Exercise 14C

Prove each of the following identities.

1 $\sin\theta\tan\theta+\cos\theta\equiv\sec\theta$

2 $\operatorname{cosec}\theta+\tan\theta\sec\theta\equiv\operatorname{cosec}\theta\sec^2\theta$

3 $\operatorname{cosec}\theta-\sin\theta\equiv\cot\theta\cos\theta$

4 $(\sin\theta+\cos\theta)^2-1\equiv2\sin\theta\cos\theta$

5 $(\sin\theta-\operatorname{cosec}\theta)^2\equiv\sin^2\theta+\cot^2\theta-1$

6 $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)\equiv1$

7 $\tan^2\theta+\sin^2\theta\equiv(\sec\theta+\cos\theta)(\sec\theta-\cos\theta)$

8 $\sec^2\theta+\cot^2\theta\equiv\operatorname{cosec}^2\theta+\tan^2\theta$

9 $(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)\equiv\sin^3\theta+\cos^3\theta$

10 $\tan^4\theta+\tan^2\theta\equiv\sec^4\theta-\sec^2\theta$

11 $\cos^4\theta-\sin^4\theta\equiv\cos^2\theta-\sin^2\theta$

12 $\sin\theta+\cos\theta\equiv\dfrac{1-2\cos^2\theta}{\sin\theta-\cos\theta}$

13 $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}\equiv2\operatorname{cosec}\theta$

14 $\dfrac{\operatorname{cosec}\theta}{\cot\theta+\tan\theta}\equiv\cos\theta$

15 $\dfrac{1}{1+\tan^2\theta}+\dfrac{1}{1+\cot^2\theta}\equiv1$

16 $\dfrac{1-\sin\theta}{\cos\theta}\equiv\dfrac{1}{\sec\theta+\tan\theta}$

17 $\dfrac{\tan\theta+\cot\theta}{\sec\theta+\operatorname{cosec}\theta}\equiv\dfrac{1}{\sin\theta+\cos\theta}$

18 $\sec^4\theta-\operatorname{cosec}^4\theta\equiv\dfrac{\sin^2\theta-\cos^2\theta}{\sin^4\theta\cos^4\theta}$

19 $\sqrt{(\sec^2\theta-1)}+\sqrt{(\operatorname{cosec}^2\theta-1)}\equiv\dfrac{1}{\sin\theta\cos\theta}$

20 $\dfrac{\sin\theta}{\sqrt{(1+\cot^2\theta)}}+\dfrac{\cos\theta}{\sqrt{(1+\tan^2\theta)}}\equiv1$

21 $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}\equiv\sec\theta-\tan\theta$

22 $\dfrac{1+\sin\theta+\cos\theta}{\cos\theta}\equiv\dfrac{1-\sin\theta+\cos\theta}{1-\sin\theta}$

***23** $\sqrt{\dfrac{\tan\theta+\sin\theta}{\cot\theta-\cos\theta}}\equiv\tan^2\theta\sqrt{\dfrac{1+\sin\theta}{1-\cos\theta}}$

***24** $\dfrac{\tan^3\theta}{1+\tan^2\theta}+\dfrac{\cot^3\theta}{1+\cot^2\theta}\equiv\dfrac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$

***25** $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}\equiv1+\sin\theta\cos\theta$

***26** $\dfrac{\cot^2\theta(\sec\theta-1)}{1+\sin\theta}\equiv\dfrac{\sec^2\theta(1-\sin\theta)}{1+\sec\theta}$