test revision (ch 2 - 6) *[140 marks]*

The functions f and g are defined such that $f\left(x\right)=\frac{x+3}{4}$ and $g\left(x\right)=8x+5$.

1a. Show that $\left(g\circ f\right)\left(x\right)=2x+11$. *[2 marks]*

Markscheme

attempt to form composition ***M1***

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ ***A1***

$\left(g\circ f\right)\left(x\right)=2x+11$ ***AG***

[2 marks]

1b. Given that $\left(g\circ f\right)^{-1}\left(a\right)=4$, find the value of a. *[3 marks]*

Markscheme

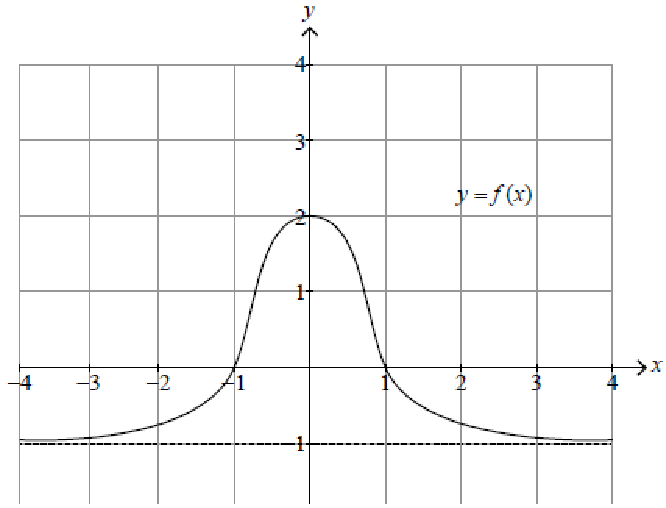
attempt to substitute 4 (seen anywhere) ***(M1)***

correct equation $a=2\times 4+11$ ***(A1)***

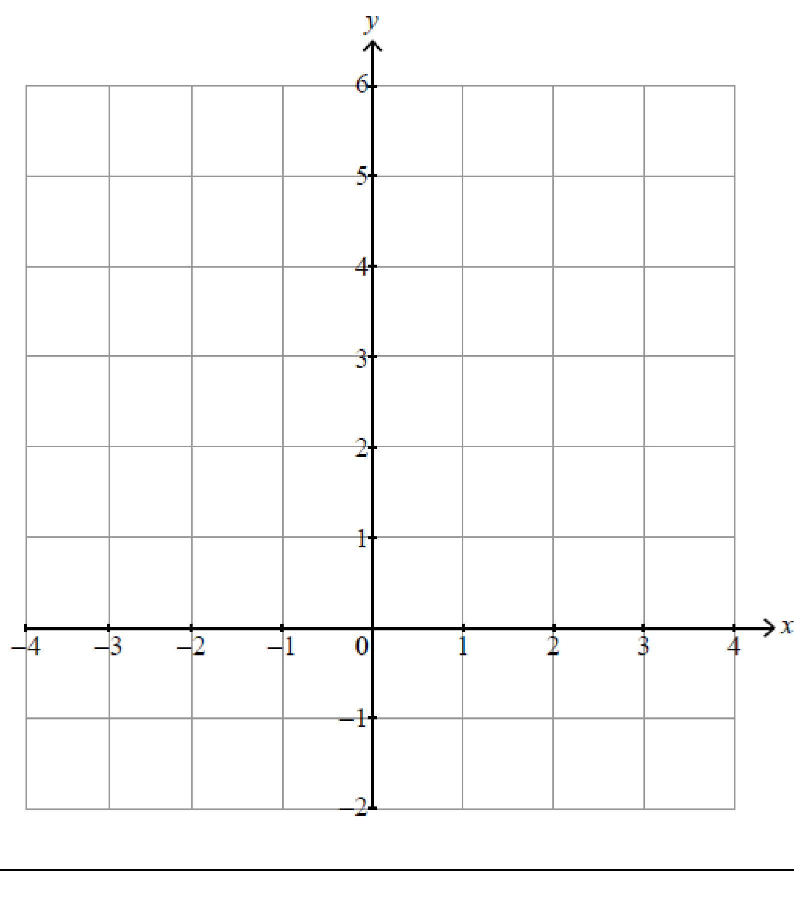
a = 19 ***A1***

[3 marks]

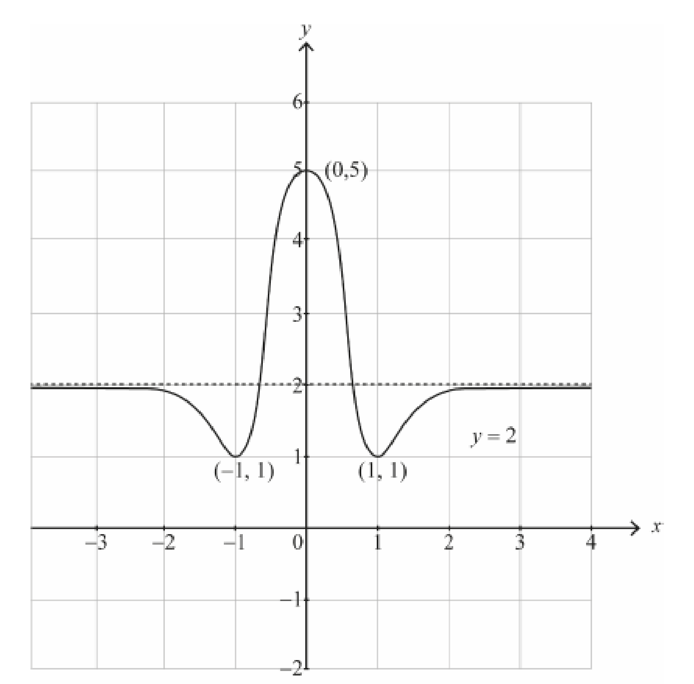
2. The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$. *[5 marks]*



On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



Markscheme



no y values below 1 ***A1***

horizontal asymptote at $y = 2$ with curve approaching from below as $x \to \pm\infty$ ***A1***

