Saturday 26.11 *[41 marks]*

1a. Use the binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^4$. Give your answer in the form $a + b\mathrm{i}$ where a and b are expressed in terms of $\sin\theta$ and $\cos\theta$. *[3 marks]*

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1b. Use de Moivre's theorem and the result from part (a) to show that *[5 marks]*

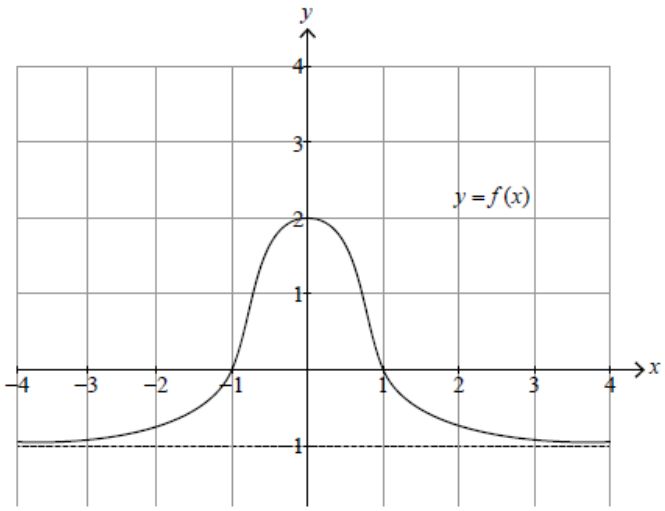
$\cot 4\theta = \frac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}$.

1c. Use the identity from part (b) to show that the quadratic equation $x^2 - 6x + 1 = 0$ has roots $\cot^2 \frac{\pi}{8}$ and $\cot^2 \frac{3\pi}{8}$. *[5 marks]*

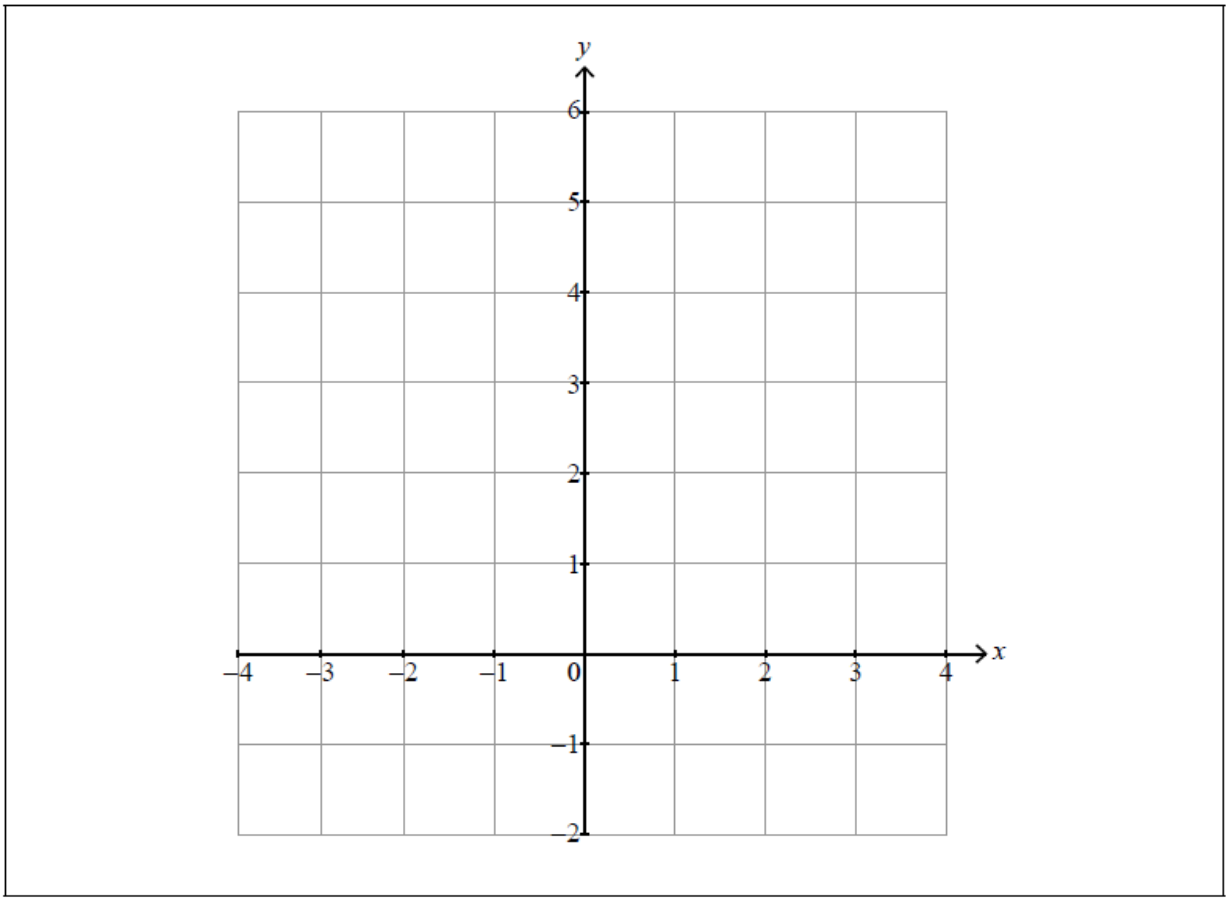
1d. Hence find the exact value of $\cot^2 \frac{3\pi}{8}$. *[4 marks]*

1e. Deduce a quadratic equation with integer coefficients, having roots $\operatorname{cosec}^2 \frac{\pi}{8}$ and $\operatorname{cosec}^2 \frac{3\pi}{8}$. *[3 marks]*

2. The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$. *[5 marks]*



On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



3. The plane Π has the Cartesian equation $2x + y + 2z = 3$ *[7 marks]*

The line L has the vector equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ p \end{pmatrix}$, $\mu, p \in \mathbb{R}$. The acute angle between the line L and the plane Π is 30°.

Find the possible values of p.

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4. A discrete random variable X has the probability distribution given by the following table. *[4 marks]*

x	0	1	2	3
$\mathrm{P}(X=x)$	p	$\frac{1}{4}$	$\frac{1}{6}$	q

Given that $\mathrm{E}(X)=\frac{19}{12}$, determine the value of p and the value of q.

5. Let $f'(x)=\frac{8x}{\sqrt{2x^2+1}}$. Given that $f(0)=5$, find $f(x)$. *[5 marks]*