Name:

1. (6 points)

   In this question all distances are expressed in kilometres and time is in hours

   Ship A starts sailing with a velocity vector $\begin{pmatrix} 30 \\ -40 \end{pmatrix}$ from a port located at $(0, 30)$.

   (a) Find a vector equation of the path of ship A.

   (b) Find the speed of ship A.

   Ship B starts sailing an hour later from a port located at $(10, -10)$. The two ships meet at a point $(90, -90)$.

   (c) Find a vector equation of the path of ship B.

2. (6 points) Let $f(x) = e^{2x}$.

   (a) Find the first three derivatives of $f(x)$.

   (b) Prove by induction that $f^n(x) = 2^n e^{2x}$, where $f^n(x)$ denotes the n-th derivative of $f(x)$.

3. (6 points) Consider the region in the first quadrant bounded by the graphs of $y = x^3$ and $y = 4x$.

   (a) Find the value of k such that the line $x = k$ divides the region into two regions of equal area.

   (b) Find the value of m such that the line $y = m$ divides the region into two regions of equal area.

4. (6 points) The velocity of a particle moving in a straight line can be modelled by $v(t) = 5\sin^2 t \cos t$ for $0 \leqslant t \leqslant 2\pi$, where v is measured in cms^{-1} and t is measured in seconds.

   (a) Find the times when the particle is at rest, and describe its movement over the entire interval.

   (b) Find the displacement of the particle in the given time interval, and interpret your answer.

   (c) Find the total distance covered by the particle.

5. (6 points) Given the numbers:

$$z_1 = \text{cis}\frac{3\pi}{4} \qquad \text{and} \qquad z_2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Find:

(a) z_1 in Cartesian form,

(b) z_2 in polar form,

(c) $z_1 \times z_2$ in both forms.

Hence find the exact values of:

(d) $\sin\dfrac{17\pi}{12}$ (e) $\tan\dfrac{17\pi}{12}$

6. (5 points) The point O is the centre of a the regular hexagon $ABCDEF$. Given that $\vec{OA} = \mathbf{a}$ and $\vec{AB} = \mathbf{b}$, express in terms of$\mathbf{a}$ and $\mathbf{b}$:

   (a) $\vec{OB}$

   (b) $\vec{BD}$

   (c) $\vec{FC}$

   What does the result in (c) tells you about the relationship between AB and FC.

7. (5 points) Find the number of 8 digit numbers which:

   (a) contain exactly 3 zeros,

   (b) contain exactly 3 zeros and the no two zeros are next to each other.