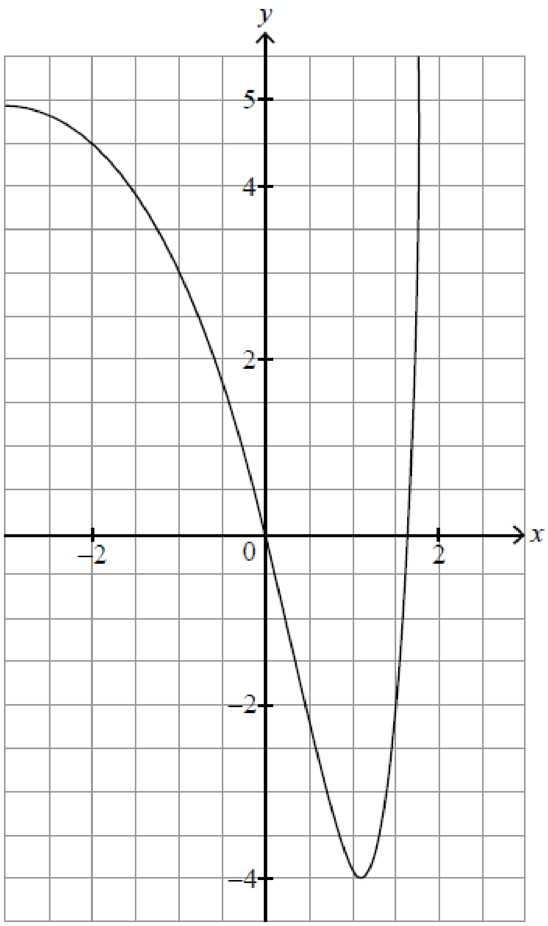
(±1,1) local minima ***A1***

(0,5) local maximum ***A1***

smooth curve and smooth stationary points ***A1***

[5 marks]

The function f is defined by $f\left(x\right) = \mathrm{e}^{2x} - 6\mathrm{e}^{x} + 5,\ x \in \mathbb{R},\ x \leqslant a$. The graph of $y = f\left(x\right)$ is shown in the following diagram.



3a. Find the largest value of a such that f has an inverse function. *[3 marks]*

Markscheme

attempt to differentiate and set equal to zero ***M1***

$f'\left(x\right) = 2\mathrm{e}^{2x} - 6\mathrm{e}^{x} = 2\mathrm{e}^{x}\left(\mathrm{e}^{x} - 3\right) = 0$ ***A1***

minimum at $x = \ln 3$

$a = \ln 3$ ***A1***

[3 marks]

3b. For this value of a, find an expression for $f^{-1}\left(x\right)$, stating its domain. *[5 marks]*

Markscheme

Note: Interchanging x and y can be done at any stage.

$y = (e^x - 3)^2 - 4$ ***(M1)***

$e^x - 3 = \pm\sqrt{y+4}$ ***A1***

as $x \leqslant \ln 3$, $x = \ln\left(3 - \sqrt{y+4}\right)$ ***R1***

so $f^{-1}(x) = \ln\left(3 - \sqrt{x+4}\right)$ ***A1***

domain of f^{-1} is $x \in \mathbb{R}$, $-4 \leqslant x < 5$ ***A1***

[5 marks]

4. The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$. *[6 marks]*

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b.

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting compared to formal exam papers.

$(f \circ g)(x) = ax + b - 2$ **(M1)**

$(f \circ g)(2) = -3 \Rightarrow 2a + b - 2 = -3(2a + b = -1)$ **A1**

$(g \circ f)(x) = a(x - 2) + b$ **(M1)**

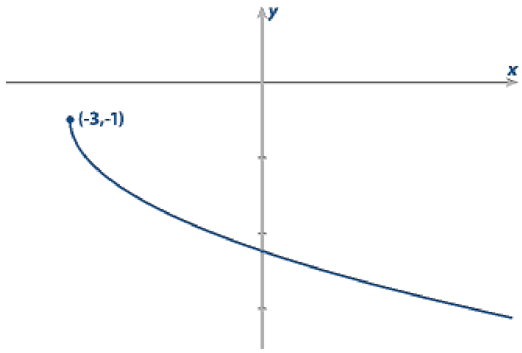
$(g \circ f)(1) = 5 \Rightarrow -a + b = 5$ **A1**

a valid attempt to solve their two linear equations for a and b **M1**

so $a = -2$ and $b = 3$ **A1**

[6 marks]

The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



5a. Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. *[3 marks]*

Markscheme

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for example,

a reflection in the x-axis (in the line $y = 0$) **A1**

a horizontal translation (shift) 3 units to the left **A1**

a vertical translation (shift) down by 1 unit **A1**

Note: Award **A1** for each correct transformation applied in a correct position in the sequence. Do not accept use of the “move” for a translation.

Note: Award **A1A1A1** for a correct alternative sequence of transformations. For example,

a vertical translation (shift) down by 1 unit, followed by a horizontal translation (shift) 3 units to the left and then a reflection in the line $y = -1$.

[3 marks]

A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

5b. State the range of f. *[1 mark]*

Markscheme

range is $f(x) \leq -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$, $(-\infty, -1]$ or $y \leq -1$.

[1 mark]

5c. Find an expression for $f^{-1}(x)$, stating its domain. *[5 marks]*

Markscheme

$-1 - \sqrt{y+3} = x$ **M1**

Note: Award **M1** for interchanging x and y (can be done at a later stage).

$\sqrt{y+3} = -x - 1 (= -(x+1))$ **A1**

$y + 3 = (x+1)^2$ **A1**

so $f^{-1}(x) = (x+1)^2 - 3 \left(f^{-1}(x) = x^2 + 2x - 2\right)$ **A1**

domain is $x \leq -1$ **A1**

Note: Correct alternative notations include $]-\infty, -1]$ or $(-\infty, -1]$.

[5 marks]

5d. Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. *[5 marks]*

Markscheme

the point of intersection lies on the line $y = x$

EITHER

$(x+1)^2 - 3 = x$ **M1**

attempts to solve their quadratic equation **M1**

for example, $(x+2)(x-1) = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \frac{-1 \pm 3}{2}\right)$

OR

$-1 - \sqrt{x+3} = x$ **M1**

$\left(-1 - \sqrt{x+3}\right)^2 = x^2 \Rightarrow 2\sqrt{x+3} + x + 4 = x^2$

substitutes $2\sqrt{x+3} = -2(x+1)$ to obtain $-2(x+1) + x + 4 = x^2$

attempts to solve their quadratic equation **M1**

for example, $(x+2)(x-1) = 0$ or $x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2}$ $\left(x = \frac{-1 \pm 3}{2}\right)$

THEN

$x = -2, 1$ **A1**

as $x \leq -1$, the only solution is $x = -2$ **R1**

so the coordinates of the point of intersection are $(-2,\ -2)$ **A1**

Note: Award **R0A1** if $(-2,\ -2)$ is stated without a valid reason given for rejecting $(1,\ 1)$.

[5 marks]

Let $f(x) = \frac{2x+6}{x^2+6x+10}$, $x \in \mathbb{R}$.

6a. Show that $f(x)$ has no vertical asymptotes. *[3 marks]*

Markscheme

$x^2+6x+10=x^2+6x+9+1=(x+3)^2+1$ ***M1A1***

So the denominator is never zero and thus there are no vertical asymptotes.
(or use of discriminant is negative) ***R1***

[3 marks]

6b. Find the equation of the horizontal asymptote. *[2 marks]*

Markscheme

$x\to\pm\infty,\ f(x)\to 0$ so the equation of the horizontal asymptote is $y=0$ ***M1A1***

[2 marks]

Let $f(x)=\frac{2x^2-5x-12}{x+2}$, $x\in\mathbb{R}$, $x\neq -2$.

