test revision (ch 2 - 6) *[140 marks]*

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

1a. Show that $(g \circ f)(x) = 2x + 11$. *[2 marks]*

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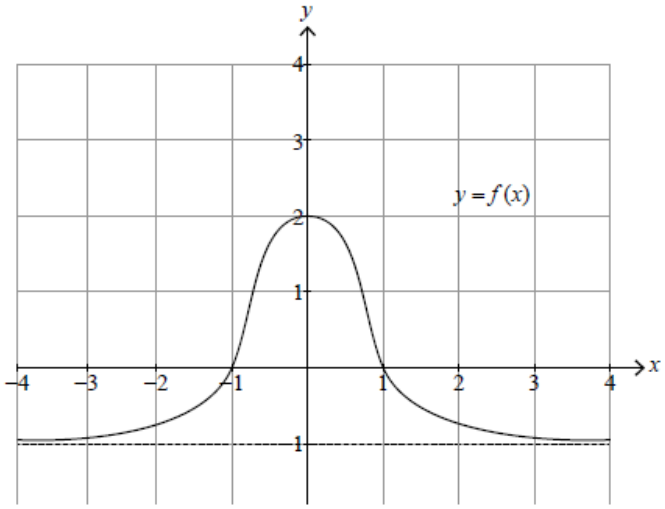
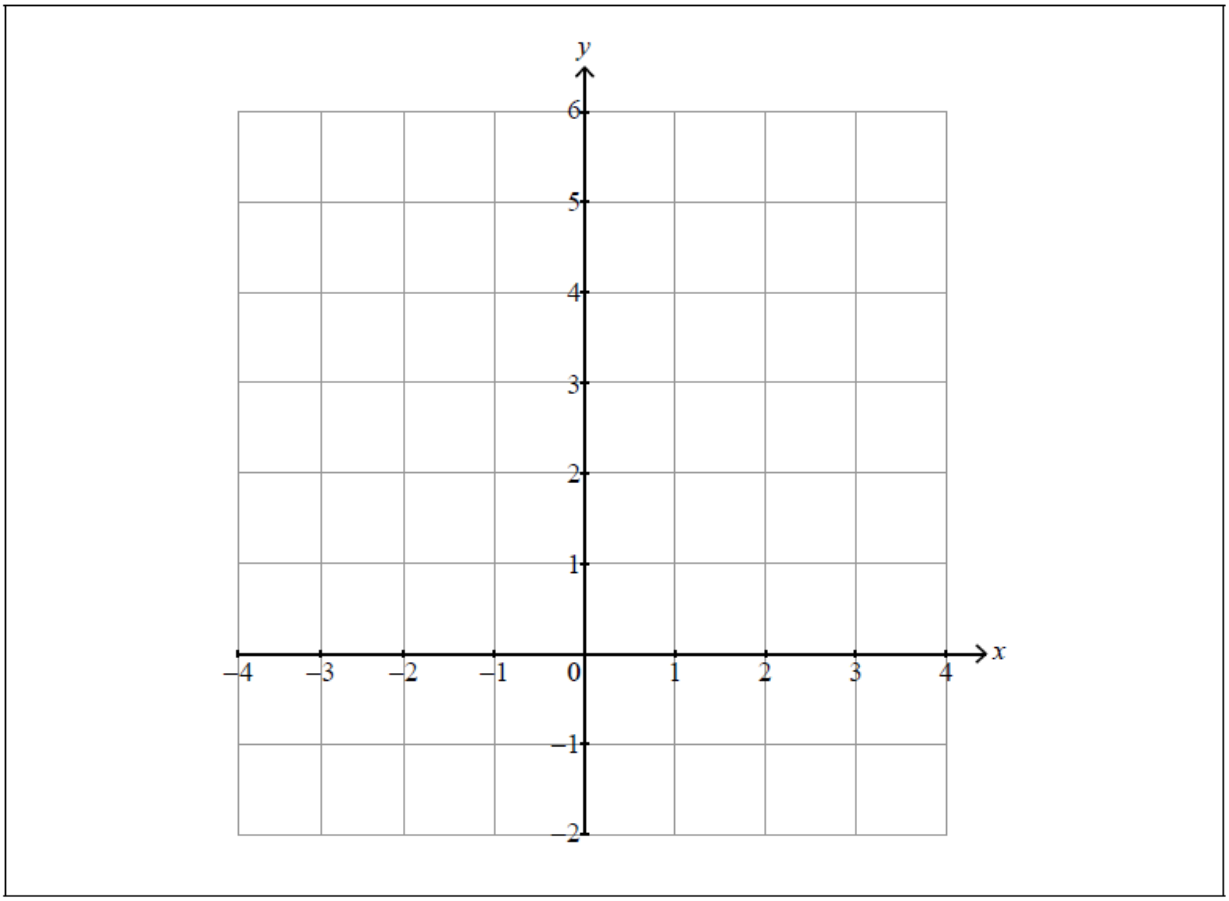
1b. Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. *[3 marks]*

