Name:

1. (17 points)

Consider points $A(4, 1, 2)$, $B(1, 2, 1)$ and $C(c, c, 4)$. The triangle ABC is a right triangle with the right angle at B.

(a) Show that $c = -1$.

(b) Find the area of the triangle ABC.

(c) Find the Cartesian equation of the plane Π containing the triangle ABC.

Point D has coordinates $(2, 0, 5)$.

(d) Show that point D does not lie on the plane Π.

(e) Find a vector equation of the line L that passes through A and D.

(f) Find the angle between the line L and the plane Π.

(g) Using parts (b) and (f) find the area of the tetrahedron $ABCD$.

2. (5 points)

Solve the equation

$$z^5 = 16 - 16\sqrt{3}i$$

Give your answers in the form $r \operatorname{cis} \theta$, where $r > 0$ and $0 \leqslant \theta < 2\pi$.

3. (6 points)

Consider the function $f(x) = 5.8\sin\left(\frac{\pi}{6}\left(x+1\right)\right) + b$, where $x \in \mathbb{R}$ and b is a positive constant.

(a) Find the period of $f(x)$.

The function has a local maximum at $(2, 21.8)$ and a local minimum at $(8, 10.2)$

(b) Find the value of b.

(c) Calculate $f(6)$.

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}\left(x-3.75\right)\right) + q$, where $x \in \mathbb{R}$ and p and q are positive constants. The graph of g passes through $(3, 2.5)$ and $(6, 15.1)$.

(d) Find the values of p and q.

4. (6 points) Solve the following equations for $-\pi \leqslant \theta \leqslant 2\pi$:

   (a) $2\cos^2\theta = \sin 2\theta$

   (b) $1 + \sin x = 2\cos^2 x$

5. (6 points) Find the number of 7-digit numbers which contain:

   (a) exactly 4 zeroes,

   (b) exactly 4 ones.