7a. Find all the intercepts of the graph of $f(x)$ with both the x and y axes. *[4 marks]*

Markscheme

$x=0\Rightarrow y=-6$ intercept on the y axes is (0, −6) ***A1***

$2x^2-5x-12=0\Rightarrow(2x+3)(x-4)=0\Rightarrow x=\frac{-3}{2}\text{ or }4$ ***M1***

intercepts on the x axes are $\left(\frac{-3}{2},\ 0\right)\text{ and }(4,\ 0)$ ***A1A1***

[4 marks]

7b. Write down the equation of the vertical asymptote. *[1 mark]*

Markscheme

$x = -2$ ***A1***

[1 mark]

7c. As $x \to \pm\infty$ the graph of $f\left(x\right)$ approaches an oblique straight line asymptote. *[4 marks]*

Divide $2x^2 - 5x - 12$ by $x + 2$ to find the equation of this asymptote.

Markscheme

$f(x) = 2x - 9 + \frac{6}{x+2}$ ***M1A1***

So equation of asymptote is $y = 2x - 9$ ***M1A1***

[4 marks]

Let $f\left(x\right) = \frac{x^2-10x+5}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

8a. Find the co-ordinates of all stationary points. *[4 marks]*

Markscheme

$f'\left(x\right) = \frac{(2x-10)(x+1)-\left(x^2-10x+5\right)1}{(x+1)^2}$ ***M1***

$f'\left(x\right) = 0 \Rightarrow x^2 + 2x - 15 = 0 \Rightarrow \left(x+5\right)\left(x-3\right) = 0$ ***M1***

Stationary points are $\left(-5,\ -20\right)\ \text{and}\ \left(3,\ -4\right)$ ***A1A1***

[4 marks]

8b. Write down the equation of the vertical asymptote. *[1 mark]*

Markscheme

$x = -1$ ***A1***

[1 mark]

8c. Sketch the graph of *y=f(x)* clearly indicating all the asymptotes and axes intercepts. *[5 marks]*

Markscheme

Looking at the nature table

x		-5		-1		3	
$f'(x)$	+ve	0	-ve	undefined	-ve	0	+ve

M1A1

$(-5, -20)$ is a max and $(3, -4)$ is a min ***A1A1***

[4 marks]

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

9a. Find $g(0)$. *[1 mark]*

Markscheme

$g(0) = -2$ ***A1***

[1 mark]

9b. Find $(f \circ g)(0)$. *[2 marks]*

Markscheme

evidence of using composite function *(M1)*

$f(g(0))$ OR $f(-2)$

$(f \circ g)(0) = 8$ ***A1***

[2 marks]

9c. Find the value of x such that $f(x) = 0$. *[2 marks]*

Markscheme

$x = 3$ ***A2***

[2 marks]

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

10a. Write down the equation of the vertical asymptote. *[1 mark]*

Markscheme

$x = -1$ ***A1***

[1 mark]

