polynomials 3IB HL [88 marks]

1. Let $f(x) = x^4 + px^3 + qx + 5$ where p, q are constants.

[5 marks]

The remainder when f(x) is divided by (x + 1) is 7, and the remainder when f(x) is divided by (x - 2) is 1. Find the value of p and the value of q.

..... 2. Consider the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where a, b, c, $d \in \mathbb{R}$ [7 marks] and $z \in \mathbb{C}$.

Two of the roots of the equation are $\log_2 6$ and $i\sqrt{3}$ and the sum of all the roots is $3 + \log_2 3$.

Show that 6a + d + 12 = 0.

Given that $lphaeta=-rac{k^2}{4}$, find the value of k.

.

4. The quadratic equation $x^2 - 2kx + (k - 1) = 0$ has roots α and β such [6 marks] that $\alpha^2 + \beta^2 = 4$. Without solving the equation, find the possible values of the real number k.

5. The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of [5 marks] (x-1), (x-2) and (x-3).

Find the values of p, q and r.

. .

Consider the polynomial $q(x) = 3x^3 - 11x^2 + kx + 8$.

6a. Given that q(x) has a factor (x-4), find the value of k.

[3 marks]

6b. Hence or otherwise, factorize q(x) as a product of linear factors. [3 marks]

It is given that $f(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$ where a and b are positive integers.

7a. Given that $x^2 - 1$ is a factor of f(x) find the value of a and the value of [4 marks] b.

7b. Factorize f(x) into a product of linear factors.

[3 marks]

7c. Using your graph state the range of values of c for which f(x) = c has [3 marks] exactly two distinct real roots.

Consider the equation $x^5-3x^4+mx^3+nx^2+px+q=0$, where m , n , p , $q\in\mathbb{R}$

The equation has three distinct real roots which can be written as $\log_2 a$, $\log_2 b$ and $\log_2 c.$

The equation also has two imaginary roots, one of which is $d\mathbf{i}$ where $d \in \mathbb{R}$.

8a. Show that abc = 8.

[5 marks]

The values a, b, and c are consecutive terms in a geometric sequence.

8b. Show that one of the real roots is equal to 1.

[3 marks]

This question asks you to investigate conditions for the existence of complex roots of polynomial equations of degree 3 and 4.

The cubic equation $x^3+px^2+qx+r=0$, where $p,\ q,\ r\ \in\ \mathbb{R}$, has roots $lpha,\ eta$ and $\gamma.$

9a. By expanding $(x-lpha)(x-eta)(x-\gamma)$ show that:

[3 marks]

 $egin{aligned} p &= -(lpha + eta + \gamma) \ q &= lpha eta + eta \gamma + \gamma lpha \ r &= -lpha eta \gamma. \end{aligned}$

⁹c. Hence show that $\left(lpha-eta
ight)^2+\left(eta-\gamma
ight)^2+\left(\gamma-lpha
ight)^2=2p^2-6q.$

[3 marks]

[2 marks]

Consider the equation $x^3-7x^2+qx+1=0$, where $q\in\mathbb{R}.$

9e. Using the result from part (c), show that when q=17, this equation has *[2 marks]* at least one complex root.

Noah believes that if $p^2 \geq 3q$ then $lpha, \ eta$ and γ are all real.

9f. By varying the value of q in the equation $x^3 - 7x^2 + qx + 1 = 0$, [2 marks] determine the smallest positive integer value of q required to show that Noah is incorrect.

9g. Explain why the equation will have at least one real root for all values of q[1 mark]

Now consider polynomial equations of degree 4.

The equation $x^4 + px^3 + qx^2 + rx + s = 0$, where $p, q, r, s \in \mathbb{R}$, has roots α, β, γ and δ . In a similar way to the cubic equation, it can be shown that: $p = -(\alpha + \beta + \gamma + \delta)$ $q = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$ $r = -(\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta)$ $s = \alpha\beta\gamma\delta$.

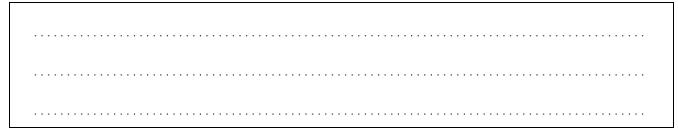
9h. Find an expression for $lpha^2+eta^2+\gamma^2+\delta^2$ in terms of p and q.

[3 marks]

9i. Hence state a condition in terms of p and q that would imply [1 mark] $x^4 + px^3 + qx^2 + rx + s = 0$ has at least one complex root.

9j. Use your result from part (f)(ii) to show that the equation $x^4 - 2x^3 + 3x^2 - 4x + 5 = 0$ has at least one complex root.

[1 mark]



The equation $x^4 - 9x^3 + 24x^2 + 22x - 12 = 0$, has one integer root.

9k. State what the result in part (f)(ii) tells us when considering this equation [1 mark] $x^4 - 9x^3 + 24x^2 + 22x - 12 = 0$.

91. Write down the integer root of this equation.

[1 mark]

9m. By writing $x^4 - 9x^3 + 24x^2 + 22x - 12$ as a product of one linear and [4 marks] one cubic factor, prove that the equation has at least one complex root.

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