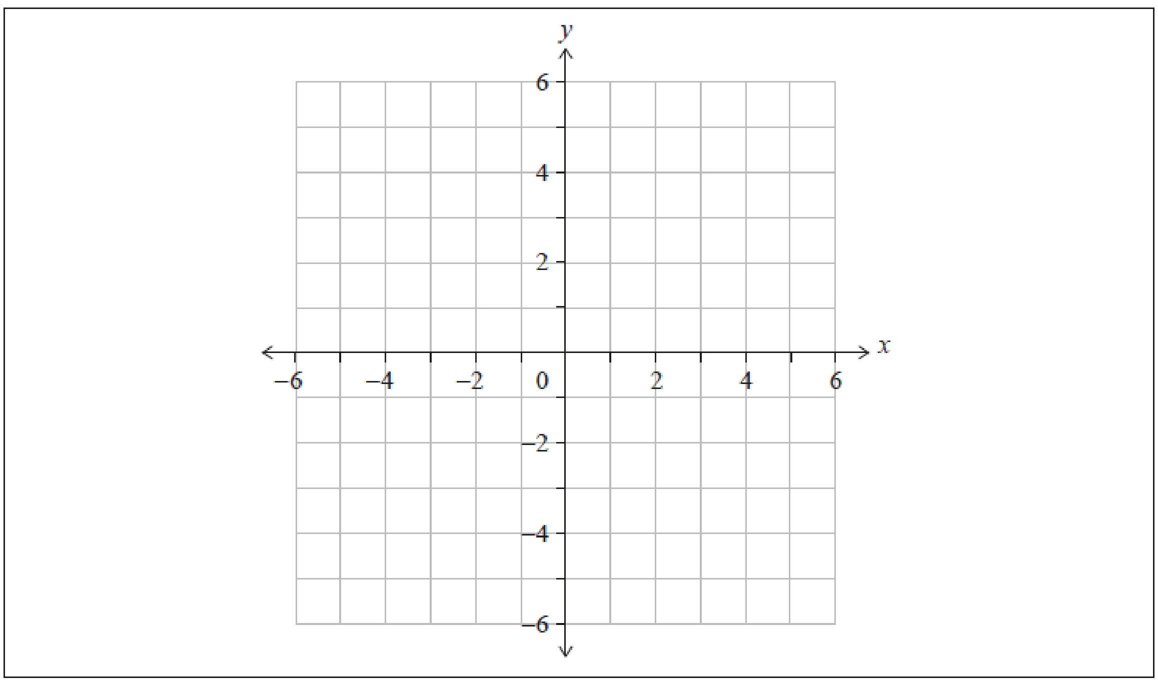
Markscheme

$y = 2$ ***A1***

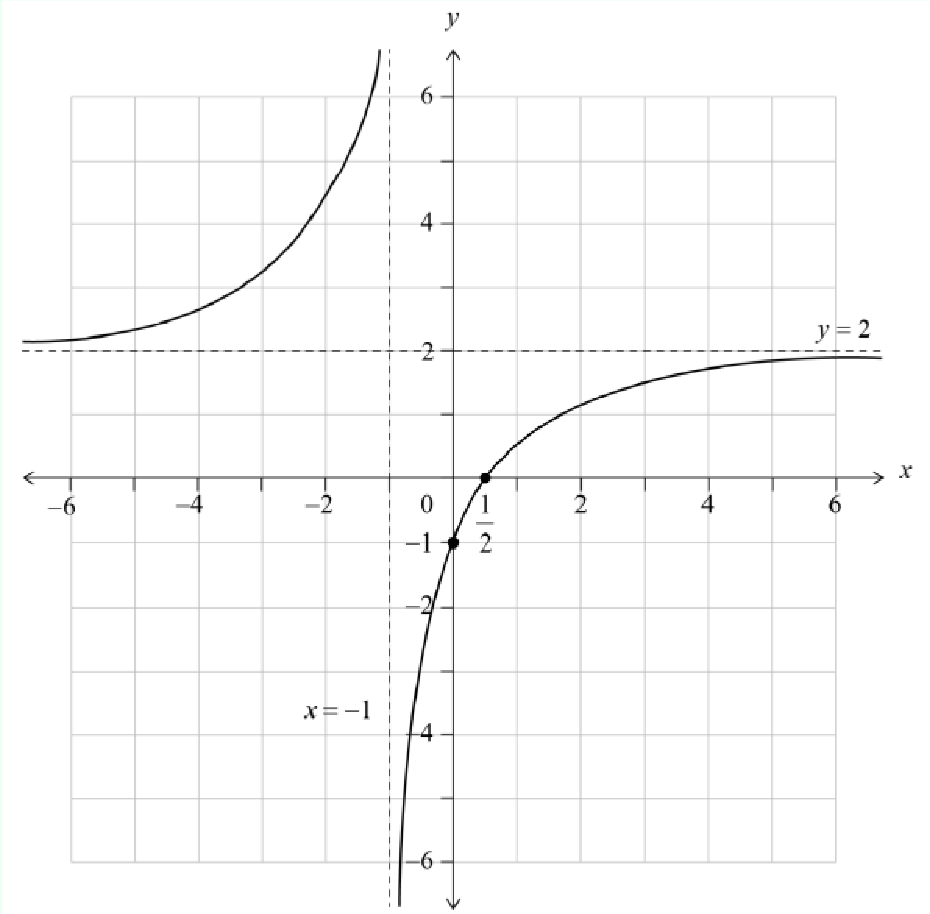
[1 mark]

2c. On the set of axes below, sketch the graph of $y = f(x)$. *[3 marks]*

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$ ***A1A1***

[3 marks]

2d. Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. *[1 mark]*

Markscheme

$x > \frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

2e. Solve the inequality $0 < \frac{2|x|-1}{|x|+1} < 2$. *[2 marks]*

Markscheme

EITHER

attempts to sketch $y = \frac{2|x|-1}{|x|+1}$ ***(M1)***

OR

attempts to solve $2|x|-1 = 0$ ***(M1)***

Note: Award the ***(M1)*** if $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ are identified.