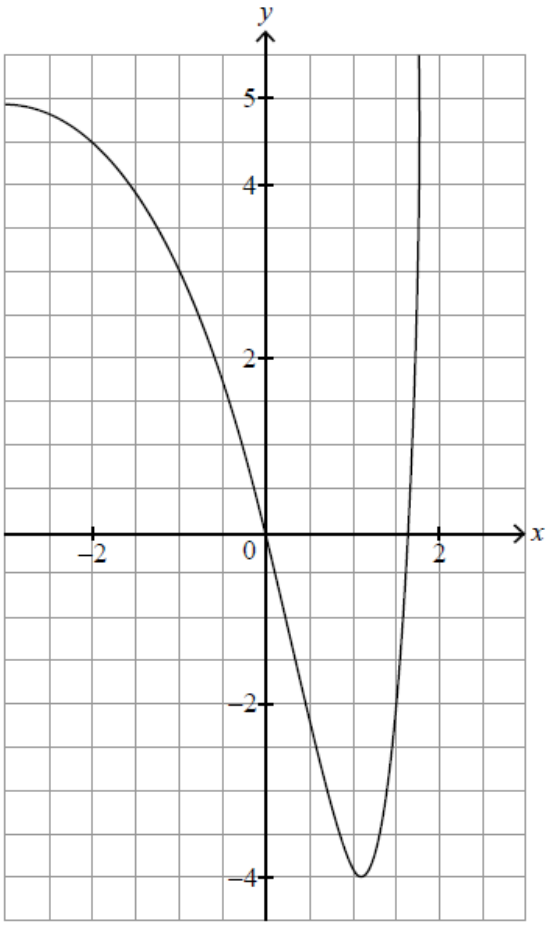
2. The following diagram shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = -1$. The graph crosses the x-axis at $x = -1$ and $x = 1$, and the y-axis at $y = 2$. *[5 marks]*



On the following set of axes, sketch the graph of $y = [f(x)]^2 + 1$, clearly showing any asymptotes with their equations and the coordinates of any local maxima or minima.



The function f is defined by $f(x) = e^{2x} - 6e^x + 5$, $x \in \mathbb{R}$, $x \leqslant a$. The graph of $y = f(x)$ is shown in the following diagram.



