1. (a) Write the vector equations of the following lines in parametric form.

$$\boldsymbol{r}_1 = \begin{pmatrix} 3 \\ 2 \\ 7 \end{pmatrix} + m\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

$$\boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} + n\begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

(2)

(b) Hence show that these two lines intersect and find the point of intersection, A.

(5)

(c) Find the Cartesian equation of the plane Π that contains these two lines.

(4)

(d) Let B be the point of intersection of the plane Π and the line $\boldsymbol{r} = \begin{pmatrix} -8 \\ -3 \\ 0 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$.

Find the coordinates of B.

(4)

(e) If C is the mid-point of AB, find the vector equation of the line perpendicular to the plane Π and passing through C.

(3)
(Total 18 marks)

2. The line L is given by the parametric equations $x = 1 – \lambda$, $y = 2 – 3\lambda$, $z = 2$.
Find the coordinates of the point on L that is nearest to the origin.

(Total 6 marks)

3. (a) Find the set of values of k for which the following system of equations has no solution.

$$x + 2y – 3z = k$$
$$3x + y + 2z = 4$$
$$5x + 7z = 5$$

(4)

(b) Describe the geometrical relationship of the three planes represented by this system of equations.

(1)
(Total 5 marks)

4. Find the vector equation of the line of intersection of the three planes represented by the following system of equations.

$$\begin{aligned} 2x - 7y + 5z &= 1 \\ 6x + 3y - z &= -1 \\ -14x - 23y + 13z &= 5 \end{aligned}$$

(Total 6 marks)

5. Consider the points A(1, −1, 4), B(2, − 2, 5) and O(0, 0, 0).

(a) Calculate the cosine of the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$.

(5)

(b) Find a vector equation of the line L_1 which passes through A and B.

(2)

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$, where $t \in \mathbb{R}$.

(c) Show that the lines L_1 and L_2 intersect and find the coordinates of their point of intersection.

(7)

(d) Find the Cartesian equation of the plane that contains both the line L_2 and the point A.

(6)
(Total 20 marks)

6. The points A, B, C have position vectors $\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $3\boldsymbol{i} + \boldsymbol{k}$ respectively and lie in the plane π.

(a) Find

(i) the area of the triangle ABC;

(ii) the shortest distance from C to the line AB;

(iii) the cartesian equation of the plane π.

(14)

The line L passes through the origin and is normal to the plane π, it intersects π at the point D.

(b) Find

(i) the coordinates of the point D;

(ii) the distance of π from the origin.

(6)
(Total 20 marks)

7. The vector equation of line l is given as $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$.

Find the Cartesian equation of the plane containing the line l and the point A(4, –2, 5).

(Total 6 marks)

8. Two lines are defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \text{ and } l_2 : \frac{x-4}{-3} = \frac{y+7}{4} = -(z+3).$$

(a) Find the coordinates of the point A on l_1 and the point B on l_2 such that $\overrightarrow{AB}$ is perpendicular to both l_1 and l_2.

(13)

(b) Find $|AB|$.

(3)

(c) Find the Cartesian equation of the plane Π that contains l_1 and does not intersect l_2.

(3)

(Total 19 marks)

9. (a) Show that a Cartesian equation of the line, l_1, containing points A(1, –1, 2) and B(3, 0, 3) has the form $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{1}$.

(2)

(b) An equation of a second line, l_2, has the form $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$. Show that the lines l_1 and l_2 intersect, and find the coordinates of their point of intersection.

(5)

(c) Given that direction vectors of l_1 and l_2 are $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ respectively, determine $\boldsymbol{d}_1 \times \boldsymbol{d}_2$.

(3)

(d) Show that a Cartesian equation of the plane, Π, that contains l_1 and l_2 is $-x - y + 3z = 6$.

(3)

(e) Find a vector equation of the line l_3 which is perpendicular to the plane Π and passes through the point T(3, 1, –4).

(2)

(f) (i) Find the point of intersection of the line l_3 and the plane Π.

(ii) Find the coordinates of T′, the reflection of the point T in the plane Π.