10b. Write down the equation of the horizontal asymptote. *[1 mark]*

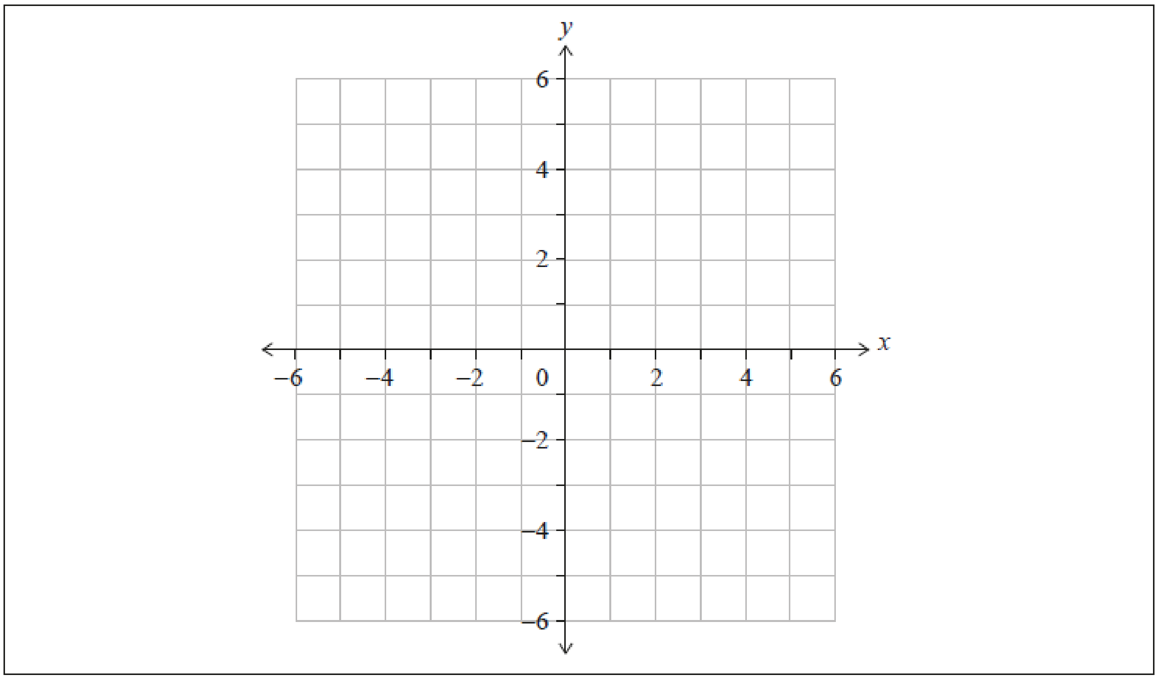
Markscheme

$y = 2$ ***A1***

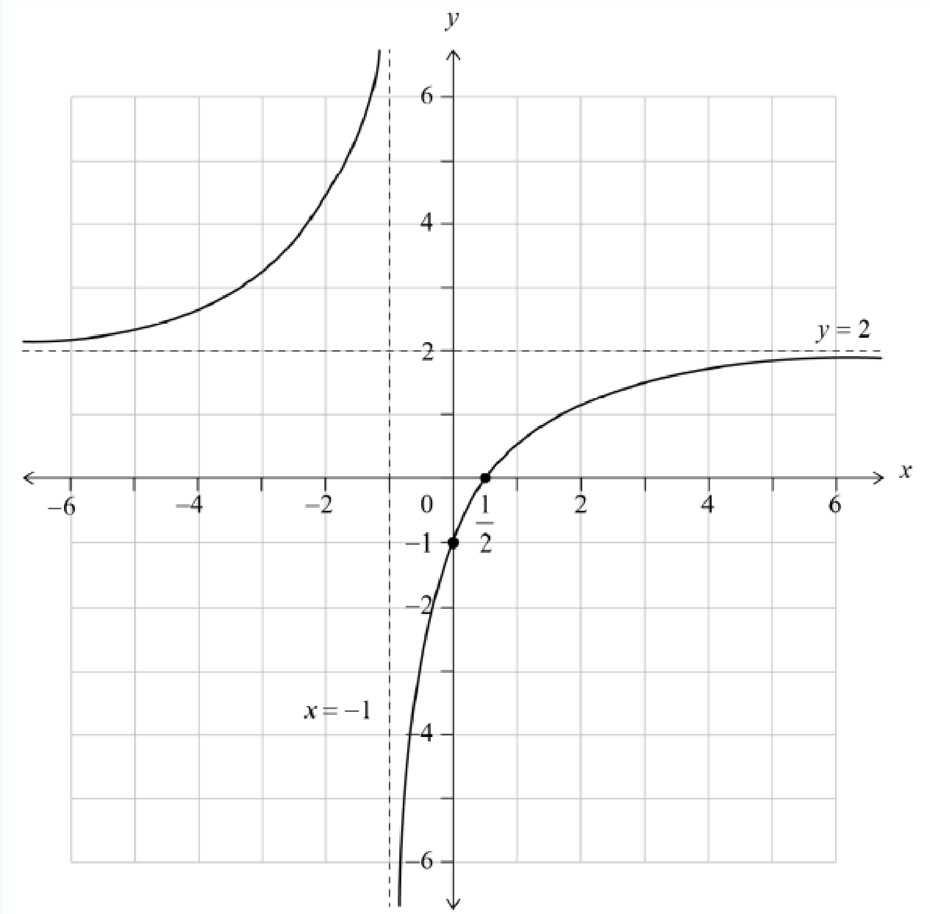
[1 mark]

10c. On the set of axes below, sketch the graph of $y = f(x)$. *[3 marks]*

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

Note: The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x = -1$ and $y = 2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$ ***A1A1***

[3 marks]

10d. Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. *[1 mark]*

Markscheme

$x > \frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

11a. Write down the equation of the vertical asymptote. *[1 mark]*

Markscheme

$x = -1$ ***A1***

[1 mark]

11b. Write down the equation of the horizontal asymptote. *[1 mark]*

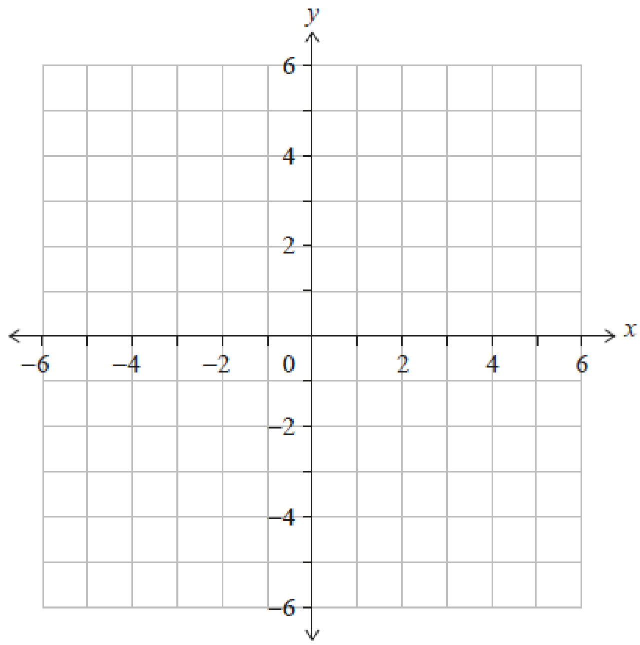
Markscheme

$y = 2$ ***A1***

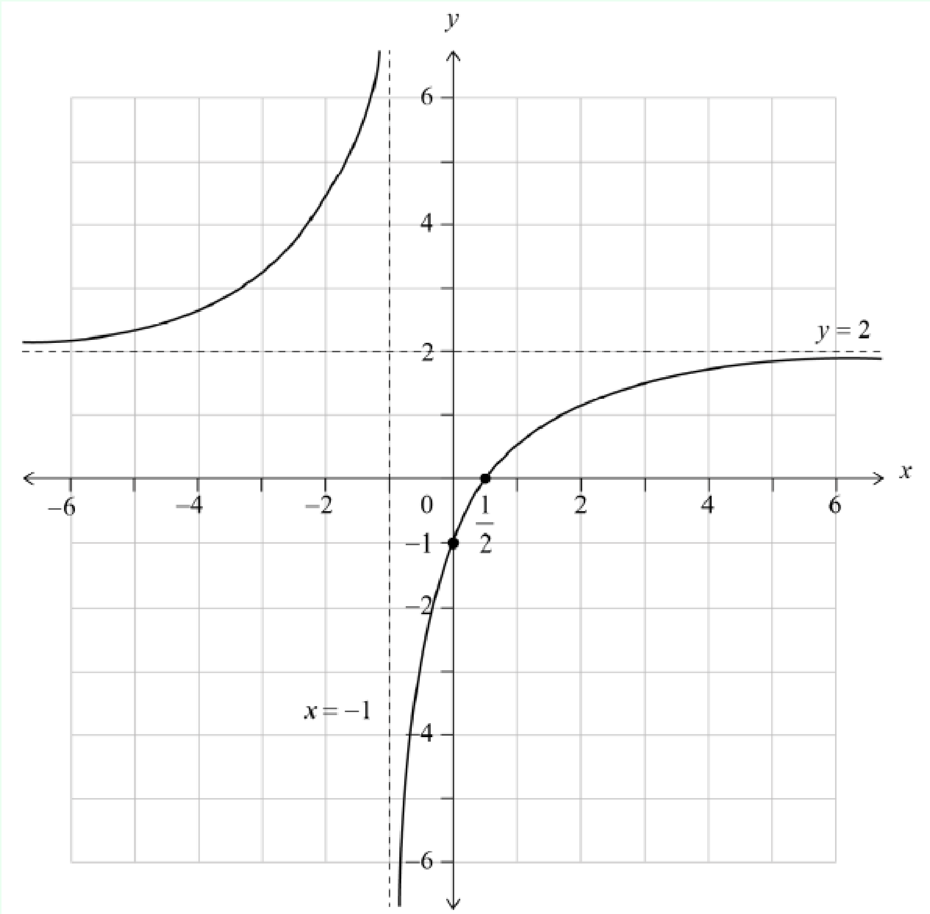
[1 mark]

11c. On the set of axes below, sketch the graph of $y = f(x)$. *[3 marks]*

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



Markscheme



rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$ ***A1A1***

[3 marks]

11d. Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. *[1 mark]*

Markscheme

$x > \frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2},\ \infty\right)$ and $\left]\frac{1}{2},\infty\right[$.

[1 mark]

11e. Solve the inequality $0 < \frac{2|x|-1}{|x|+1} < 2$. *[2 marks]*

Markscheme

EITHER

attempts to sketch $y = \frac{2|x|-1}{|x|+1}$ ***(M1)***

OR

attempts to solve $2|x|-1 = 0$ ***(M1)***

Note: Award the ***(M1)*** if $x = \frac{1}{2}$ and $x = -\frac{1}{2}$ are identified.