3a. Find the largest value of a such that f has an inverse function. *[3 marks]*

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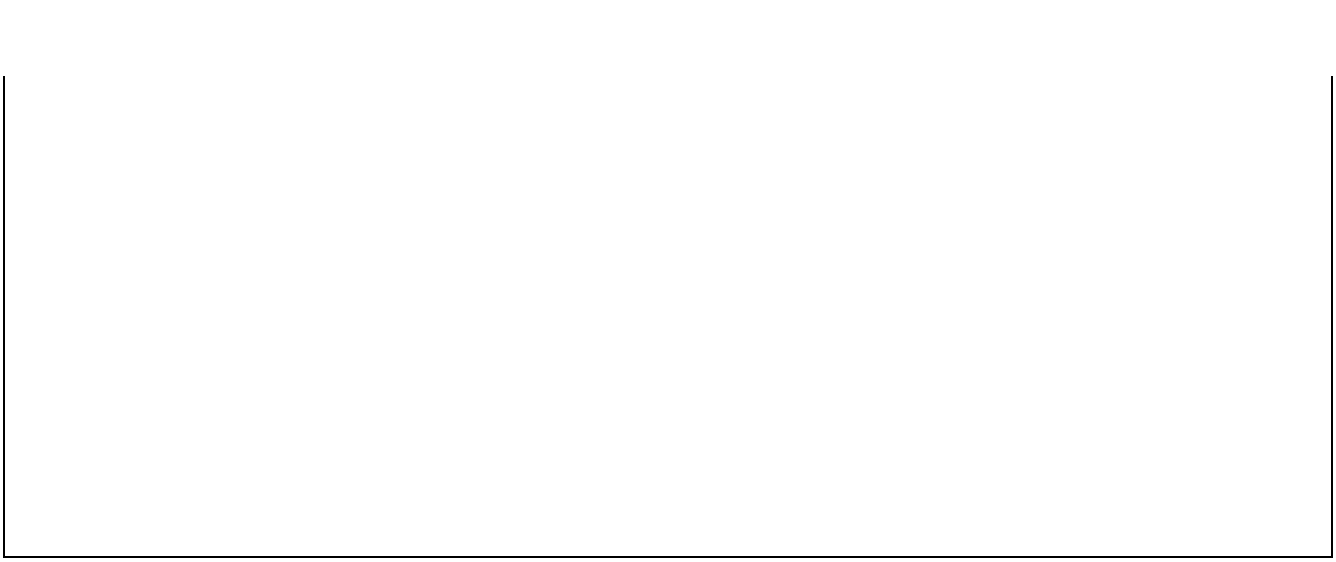

3b. For this value of a, find an expression for $f^{-1}(x)$, stating its domain. *[5 marks]*

4. The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = x - 2$ and $g(x) = ax + b$, where $a,\ b \in \mathbb{R}$. *[6 marks]*

Given that $(f \circ g)(2) = -3$ and $(g \circ f)(1) = 5$, find the value of a and the value of b.

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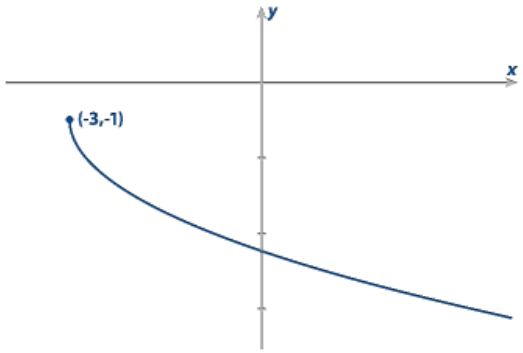
The following diagram shows the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$.



5a. Describe a sequence of transformations that transforms the graph of $y = \sqrt{x}$ for $x \geq 0$ to the graph of $y = -1 - \sqrt{x+3}$ for $x \geq -3$. *[3 marks]*

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A function f is defined by $f(x) = -1 - \sqrt{x+3}$ for $x \geq -3$.

5b. State the range of f. *[1 mark]*

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5c. Find an expression for $f^{-1}(x)$, stating its domain. *[5 marks]*

5d. Find the coordinates of the point(s) where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. *[5 marks]*

Let $f(x) = \frac{2x+6}{x^2+6x+10}$, $x \in \mathbb{R}$.

6a. Show that $f(x)$ has no vertical asymptotes. *[3 marks]*

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6b. Find the equation of the horizontal asymptote. *[2 marks]*

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Let $f(x) = \frac{2x^2-5x-12}{x+2}$, $x \in \mathbb{R}$, $x \neq -2$.

