Name:

1. (3 points) Solve the equation

$$2^{2x+1} + 4 = 9 \times 2^x$$

2. (7 points) Consider the function

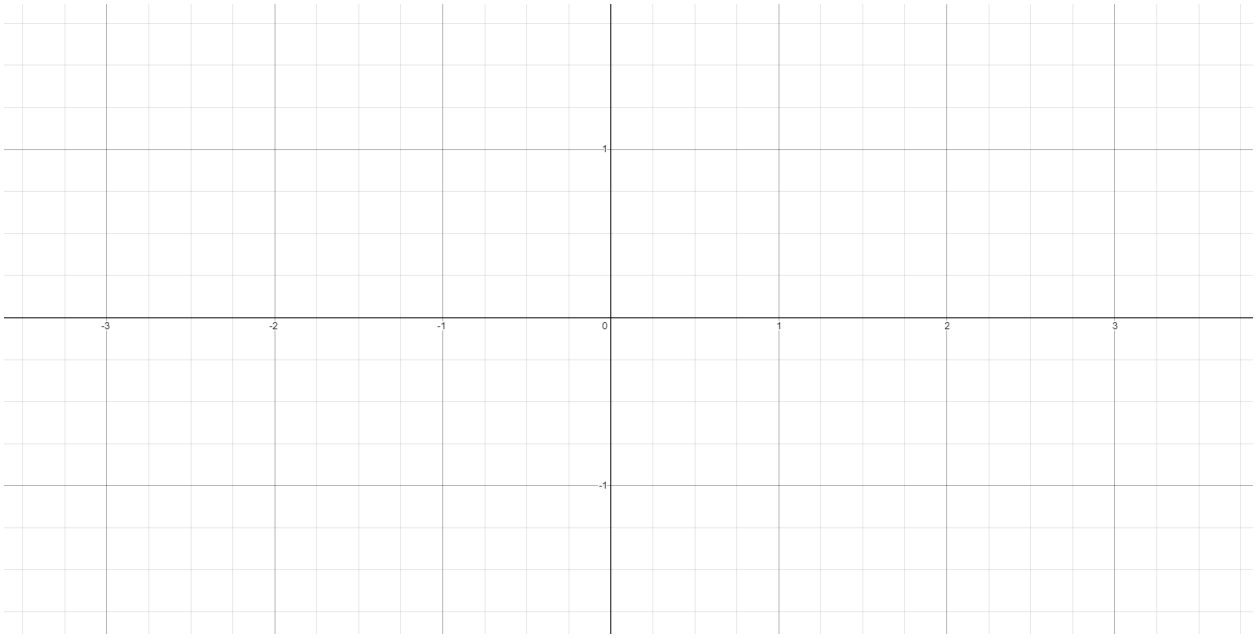
$$f(x) = \frac{x^2}{2x+1}$$

(a) Write down the equations of all the asymptotes of the graph of $y = f(x)$.

(b) Prove algebraically that the range of $y = f(x)$ is $y \in]-\infty, -1] \cup [0, \infty[$.

(c) Solve $f(x) = 0$ and $f(x) = -1$.

(d) Sketch the graph of $y = f(x)$.



3. (10 points)

(a) Use the formula

$$\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

to solve the equation

$$\sin 4x - \cos x = 0$$

for $0 \leqslant x \leqslant \frac{\pi}{2}$.

(b) Show that

$$\sin 4x = 4\sin x \cos x - 8\sin^3 x \cos x$$

for all x.

(c) Show that $x = \frac{1}{2}$ is a solution to the equation

$$-8x^3 + 4x - 1 = 0$$

and find the other solutions.

(d) Using all the previous parts, or otherwise, find the exact value of $\sin\left(\frac{\pi}{10}\right)$. Justify your answer.