THEN

$x < -\frac{1}{2}$ or $x > \frac{1}{2}$ ***A1***

Note: Accept the use of a comma. Condone the use of ‘and’. Accept correct alternative notation.

[2 marks]

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

12a. the vertical asymptote of the graph of f. *[1 mark]*

Markscheme

$x = 3$ ***A1***

[1 mark]

12b. the horizontal asymptote of the graph of f. *[1 mark]*

Markscheme

$y = -2$ ***A1***

[1 mark]

Find the coordinates where the graph of f crosses

12c. the x-axis. *[1 mark]*

Markscheme

$(-2,\ 0)$ (accept $x = -2$) ***A1***

[1 mark]

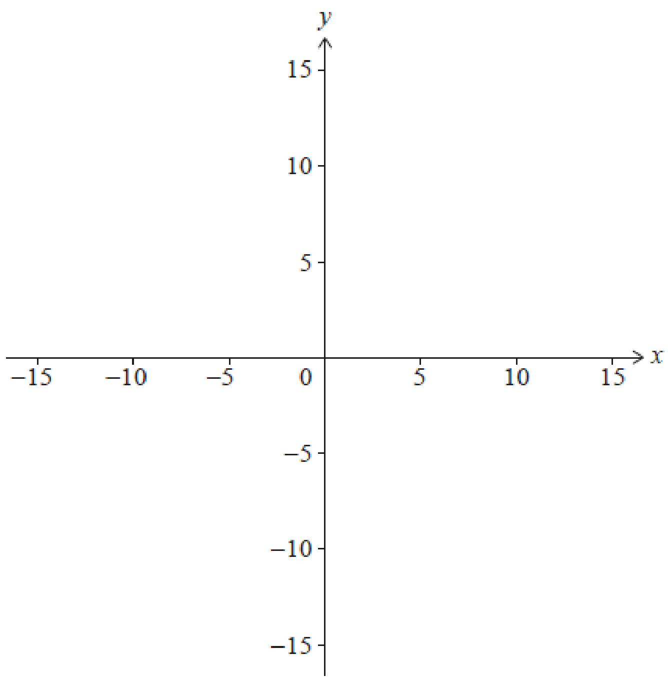
12d. the y-axis. *[1 mark]*

Markscheme

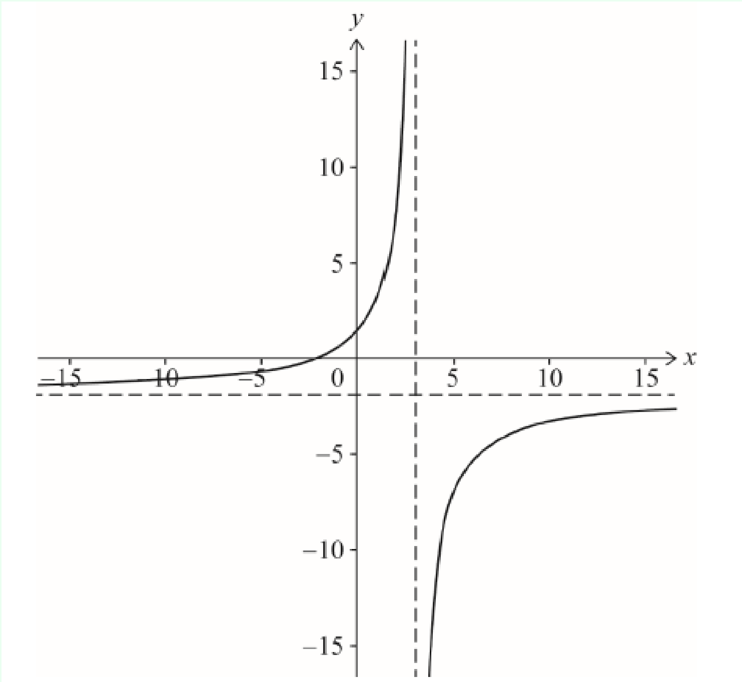
$\left(0,\ \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) ***A1***

[1 mark]

12e. Sketch the graph of f on the axes below. *[1 mark]*



Markscheme



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

Consider the function $f(x) = a^x$ where $x,\ a \in \mathbb{R}$ and $x > 0,\ a > 1$.

The graph of f contains the point $\left(\frac{2}{3},\ 4\right)$.

13a. Show that $a = 8$. *[2 marks]*

Markscheme

$f\left(\frac{2}{3}\right) = 4$ OR $a^{\frac{2}{3}} = 4$ ***(M1)***

$a = 4^{\frac{3}{2}}$ OR $a = \left(2^2\right)^{\frac{3}{2}}$ OR $a^2 = 64$ OR $\sqrt[3]{a} = 2$ ***A1***

$a = 8$ ***AG***

[2 marks]

13b. Write down an expression for $f^{-1}(x)$. *[1 mark]*

Markscheme

$f^{-1}(x) = \log_8 x$ ***A1***

Note: Accept $f^{-1}(x) = \log_a x$.

Accept any equivalent expression for f^{-1} e.g. $f^{-1}(x) = \frac{\ln x}{\ln 8}$.

[1 mark]

13c. Find the value of $f^{-1}\left(\sqrt{32}\right)$. *[3 marks]*

Markscheme

correct substitution ***(A1)***

$\log_8 \sqrt{32}$ OR $8^x = 32^{\frac{1}{2}}$

correct working involving log/index law ***(A1)***

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2 = \frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8 = 3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x} = 2^{\frac{5}{2}}$

$f^{-1}\left(\sqrt{32}\right) = \frac{5}{6}$ ***A1***

[3 marks]

Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

13d. Show that $27,\ p,\ q$ and 125 are four consecutive terms in a geometric sequence. *[4 marks]*

Markscheme

METHOD 1

equating a pair of differences ***(M1)***

$u_2 - u_1 = u_4 - u_3 (= u_3 - u_2)$

$\log_8 p - \log_8 27 = \log_8 125 - \log_8 q$

$\log_8 125 - \log_8 q = \log_8 q - \log_8 p$

$\log_8\left(\frac{p}{27}\right) = \log_8\left(\frac{125}{q}\right)$, $\log_8\left(\frac{125}{q}\right) = \log_8\left(\frac{q}{p}\right)$ ***A1A1***

$\frac{p}{27} = \frac{125}{q}$ and $\frac{125}{q} = \frac{q}{p}$ ***A1***

$27,\ p,\ q$ and 125 are in geometric sequence ***AG***

Note: If candidate assumes the sequence is geometric, award no marks for part (i). If $r = \frac{5}{3}$ has been found, this will be awarded marks in part (ii).