7a. Find all the intercepts of the graph of $f(x)$ with both the x and y axes. *[4 marks]*

7b. Write down the equation of the vertical asymptote. *[1 mark]*

7c. As $x \to \pm\infty$ the graph of $f(x)$ approaches an oblique straight line asymptote. *[4 marks]*

Divide $2x^2 - 5x - 12$ by $x + 2$ to find the equation of this asymptote.

Let $f(x) = \frac{x^2-10x+5}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$.

8a. Find the co-ordinates of all stationary points. *[4 marks]*

8b. Write down the equation of the vertical asymptote. *[1 mark]*

8c. Sketch the graph of *y=f(x)* clearly indicating all the asymptotes and axes intercepts. *[5 marks]*

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	−2	0	3	4
$f(x)$	8	4	0	−3
$g(x)$	−5	−2	4	0

9a. Find $g(0)$. *[1 mark]*

9b. Find $(f \circ g)(0)$. *[2 marks]*

9c. Find the value of x such that $f(x) = 0$. *[2 marks]*

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R},\ x \neq -1$.

The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

10a. Write down the equation of the vertical asymptote. *[1 mark]*

10b. Write down the equation of the horizontal asymptote. *[1 mark]*

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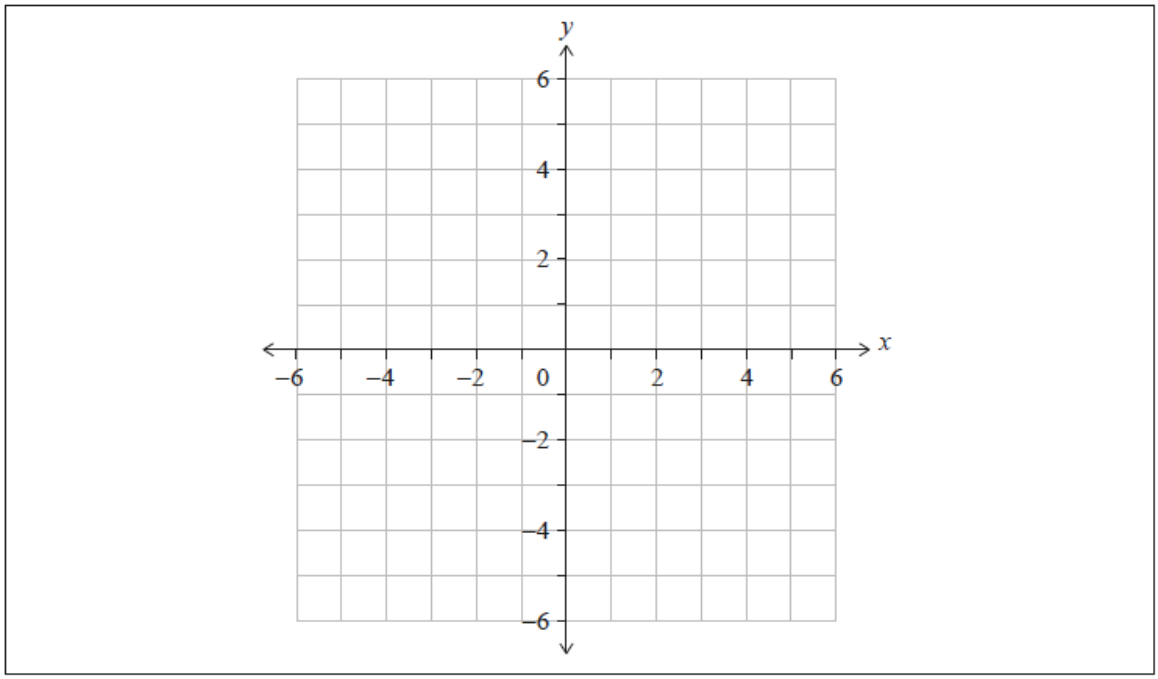
10c. On the set of axes below, sketch the graph of $y = f(x)$. *[3 marks]*