THEN

$x < -\frac{1}{2}$ or $x > \frac{1}{2}$ ***A1***

Note: Accept the use of a comma. Condone the use of ‘and’. Accept correct alternative notation.

[2 marks]

3. A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$. *[7 marks]*

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

Markscheme

attempt to use $\cos(2A + B) = \cos 2A \cos B - \sin 2A \sin B$ (may be seen later) ***M1***

attempt to use any double angle formulae (seen anywhere) ***M1***

attempt to find either $\sin A$ or $\cos B$ (seen anywhere) ***M1***

$\cos A = \frac{2}{3} \Rightarrow \sin A \left(= \sqrt{1 - \frac{4}{9}}\right) = \frac{\sqrt{5}}{3}$ ***(A1)***

$\sin B = \frac{1}{3} \Rightarrow \cos B \left(= \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}\right) = \frac{2\sqrt{2}}{3}$ ***A1***

$\cos 2A \left(= 2\cos^2 A - 1\right) = -\frac{1}{9}$ ***A1***

$\sin 2A \left(= 2 \sin A \cos A\right) = \frac{4\sqrt{5}}{9}$ ***A1***

So $\cos(2A + B) = \left(-\frac{1}{9}\right)\left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{4\sqrt{5}}{9}\right)\left(\frac{1}{3}\right)$

$= -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$ ***AG***

[7 marks]

The faces of a fair six-sided die are numbered 1, 2, 2, 4, 4, 6. Let X be the discrete random variable that models the score obtained when this die is rolled.

4a. Complete the probability distribution table for X. *[2 marks]*

x				
$P(X = x)$				

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

x	1	2	4	6
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

4b. Find the expected value of X. *[2 marks]*

Markscheme

$\mathrm{E}(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{3} + 4 \times \frac{1}{3} + 6 \times \frac{1}{6}$ ***(M1)***

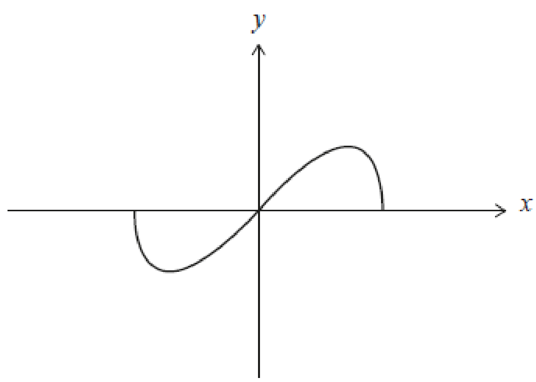
$= \frac{19}{6} \left(= 3\frac{1}{6}\right)$ ***A1***

Note: If the probabilities in (a) are not values between 0 and 1 or lead to $\mathrm{E}(X) > 6$ award ***M1A0*** to correct method using the incorrect probabilities; otherwise allow ***FT*** marks.

[2 marks]

A function f is defined by $f(x) = x\sqrt{1 - x^2}$ where $-1 \le x \le 1$.

The graph of $y = f(x)$ is shown below.



5a. Show that f is an odd function. *[2 marks]*

Markscheme

attempts to replace x with $-x$ ***M1***

$f(-x) = -x\sqrt{1-(-x)^2}$

$= -x\sqrt{1-(-x)^2}(= -f(x))$ ***A1***

Note: Award ***M1A1*** for an attempt to calculate both $f(-x)$ and $-f(-x)$ independently, showing that they are equal.
Note: Award ***M1A0*** for a graphical approach including evidence that **either** the graph is invariant after rotation by 180° about the origin **or** the graph is invariant after a reflection in the y-axis and then in the x-axis (or vice versa).

so f is an odd function ***AG***

[2 marks]

5b. The range of f is $a \le y \le b$, where $a,\ b \in \mathbb{R}$. *[6 marks]*

Find the value of a and the value of b.

Markscheme

attempts both product rule and chain rule differentiation to find $f'(x)$ ***M1***

$f'(x) = x \times \frac{1}{2} \times (-2x) \times \left(1 - x^2\right)^{-\frac{1}{2}} + \left(1 - x^2\right)^{\frac{1}{2}} \times 1 \left(= \sqrt{1 - x^2} - \frac{x^2}{\sqrt{1-x^2}}\right)$

A1

$= \frac{1-2x^2}{\sqrt{1-x^2}}$

sets their $f'(x) = 0$ ***M1***

$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$ ***A1***

attempts to find at least one of $f\left(\pm \frac{1}{\sqrt{2}}\right)$ ***(M1)***

Note: Award **M1** for an attempt to evaluate $f(x)$ at least at one of their $f'(x) = 0$ roots.

$a = -\frac{1}{2}$ and $b = \frac{1}{2}$ ***A1***

Note: Award ***A1*** for $-\frac{1}{2} \leq y \leq \frac{1}{2}$.

[6 marks]