METHOD 2

expressing a pair of consecutive terms, in terms of d ***(M1)***

$p = 8^d \times 27$ and $q = 8^{2d} \times 27$ OR $q = 8^{2d} \times 27$ and $125 = 8^{3d} \times 27$

two correct pairs of consecutive terms, in terms of d ***A1***

$\frac{8^d \times 27}{27} = \frac{8^{2d} \times 27}{8^d \times 27} = \frac{8^{3d} \times 27}{8^{2d} \times 27}$ (must include 3 ratios) ***A1***

all simplify to 8^d ***A1***

$27,\ p,\ q$ and 125 are in geometric sequence ***AG***

[4 marks]

13e. Find the value of p and the value of q. *[5 marks]*

Markscheme

METHOD 1 (geometric, finding r)

$u_4 = u_1 r^3$ OR $125 = 27(r)^3$ ***(M1)***

$r = \frac{5}{3}$ (seen anywhere) ***A1***

$p = 27r$ OR $\frac{125}{q} = \frac{5}{3}$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 2 (arithmetic)

$u_4 = u_1 + 3d$ OR $\log_8 125 = \log_8 27 + 3d$ ***(M1)***

$d = \log_8\left(\frac{5}{3}\right)$ (seen anywhere) ***A1***

$\log_8 p = \log_8 27 + \log_8\left(\frac{5}{3}\right)$ OR $\log_8 q = \log_8 27 + 2\log_8\left(\frac{5}{3}\right)$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 3 (geometric using proportion)

recognizing proportion ***(M1)***

$pq = 125 \times 27$ OR $q^2 = 125p$ OR $p^2 = 27q$

two correct proportion equations ***A1***

attempt to eliminate either p or q ***(M1)***

$q^2 = 125 \times \frac{125 \times 27}{q}$ OR $p^2 = 27 \times \frac{125 \times 27}{p}$

$p = 45,\ q = 75$ ***A1A1***

[5 marks]

14. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3\left(4x^3\right)$, where $x > 0$. *[5 marks]*

Markscheme

attempt to use change the base ***(M1)***

$\log_3 \sqrt{x} = \frac{\log_3 2}{2} + \log_3 \left(4x^3\right)$

attempt to use the power rule ***(M1)***

$\log_3 \sqrt{x} = \log_3 \sqrt{2} + \log_3 \left(4x^3\right)$

attempt to use product or quotient rule for logs, $\ln a + \ln b = \ln ab$ ***(M1)***

$\log_3 \sqrt{x} = \log_3 \left(4\sqrt{2}x^3\right)$

Note: The ***M*** marks are for attempting to use the relevant log rule and may be applied in any order and at any time during the attempt seen.

$\sqrt{x} = 4\sqrt{2}x^3$

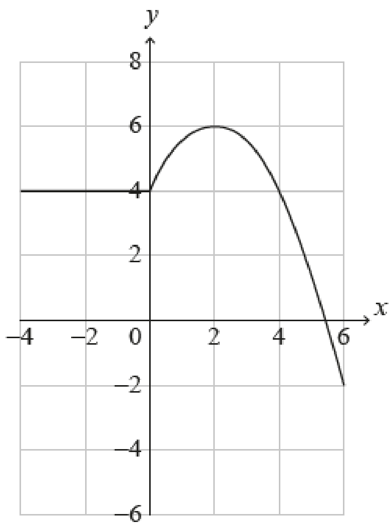
$x = 32x^6$

$x^5 = \frac{1}{32}$ ***(A1)***

$x = \frac{1}{2}$ ***A1***

[5 marks]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



15a. Write down the value of $f(2)$. *[1 mark]*

Markscheme

$f(2) = 6$ ***A1***

[1 mark]

15b. Write down the value of $(f \circ f)(2)$. *[1 mark]*

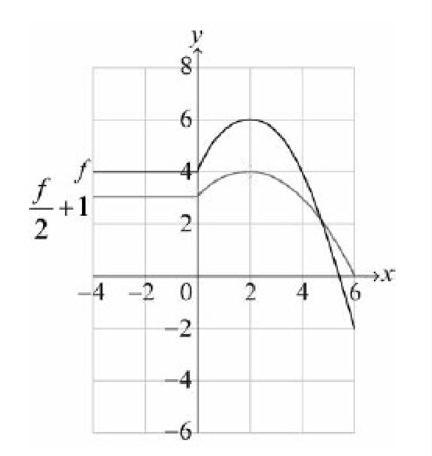
Markscheme

$(f \circ f)(2) = -2$ ***A1***

[1 mark]

15c. Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g. *[3 marks]*

Markscheme



M1A1A1

Note: Award ***M1*** for an attempt to apply any vertical stretch or vertical translation, ***A1*** for a correct horizontal line segment between -4 and 0 (located roughly at $y = 3$),
A1 for a correct concave down parabola including max point at $(2, 4)$ and for correct end points at $(0, 3)$ and $(6, 0)$ (within circles). Points do not need to be labelled.

[3 marks]

Let $f(x) = a\log_3(x-4)$, for $x > 4$, where $a > 0$.

Point $\mathrm{A}(13, 7)$ lies on the graph of f.

16a. Find the value of a. *[3 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