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



10d. Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. *[1 mark]*

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11a. Write down the equation of the vertical asymptote. *[1 mark]*

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11b. Write down the equation of the horizontal asymptote. *[1 mark]*

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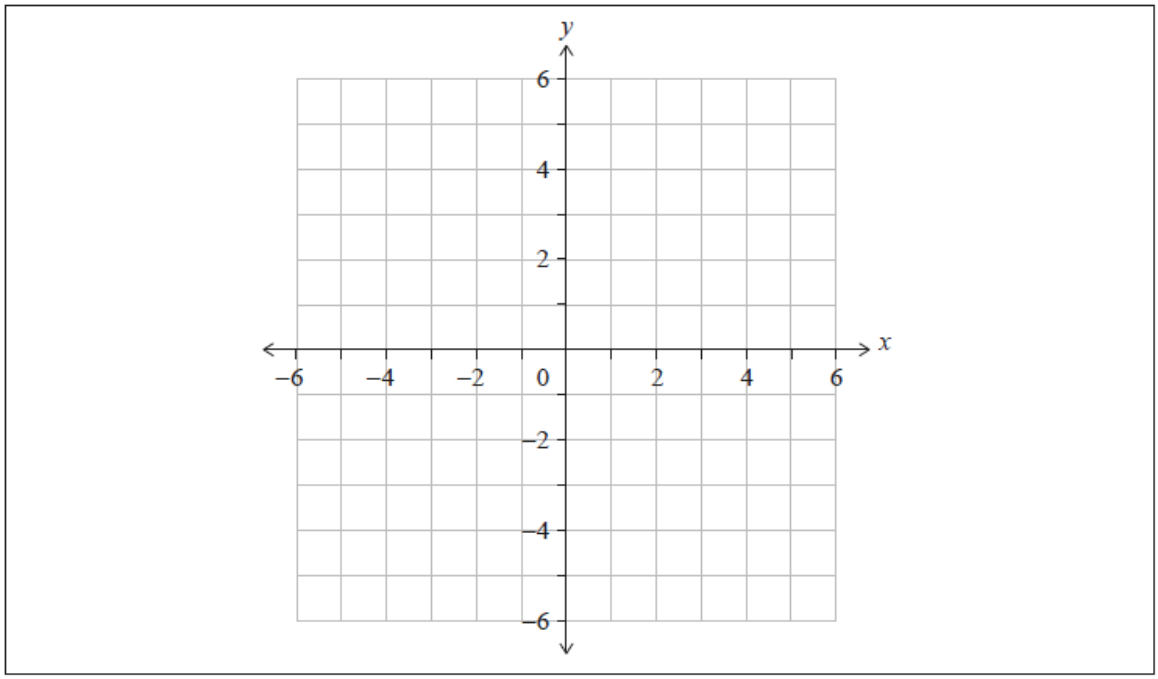
11c. On the set of axes below, sketch the graph of $y = f(x)$. *[3 marks]*

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.



11d. Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$. *[1 mark]*

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11e. Solve the inequality $0 < \frac{2|x|-1}{|x|+1} < 2$. *[2 marks]*

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The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R},\ x \neq 3$.

Write down the equation of

12a. the vertical asymptote of the graph of f. *[1 mark]*

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12b. the horizontal asymptote of the graph of f. *[1 mark]*

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Find the coordinates where the graph of f crosses

