Sunday 27.11 *[34 marks]*

1. Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all *[5 marks]* possible values of n for which the expansion has a non-zero constant term.

Let $f(x) = \dfrac{2x+6}{x^2+6x+10}$, $x \in \mathbb{R}$.

2a. Show that $f(x)$ has no vertical asymptotes.   *[3 marks]*

2b. Find the equation of the horizontal asymptote.   *[2 marks]*

2c. [3 marks]

Find the exact value of $\int_0^1 f(x)\,dx$, giving the answer in the form $\ln q$, $q \in \mathbb{Q}$.

The lines l_1 and l_2 have the following vector equations where λ, $\mu \in \mathbb{R}$ and $m \in \mathbb{R}$.

$$l_1 : r_1 = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ m \end{pmatrix} \quad l_2 : r_2 = \begin{pmatrix} -1 \\ -4 \\ -2m \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -5 \\ -m \end{pmatrix}$$

3a. Show that l_1 and l_2 are never perpendicular to each other. [3 marks]

The plane Π has Cartesian equation $x + 4y - z = p$ where $p \in \mathbb{R}$.

Given that l_1 and Π have no points in common, find

3b. the value of m. *[2 marks]*

3c. the condition on the value of p. *[2 marks]*

Consider two events A and A defined in the same sample space.

4a. Show that $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(A' \cap B)$.   *[3 marks]*

Given that $\mathrm{P}(A \cup B) = \frac{4}{9}, \mathrm{P}(B|A) = \frac{1}{3}$ and $\mathrm{P}(B|A') = \frac{1}{6}$,

4b. (i)   show that $\mathrm{P}(A) = \frac{1}{3}$;   *[6 marks]*

(ii)   hence find $\mathrm{P}(B)$.

5. Use l'Hôpital's rule to determine the value of $\lim\limits_{x\to 0}\left(\dfrac{2x\cos\left(x^2\right)}{5\tan x}\right)$.   [5 marks]