attempt to substitute coordinates (in any order) into f ***(M1)***

eg $a\log_3(13-4) = 7$, $a\log_3(7-4) = 13$, $a\log 9 = 7$

finding $\log_3 9 = 2$ (seen anywhere) ***(A1)***

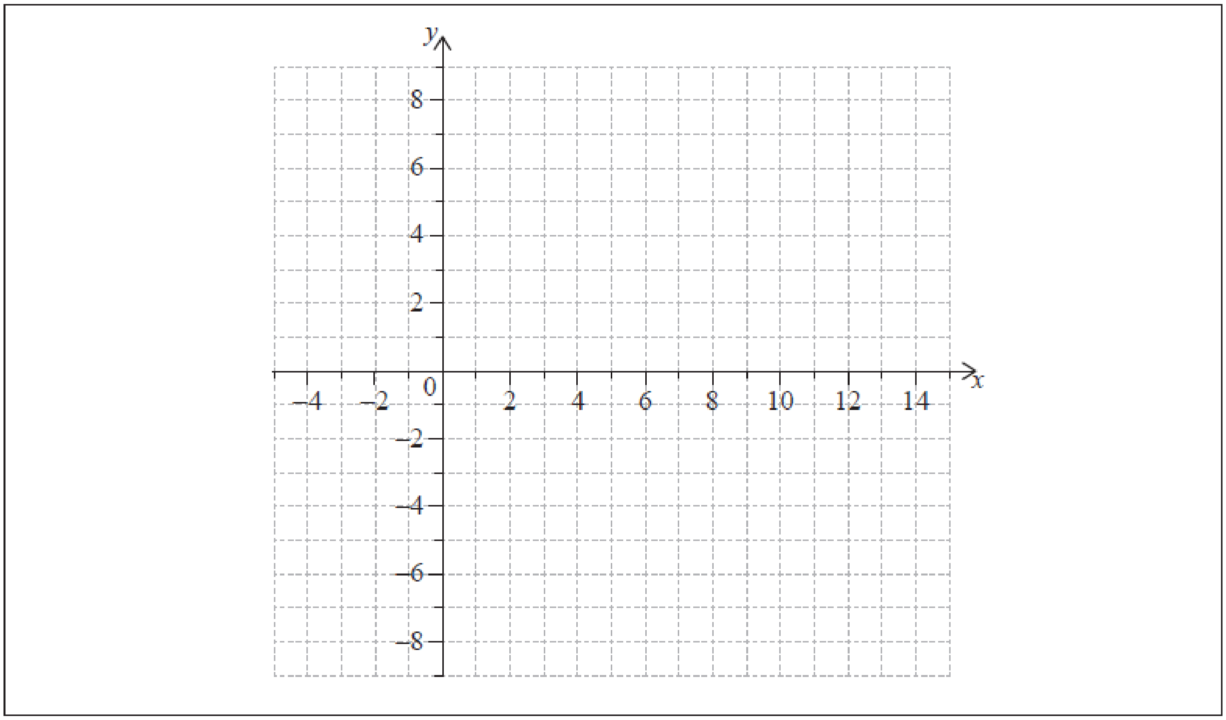
eg $\log_3 9 = 2$, $2a = 7$

$a = \frac{7}{2}$ ***A1 N2***

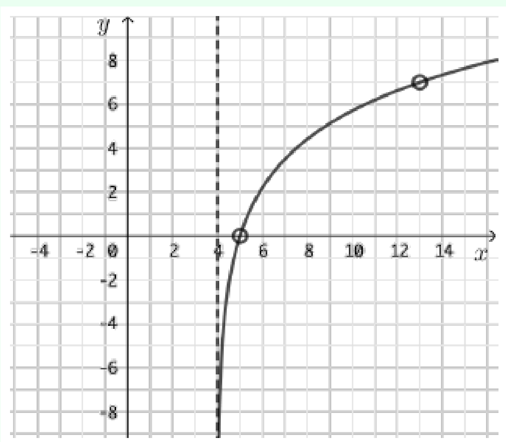
[3 marks]

16b. The x-intercept of the graph of f is $(5,\ 0)$. *[3 marks]*

On the following grid, sketch the graph of f.



Markscheme



A1A1A1 N3

Note: Award ***A1*** for correct shape of logarithmic function (must be increasing and concave down).

Only if the shape is correct, award the following:

A1 for being asymptotic to $x = 4$

A1 for curve including both points in circles.

[3 marks]

Consider the functions f and g defined by $f(x) = \ln|x|$, $x \in \mathbb{R} \setminus \{0\}$, and $g(x) = \ln|x + k|$, $x \in \mathbb{R} \setminus \{-k\}$, where $k \in \mathbb{R}$, $k > 2$.

17a. Describe the transformation by which $f(x)$ is transformed to $g(x)$. *[1 mark]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

translation k units to the left (or equivalent) ***A1***

[1 mark]

17b. State the range of g. *[1 mark]*

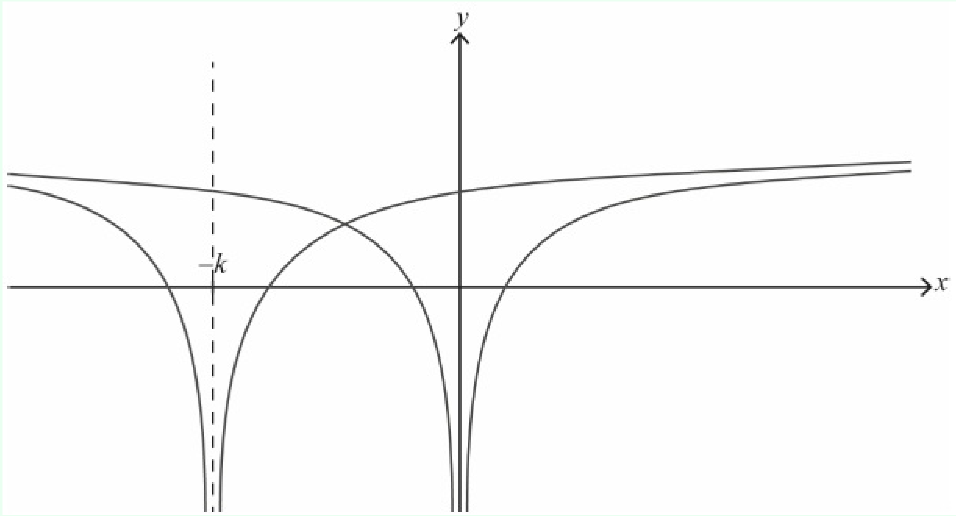
Markscheme

range is $(g(x) \in) \mathbb{R}$ ***A1***

[1 mark]

17c. Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes, clearly stating the points of intersection with any axes. *[6 marks]*

Markscheme



correct shape of $y = f(x)$ ***A1***

their $f(x)$ translated k units to left (possibly shown by $x = -k$ marked on x-axis) ***A1***

asymptote included and marked as $x = -k$ ***A1***

$f(x)$ intersects x-axis at $x = -1$, $x = 1$ ***A1***

$g(x)$ intersects x-axis at $x = -k - 1$, $x = -k + 1$ ***A1***

$g(x)$ intersects y-axis at $y = \ln k$ ***A1***

Note: Do not penalise candidates if their graphs "cross" as $x \to \pm\infty$.

Note: Do not award ***FT*** marks from the candidate's part (a) to part (c).

[6 marks]

The graphs of f and g intersect at the point P .

