EXAM-STYLE QUESTIONS

1 When the polynomial $f(x) = x^4 - 3x^3 + ax^2 - 4x + 7$ is divided by $(x + 2)$ the remainder is 7. Find the value of a.

2 Solve the simultaneous equations:

$$\begin{cases} 3x - 2y = i - 2 \\ 4y - (1 - i)x = 3 + 3i \end{cases}$$

4 Given that $1 - 2i$ is a complex root of the equation $z^4 - 2z^3 + 14z^2 - 18z + 45 = 0$, find the remaining roots.

6 Find the value of a such that the roots α and β of the quadratic equation $x^2 + ax + a + 1 = 0$ satisfy $\alpha^3 + \beta^3 = 9$

9 The cubic equation $x^3 - 5x^2 + 6x - 3 = 0$ has solutions α, β and γ. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$

6 Solve the equation $\log_3 x + \log_x 9 - 3 = 0$

9 Solve these simultaneous equations.

$2\log_x y = 1 \qquad xy = 125$

5 Solve the equation $5^{x+1} + \frac{4}{5^x} - 21 = 0$

10 Find the value of x which satisfies the equation $e^x - e^{-x} = 4$

Hence, show that for this value of x

$e^x + e^{-x} = 2\sqrt{5}$

2 For events A and B it is known that: $P(A' \cap B') = 0.35$, $P(A) = 0.25$ and $P(B) = 0.6$ Find

a $P(A \cap B)$ **b** $P(A|B)$ **c** $P(B'|A')$

4 In one box there are 14 white and 16 black balls, while in a second box there are 7 white and 12 black balls. A ball is drawn from the first box and placed in the second box, and then two balls are drawn from the second box.

a What is the probability that both balls are black?

b What is the probability that the ball drawn from the first box was white, given that both balls drawn from the second box were white?

5 A sample space U contains the events A and B. These probabilities are given: $\mathrm{P}(B)=\frac{2}{3}, \mathrm{P}(A|B)=\frac{5}{6}$ and $\mathrm{P}(A'|B')=\frac{1}{4}$

a Draw a probability tree diagram representing this information

b Find: **i** $\mathrm{P}(A)$ **ii** $\mathrm{P}(B|A)$ **iii** $\mathrm{P}(B'|A')$.

6 A pair of dice is in a non-transparent bag. One dice is biased and the probability of obtaining a 6 on that dice is $\frac{2}{3}$, while the other dice is unbiased. A dice is taken from the bag at random and roll it.

a What is the probability of obtaining a 6?

b What is the probability that the unbiased dice was taken given that you did not obtain a 6?

3 Water tides can be modeled by the function

$$h(t) = a \sin [b(t + c)] + d$$

where $h(t)$ is the height of water at time t, measured in hours after midnight.

At Blue Harbor on Sunny Island the time between consecutive high tides is 12 hours. The height of the water at high tide is 14.4 m and the height of the water at low tide is 1.2 m.

On a particular day the first high tide occurs at 08:15.

a Use the information given to find the values of a, b, c and d.

b Plot the graph of the function and calculate the time of the first low tide.

A fishing boat is only allowed to leave or enter the harbor if the height of the water is at least 5 m.

c Find the time intervals during which a boat could enter or leave the harbor on that particular day.

16 Solve $\log_{16} \sqrt[3]{100 - x^2} = \frac{1}{2}$. [IB Nov 03 P1 Q10]

17 Find the exact value of x satisfying the equation $(3^x)(4^{2x+1}) = 6^{x+2}$

Give your answer in the form $\frac{\ln a}{\ln b}$, where $a, b \in \mathbb{Z}$. [IB May 03 P1 Q12]

18 Solve $2(5^{x+1}) = 1 + \frac{3}{5^x}$, giving your answer in the form $a + \log_5 b$, where $a, b \in \mathbb{Z}$. [IB Nov 03 P1 Q19]

19 Solve the simultaneous equations $\log_x y = 1$ and $xy = 16$ for $x, y > 0$.

20 Solve the simultaneous equations $\log_a(x + y) = 0$ and $2 \log_a x = \log(4y + 1)$.

21 Solve the system of simultaneous equations:

$$x + 2y = 5$$
$$4^x = 8^y$$

[IB Nov 98 P1 Q2]

22 If $f(x) = \ln(6x^2 - 5x - 6)$, find

a the exact domain of $f(x)$

b the range of $f(x)$. [IB Nov 98 P1 Q7]

23 Find all real values of x so that $3^{x^2-1} = (\sqrt{3})^{126}$. [IB May 98 P1 Q3]

24 **a** Given that $\log_a b = \frac{\log_c b}{\log_c a}$, find the real numbers k and m such that

$\log_9 x^3 = k \log_3 x$ and $\log_{27} 512 = m \log_3 8$.

b Find all values of x for which $\log_9 x^3 + \log_3 x^{\frac{1}{2}} = \log_{27} 512$.

[IB Nov 97 P1 Q4]

18 Let z_1 and z_2 be complex numbers. Solve the simultaneous equations

$z_1 + 2z_2 = 4$

$2z_1 + iz_2 = 3 + i$

Give your answer in the form $x + iy$ where $x, y \in \mathbb{Q}$.

19 If $z = 1 + \frac{2}{1 + i\sqrt{3}}$, find z in the form $x + iy$ where $x, y \in \mathbb{R}$.

20 Consider the equation $4(p - iq) = 2q - 3ip - 3(2 + 3i)$ where p and q are real numbers. Find the values of p and q.

21 If $\sqrt{z} = \frac{3}{1 + 2i} + 4 - 3i$, find z in the form $x + iy$ where $x, y \in \mathbb{R}$.

2 Find the real number k for which $1 + ki$, $(i = \sqrt{-1})$, is a zero of the polynomial $z^2 + kz + 5$. [IB Nov 00 P1 Q10]

3 If $z = 1 + 2i$ is a root of the equation $z^2 + az + b$, find the values of a and b.

4 If z is a complex number and $|z + 16| = 4|z + 1|$, find the value of $|z|$. [IB Nov 00 P1 Q18]

5 **a** Show that $(1 + i)^4 = -4$.

b Hence or otherwise, find $(1 + i)^{64}$.

6 Solve the equation $\dfrac{-i}{x - iy} = \dfrac{4 + 7i}{5 - 3i}$ for x and y, leaving your answers as rational numbers. [IB May 94 P1 Q15]

7 Find a cubic equation with real coefficients, given that two of its roots are 3 and $1 - i\sqrt{3}$.

8 If $z = x + iy$, find the real part and the imaginary part of $z + \dfrac{1}{z}$.

9 Given that $z = (b + i)^2$, where b is real and positive, find the exact value of b when $\arg z = 60°$. [IB May 01 P1 Q14]

1. Solve the equation $4z^* + 3iz = 7i$, where z is a complex number.

2. Solve the equation $(5i - 2)z = 8 + 9i$.

3. Given that $|z + i| = |z|$, find the imaginary part of z.

4. (a) Given that $|z + i| = |z - 3|$ where $z = a + bi$, show that $3a + b = 4$.

(b) Given also that $|z| = \sqrt{2}$, find the possible values of z.

5. Solve the equation $3z^2 + (2 + 3i)z + (5i - 5) = 0$, giving your answers in the form $a + bi$.

6. (a) By writing $z = a + bi$, show that $zz^* = |z|^2$.

(b) Given that $|z| = 5$, solve the equation $z^* + \dfrac{60i}{z} = 13$.