12c. the x-axis. *[1 mark]*

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12d. the y-axis. *[1 mark]*

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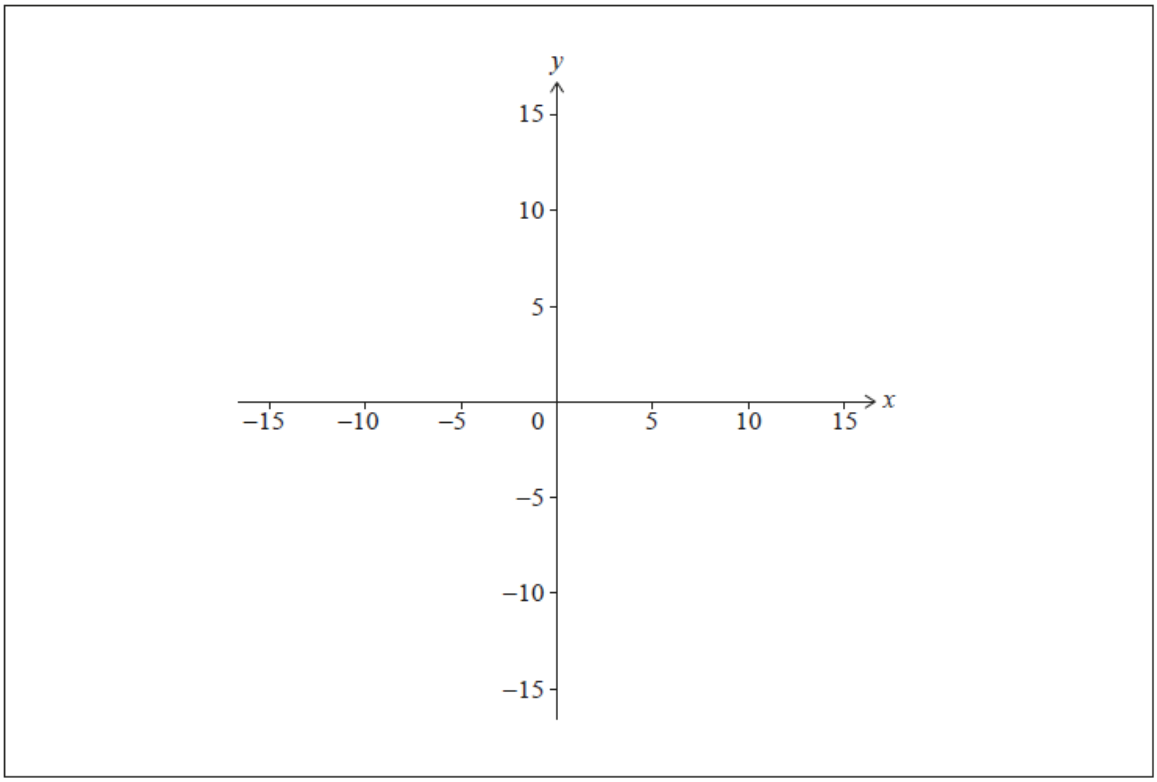
12e. Sketch the graph of f on the axes below. *[1 mark]*



Consider the function $f(x) = a^x$ where $x,\ a \in \mathbb{R}$ and $x > 0,\ a > 1$.

The graph of f contains the point $\left(\frac{2}{3},\ 4\right)$.

13a. Show that $a = 8$. *[2 marks]*

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13b. Write down an expression for $f^{-1}(x)$. *[1 mark]*

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13c. Find the value of $f^{-1}\left(\sqrt{32}\right)$. *[3 marks]*

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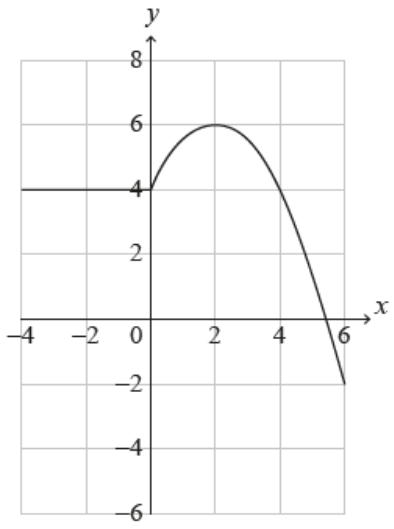
Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

13d. Show that $27,\ p,\ q$ and 125 are four consecutive terms in a geometric sequence. *[4 marks]*

13e. Find the value of p and the value of q. *[5 marks]*

14. Solve the equation $\log_3 \sqrt{x} = \frac{1}{2\log_2 3} + \log_3\left(4x^3\right)$, where $x > 0$. *[5 marks]*

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



15a. Write down the value of $f(2)$. *[1 mark]*

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15b. Write down the value of $(f \circ f)(2)$. *[1 mark]*

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15c. Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g. *[3 marks]*

Let $f(x) = a\log_3(x-4)$, for $x > 4$, where $a > 0$.

