Tuesday 29.11 *[58 marks]*

The first three terms of an arithmetic sequence are $u_1,\ 5u_1 - 8$ and $3u_1 + 8$.

1a. Show that $u_1 = 4$. *[2 marks]*

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1b. Prove that the sum of the first n terms of this arithmetic sequence is a square number. *[4 marks]*

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Consider the function defined by $f(x) = \frac{kx-5}{x-k}$, where $x \in \mathbb{R} \setminus \{k\}$ and $k^2 \neq 5$.

2a. State the equation of the vertical asymptote on the graph of $y = f(x)$. *[1 mark]*

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2b. State the equation of the horizontal asymptote on the graph of $y = f(x)$. *[1 mark]*

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2c. Use an algebraic method to determine whether f is a self-inverse function. *[4 marks]*

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Consider the case where $k = 3$.

2d. Sketch the graph of $y = f(x)$, stating clearly the equations of any asymptotes and the coordinates of any points of intersections with the coordinate axes. *[3 marks]*

2e. The region bounded by the x-axis, the curve $y = f(x)$, and the lines $x = 5$ and $x = 7$ is rotated through 2π about the x-axis. Find the volume of the solid generated, giving your answer in the form $\pi(a + b \ln 2)$, where $a,\ b \in \mathbb{Z}$. *[6 marks]*

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

3a. Find and simplify an expression for $\boldsymbol{a} \bullet \boldsymbol{b}$ in terms of t. *[2 marks]*

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3b. Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse . *[4 marks]*

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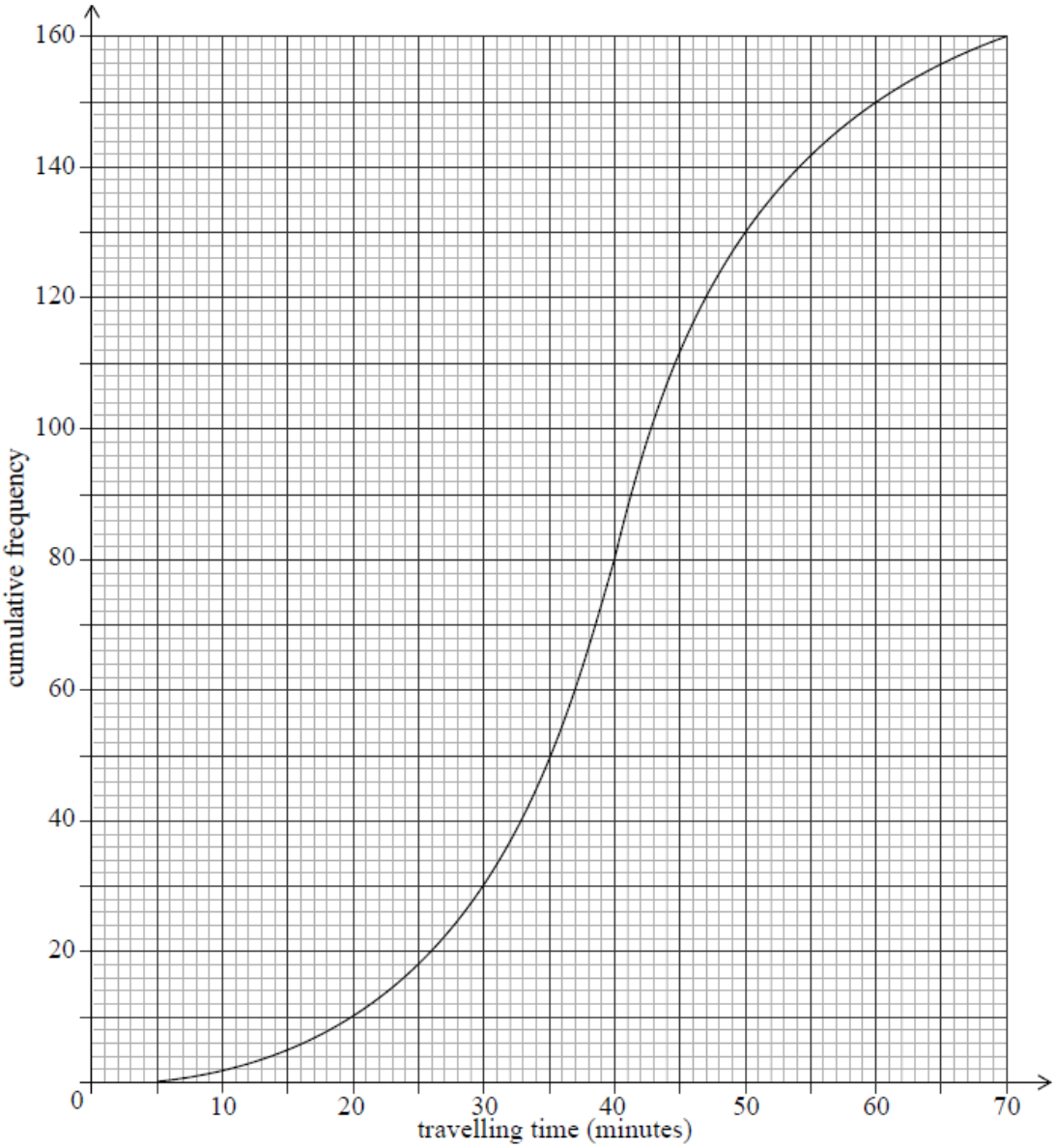
A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



4a. Find the median number of minutes spent traveling to work. *[2 marks]*

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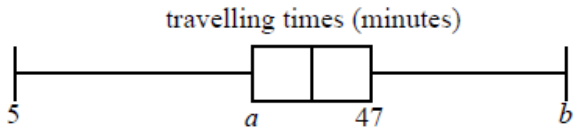
4b. Find the number of employees whose travelling time is within 15 minutes of the median. *[3 marks]*

Only 10% of the employees spent more than k minutes traveling to work.

4c. Find the value of k. *[3 marks]*

The results of the survey can also be displayed on the following box-and-whisker diagram.



4d. Write down the value of b. *[1 mark]*

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4e. Find the value of a. *[2 marks]*

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4f. Hence, find the interquartile range. *[2 marks]*

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4g. Travelling times of less than p minutes are considered outliers. *[2 marks]*

Find the value of p.

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

5a. Find $f'(x)$. *[2 marks]*

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

5b. Find the value of a and the value of b. *[3 marks]*

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5c. Sketch the graph of $y = f'(x)$. *[1 mark]*

5d. Hence explain why the graph of f has a local maximum point at $x = a$. *[1 mark]*

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5e. Find $f''(b)$. *[3 marks]*

5f. Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. *[1 mark]*

5g. The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q) . *[5 marks]*

Find the value of p and the value of q.