# Trigonometric modelling

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If you understand graphs of trig functions and their transformations, this should all be very easy.

# Exercise 17D question 1

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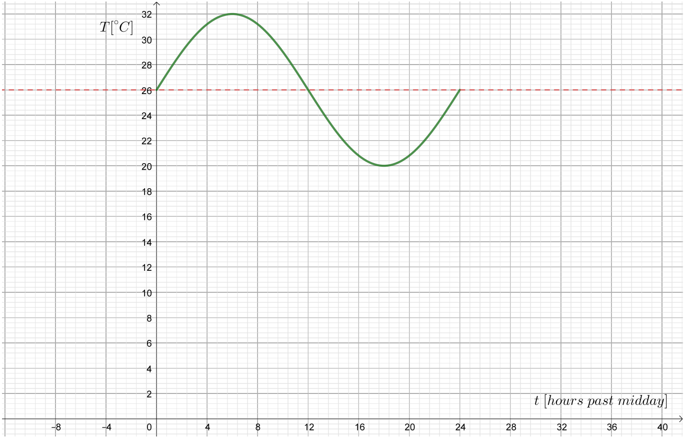
We want to sketch the graph for $0 \leqslant t \leqslant 24$, so for one full period.

# Exercise 17D question 1 (a)

We get the following graph:



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i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$. We could've also found this temperature from the graph.

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i. at midnight: $T(12) = 6\sin(\frac{\pi}{12} \times 12) + 26 = 6\sin\pi + 26 = 26^\circ C$. We could've also found this temperature from the graph.

ii. at 2 pm.: $T(2) = 6\sin(\frac{\pi}{12} \times 2) + 26 = 6\sin\frac{\pi}{6} + 26 = 29^\circ C$.

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The maximum of $\sin x$ is 1, so the maximum of $6\sin x + 26$ is 32. The maximum for the first positive argument occurs when $x = \frac{\pi}{2}$. So the maximum of $6\sin(\frac{\pi}{12}t) + 26$ will occur when

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this gives $t = 6$. So the maximum temperature of $32^\circ C$ occurs at 6 pm.

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$$h(t) = 6\cos\left(\frac{\pi}{6} \times t\right)$$

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$$d(t) = 12\sin(2\pi t)$$

Now we move on to finding trigonometric models based on real-life data.

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# Trigonometric model

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## Sunrise and sunset by month (Warsaw)

| Month | Sunrise | Sunset | Hours of daylight |
|---|---|---|---|
| Januar | 07:36 am | 03:54 pm | 8:18 hours |
| Februar | 06:50 am | 04:49 pm | 9:59 hours |
| März | 05:47 am | 05:42 pm | 11:55 hours |
| April | 05:35 am | 07:36 pm | 14:00 hours |
| Mai | 04:38 am | 08:26 pm | 15:49 hours |
| Juni | 04:11 am | 09:01 pm | 16:50 hours |
| Juli | 04:31 am | 08:52 pm | 16:21 hours |
| August | 05:18 am | 08:02 pm | 14:44 hours |
| September | 06:09 am | 06:52 pm | 12:43 hours |
| Oktober | 07:00 am | 05:43 pm | 10:43 hours |
| November | 06:55 am | 03:45 pm | 8:50 hours |
| Dezember | 07:37 am | 03:25 pm | 7:49 hours |

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The maximum $D_{max} = 16.8(3)$, the minimum $D_{min} = 7.81(6)$. This gives the principle axis $D = 12.325$ and the amplitude of $4.508(3)$.

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$$\frac{\pi}{6}(6 - c) = \frac{\pi}{2}$$

which gives $c = 3$.

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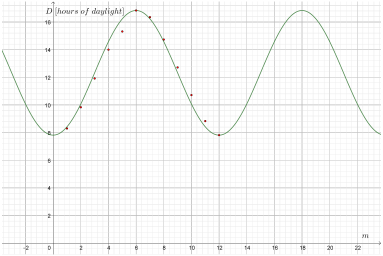

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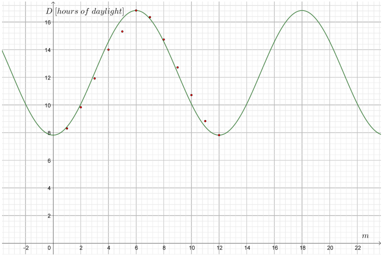


Looks like we could've done a slightly better job with horizontal shift by choosing a different point.

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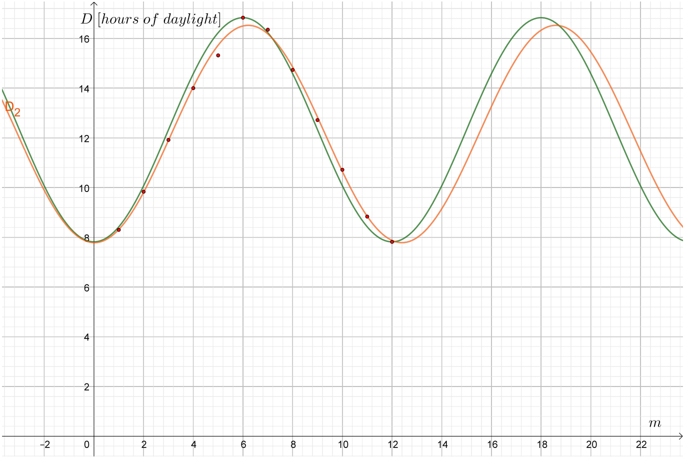
The function we got is:

$$D(m) = 4.3711\sin(0.5075m - 1.5814) + 12.1548$$

Graphs of both functions:




Admittedly the GDC has done a slightly better job.

In case of any questions you can email me at T.J.Lechowski@gmail.com.