Point $\mathrm{A}(13, 7)$ lies on the graph of f.

16a. Find the value of a. *[3 marks]*

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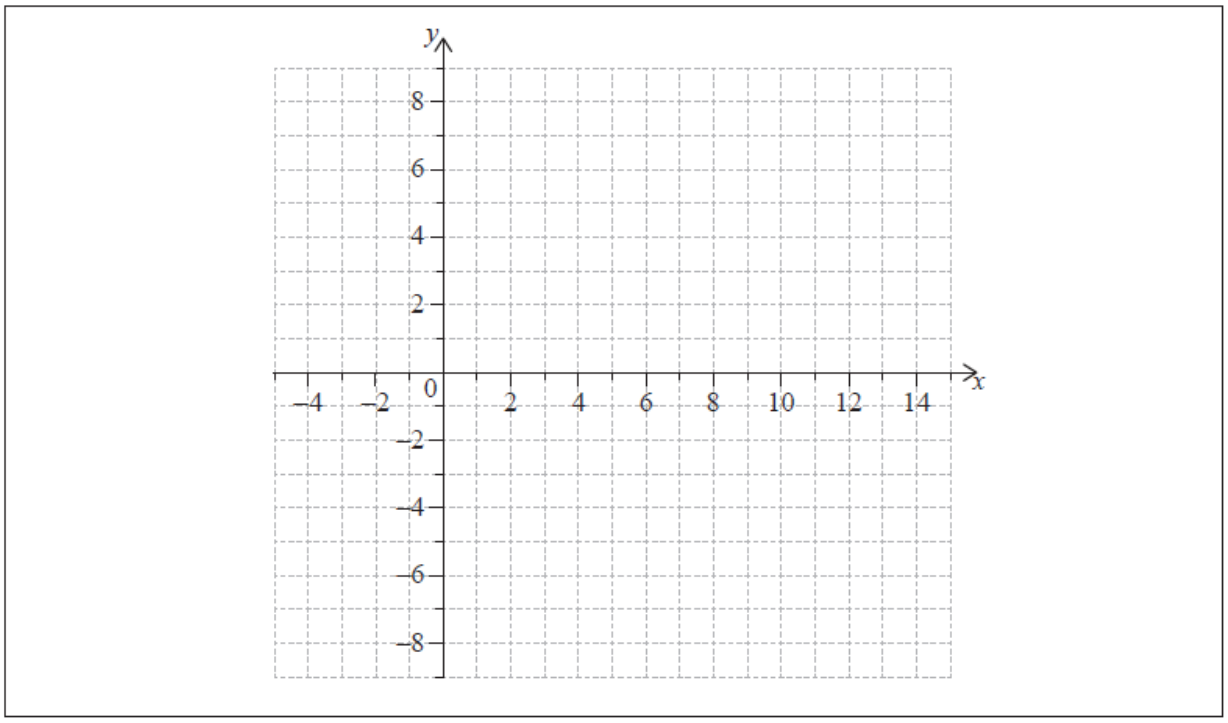
16b. The x-intercept of the graph of f is $(5,\ 0)$. *[3 marks]*

On the following grid, sketch the graph of f.



Consider the functions f and g defined by $f(x) = \ln|x|$, $x \in \mathbb{R} \setminus \{0\}$, and $g(x) = \ln|x + k|$, $x \in \mathbb{R} \setminus \{-k\}$, where $k \in \mathbb{R}$, $k > 2$.