17d. Find the coordinates of P. *[2 marks]*

Markscheme

at P $\ln(x+k) = \ln(-x)$

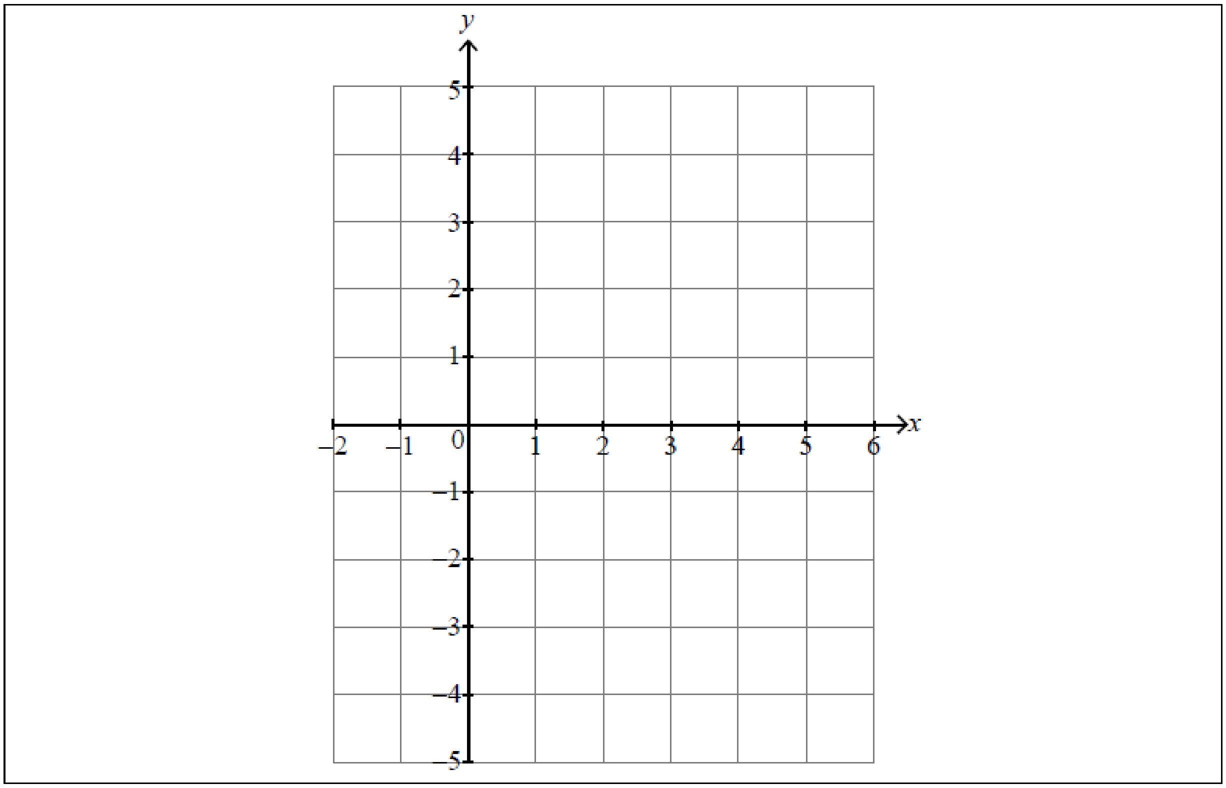
attempt to solve $x + k = -x$ (or equivalent) ***(M1)***

$x = -\frac{k}{2} \Rightarrow y = \ln\left(\frac{k}{2}\right)$ (or $y = \ln\left|\frac{k}{2}\right|$) ***A1***

P$\left(-\frac{k}{2},\ \ln\frac{k}{2}\right)$ (or P$\left(-\frac{k}{2},\ \ln\left|\frac{k}{2}\right|\right)$)

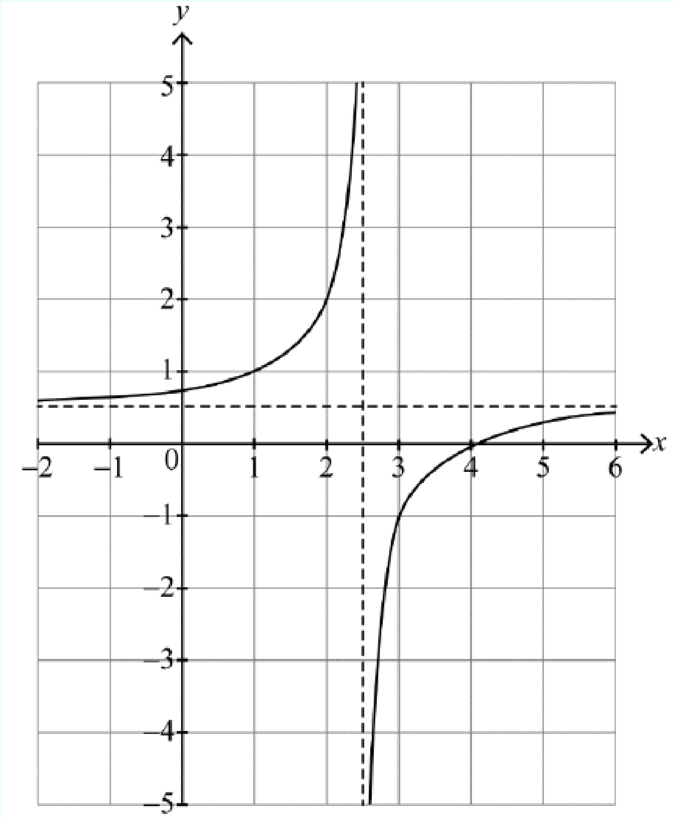
[2 marks]

18. Sketch the graph of $y = \frac{x-4}{2x-5}$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. *[5 marks]*



Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



correct shape: two branches in correct quadrants with asymptotic behaviour ***A1***

crosses at (4, 0) and $\left(0,\ \frac{4}{5}\right)$ ***A1A1***

asymptotes at $x = \frac{5}{2}$ and $y = \frac{1}{2}$ ***A1A1***

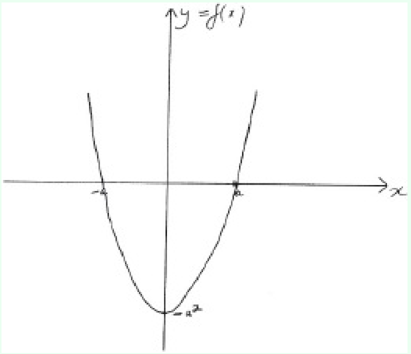
[5 marks]

Consider the function f defined by $f(x) = x^2 - a^2, x \in \mathbb{R}$ where a is a positive constant.

19a. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. *[2 marks]*

$y = f(x)$;

Markscheme



A1 for correct shape

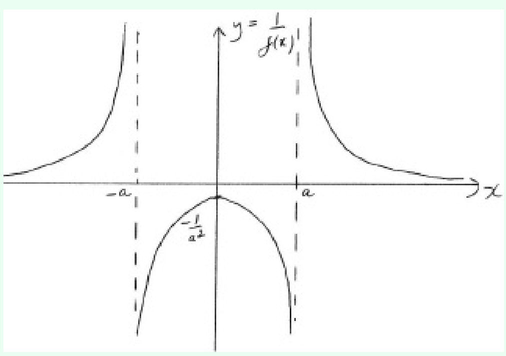
A1 for correct x and y intercepts and minimum point

[2 marks]

19b. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. *[4 marks]*

$y = \frac{1}{f(x)}$;

Markscheme



A1 for correct shape

A1 for correct vertical asymptotes

A1 for correct implied horizontal asymptote

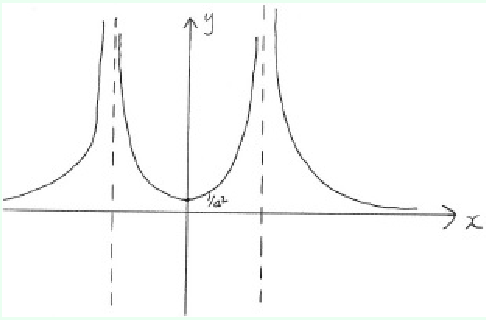
A1 for correct maximum point

[??? marks]

19c. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. *[2 marks]*

$y = \left| \frac{1}{f(x)} \right|$.

Markscheme



A1 for reflecting negative branch from (ii) in the x-axis

A1 for correctly labelled minimum point

[2 marks]