Sunday 27.11 *[34 marks]*

1. Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all possible values of n for which the expansion has a non-zero constant term. *[5 marks]*

Markscheme

EITHER

attempt to obtain the general term of the expansion

$T_{r+1} = {}_nC_r\left(8x^3\right)^{n-r}\left(-\frac{1}{2x}\right)^r$ OR $T_{r+1} = {}_nC_{n-r}\left(8x^3\right)^r\left(-\frac{1}{2x}\right)^{n-r}$ ***(M1)***

OR

recognize power of x starts at $3n$ and goes down by 4 each time ***(M1)***

THEN

recognizing the constant term when the power of x is zero (or equivalent) ***(M1)***

$r = \frac{3n}{4}$ or $n = \frac{4}{3}r$ or $3n - 4r = 0$ OR $3r - (n - r) = 0$ (or equivalent) ***A1***

r is a multiple of 3 $(r = 3, 6, 9, \ldots)$ or one correct value of n (seen anywhere) ***(A1)***

$n = 4k,\ k \in \mathbb{Z}^+$ ***A1***

Note: Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \ldots$
Do not accept $n = 4, 8, 12$

Note: Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \ldots$ and for recognizing that this pattern continues.

[5 marks]

Let $f(x) = \frac{2x+6}{x^2+6x+10}$, $x \in \mathbb{R}$.

2a. Show that $f(x)$ has no vertical asymptotes. *[3 marks]*

Markscheme

$x^2+6x+10=x^2+6x+9+1=(x+3)^2+1$ ***M1A1***

So the denominator is never zero and thus there are no vertical asymptotes.
(or use of discriminant is negative) ***R1***

[3 marks]

2b. Find the equation of the horizontal asymptote. *[2 marks]*

Markscheme

$x\to\pm\infty,\ f(x)\to 0$ so the equation of the horizontal asymptote is $y=0$
M1A1

[2 marks]

2c. *[3 marks]*

Find the exact value of $\int\limits_0^1 f(x)\ dx$, giving the answer in the form $\ln q,\ q\in\mathbb{Q}$.

Markscheme

$\int\limits_0^1 \frac{2x+6}{x^2+6x+10}\,dx=\left[\ln\left(x^2+6x+10\right)\right]_0^1=\ln 17-\ln 10=\ln\frac{17}{10}$ ***M1A1A1***

[3 marks]

The lines l_1 and l_2 have the following vector equations where $\lambda,\ \mu\in\mathbb{R}$ and $m\in\mathbb{R}$.

$$l_1: r_1=\begin{pmatrix}3\\-2\\0\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\m\end{pmatrix} l_2: r_2=\begin{pmatrix}-1\\-4\\-2m\end{pmatrix}+\mu\begin{pmatrix}2\\-5\\-m\end{pmatrix}$$

3a. Show that l_1 and l_2 are never perpendicular to each other. *[3 marks]*

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

attempts to calculate $\begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ -m \end{pmatrix}$ **(M1)**

$= -1 - m^2$ **A1**

since $m^2 \geq 0,\ -1 - m^2 < 0$ for $m \in \mathbb{R}$ **R1**

so l_1 and l_2 are never perpendicular to each other **AG**

[3 marks]

The plane Π has Cartesian equation $x + 4y - z = p$ where $p \in \mathbb{R}$.

Given that l_1 and Π have no points in common, find

3b. the value of m. *[2 marks]*

Markscheme

(since l_1 is parallel to Π, l_1 is perpendicular to the normal of Π and so)

$\begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} = 0$ **R1**

$2 + 4 - m = 0$

$m = 6$ **A1**

[2 marks]

3c. the condition on the value of p. *[2 marks]*

Markscheme

since there are no points in common, $(3,\ -2,\ 0)$ does not lie in Π

EITHER

substitutes $(3,\ -2,\ 0)$ into $x + 4y - z (\neq p)$ **(M1)**

OR

$\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} (\neq p)$ **(M1)**

THEN

$p \neq -5$ **A1**

[2 marks]

Consider two events A and A defined in the same sample space.

4a. Show that $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$= \mathrm{P}(A) + \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B) - \mathrm{P}(A \cap B)$ ***M1A1***

$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ ***AG***

METHOD 2

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***M1***

$= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A|B) \times \mathrm{P}(B)$ ***M1***

$= \mathrm{P}(A) + (1 - \mathrm{P}(A|B)) \times \mathrm{P}(B)$

$= \mathrm{P}(A) + \mathrm{P}(A'|B) \times \mathrm{P}(B)$ ***A1***

$= \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ ***AG***

[3 marks]

Given that $\mathrm{P}(A \cup B) = \frac{4}{9}, \mathrm{P}(B|A) = \frac{1}{3}$ and $\mathrm{P}(B|A') = \frac{1}{6}$,

4b. (i) show that $\mathrm{P}(A) = \frac{1}{3}$; *[6 marks]*

(ii) hence find $\mathrm{P}(B)$.

Markscheme

(i) use $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$ and $\mathrm{P}(A' \cap B) = \mathrm{P}(B|A')\mathrm{P}(A')$ ***(M1)***

$\frac{4}{9} = \mathrm{P}(A) + \frac{1}{6}(1 - \mathrm{P}(A))$ ***A1***

$8 = 18\mathrm{P}(A) + 3\,(1 - \mathrm{P}(A))$ ***M1***

$\mathrm{P}(A) = \frac{1}{3}$ ***AG***

(ii) **METHOD 1**

$\mathrm{P}(B) = \mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)$ ***M1***

$= \mathrm{P}(B|A)\mathrm{P}(A) + \mathrm{P}(B|A')\mathrm{P}(A')$ ***M1***

$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{2}{3} = \frac{2}{9}$ ***A1***

METHOD 2

$\mathrm{P}(A \cap B) = \mathrm{P}(B|A)\mathrm{P}(A) \Rightarrow \mathrm{P}(A \cap B) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ ***M1***

$\mathrm{P}(B) = \mathrm{P}(A \cup B) + \mathrm{P}(A \cap B) - \mathrm{P}(A)$ ***M1***

$\mathrm{P}(B) = \frac{4}{9} + \frac{1}{9} - \frac{1}{3} = \frac{2}{9}$ ***A1***

[6 marks]

5. Use l'Hôpital's rule to determine the value of $\lim_{x \to 0}\left(\frac{2x\cos\left(x^2\right)}{5\tan x}\right)$. *[5 marks]*

Markscheme

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attempts to apply l’Hôpital’s rule on $\lim_{x\to 0}\left(\frac{2x\cos\left(x^2\right)}{5\tan x}\right)$ **M1**

$= \lim_{x\to 0}\left(\frac{2\cos\left(x^2\right)-4x^2\sin\left(x^2\right)}{5\sec^2 x}\right)$ **M1A1A1**

Note: Award **M1** for attempting to use product and chain rule differentiation on the numerator, **A1** for a correct numerator and **A1** for a correct denominator. The awarding of **A1** for the denominator is independent of the **M1**.

$= \frac{2}{5}$ **A1**

[5 marks]