17a. Describe the transformation by which $f(x)$ is transformed to $g(x)$. *[1 mark]*

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17b. State the range of g. *[1 mark]*

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17c. Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes, clearly stating the points of intersection with any axes. *[6 marks]*

The graphs of f and g intersect at the point P .

17d. Find the coordinates of P. *[2 marks]*

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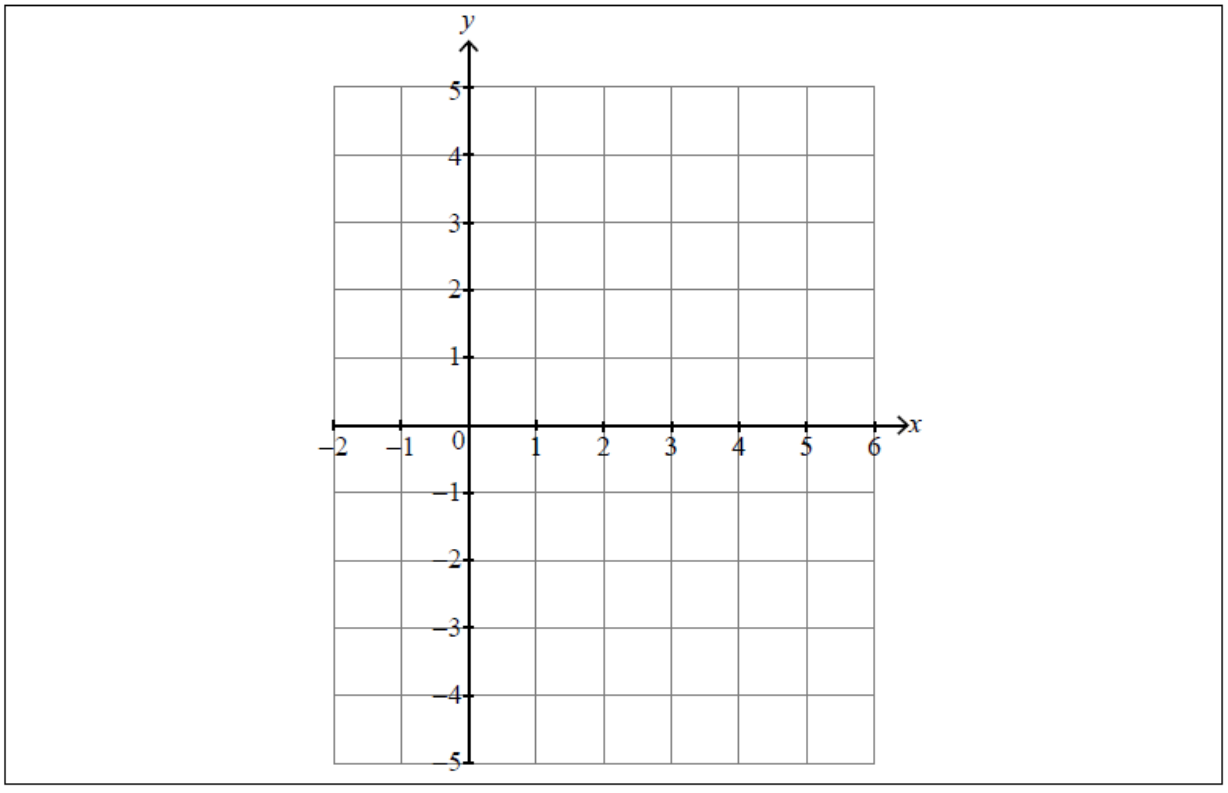
18. Sketch the graph of $y = \frac{x-4}{2x-5}$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. *[5 marks]*



Consider the function f defined by $f(x) = x^2 - a^2, x \in \mathbb{R}$ where a is a positive constant.

19a. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. *[2 marks]*

$y = f(x)$;

19b. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. *[4 marks]*

$y = \frac{1}{f(x)}$;

19c. Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes. *[2 marks]*

$y = \left| \frac{1}{f(x)} \right|$.