

Name:

1. (12 points) Find the following anti-derivatives:

(a) $\int 3x\sqrt{x^2 - 1}dx =$

(b) $\int \tan(3x)dx =$

(c) $\int \cos(2x)e^{3x}dx =$

(d) $\int \frac{2x + 1}{x^2 + 3} dx =$

2. (5 points) Find the constant term in the expansion of $\left(2 + x^2\right)\left(3x + \frac{1}{x^2}\right)^7$

3. (7 points) (a) Show that the derivative of $\ln |\tan \theta + \sec \theta|$ is $\sec \theta$.

(b) Use the substitution $x = \tan \theta$ to find $\int \frac{1}{\sqrt{1+x^2}} dx$.

4. (5 points) Prove that $2^{n+2} + 3^{2n+1}$ is exactly divisible by 7 for all $n \in \mathbb{Z}^+$.

5. (4 points) Show that $n^2 + 2$ is not a multiple of 4 for any $n \in \mathbb{Z}$.

6. (7 points)

(a) Find the first three terms of the binomial expansion of $\frac{1}{\sqrt[3]{3-x}}$ where $|x| < 3$.

(b) Hence, or otherwise, obtain the expansion of $\frac{(2+x)^3}{\sqrt[3]{3-x}}$ up to and including the term in x^2 .