(iii) Hence find the magnitude of the vector $\overrightarrow{TT'}$.

(7)
(Total 22 marks)

10. Consider the plane with equation $4x - 2y - z = 1$ and the line given by the parametric equations

$$x = 3 - 2\lambda$$
$$y = (2k - 1) + \lambda$$
$$z = -1 + k\lambda.$$

Given that the line is perpendicular to the plane, find

(a) the value of k;

(4)

(b) the coordinates of the point of intersection of the line and the plane.

(4)
(Total 8 marks)

11. The angle between the vector $\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and the vector $\boldsymbol{b} = 3\boldsymbol{i} - 2\boldsymbol{j} + m\boldsymbol{k}$ is 30°.

Find the values of m.

(Total 6 marks)

12. (a) Show that the two planes

$$\pi_1 : x + 2y - z = 1$$
$$\pi_2 : x + z = -2$$

are perpendicular.

(3)

(b) Find the equation of the plane π_3 that passes through the origin and is perpendicular to both π_1 and π_2.

(4)
(Total 7 marks)

13. Consider the points A(1, 2, 1), B(0, –1, 2), C(1, 0, 2) and D(2, –1, –6).

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

(2)

(b) Calculate $\overrightarrow{AB} \times \overrightarrow{BC}$.

(2)

(c) Hence, or otherwise find the area of triangle ABC.

(3)

(d) Find the Cartesian equation of the plane *P* containing the points A, B and C.

(3)

(e) Find a set of parametric equations for the line *L* through the point D and perpendicular to the plane *P*.

(3)

(f) Find the point of intersection E, of the line *L* and the plane *P*.

(4)

(g) Find the distance from the point D to the plane *P*.

(2)

(h) Find a unit vector that is perpendicular to the plane *P*.

(2)

(i) The point F is a reflection of D in the plane *P*. Find the coordinates of F.

(4)
(Total 25 marks)

14. Find the angle between the lines $\frac{x-1}{2} = 1 - y = 2z$ and $x = y = 3z$.

(Total 6 marks)

15. The points A(1, 2, 1), B(–3, 1, 4), C(5, –1, 2) and D(5, 3, 7) are the vertices of a tetrahedron.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(2)

(b) Find the Cartesian equation of the plane Π that contains the face ABC.

(4)

(c) Find the vector equation of the line that passes through D and is perpendicular to Π. Hence, or otherwise, calculate the shortest distance to D from Π.

(5)

(d) (i) Calculate the area of the triangle ABC.

(ii) Calculate the volume of the tetrahedron ABCD.

(4)

(e) Determine which of the vertices B or D is closer to its opposite face.

(4)
(Total 19 marks)

16. The points P(–1, 2, –3), Q(–2, 1, 0), R(0, 5, 1) and S form a parallelogram, where S is diagonally opposite Q.

(a) Find the coordinates of S.

(2)

(b) The vector product $\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{pmatrix} -13 \\ 7 \\ m \end{pmatrix}$. Find the value of *m*.

(2)

(c) Hence calculate the area of parallelogram PQRS.

(2)

(d) Find the Cartesian equation of the plane, Π_1, containing the parallelogram PQRS.

(3)

(e) Write down the vector equation of the line through the origin (0, 0, 0) that is perpendicular to the plane Π_1.

(1)

(f) Hence find the point on the plane that is closest to the origin.

(3)

(g) A second plane, Π_2, has equation $x - 2y + z = 3$. Calculate the angle between the two planes.

(4)
(Total 17 marks)

17. The equations of three planes, are given by

$$ax + 2y + z = 3$$
$$-x + (a + 1)y + 3z = 1$$
$$-2x + y + (a + 2)z = k$$

where $a \in \mathbb{R}$.

(a) Given that $a = 0$, show that the three planes intersect at a point.

(3)

(b) Find the value of *a* such that the three planes do **not** meet at a point.

(5)

(c) Given *a* such that the three planes do **not** meet at a point, find the value of *k* such that the planes meet in one line and find an equation of this line in the form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix}.$$

(6)
(Total 14 marks)

18. The system of equations

$$2x - y + 3z = 2$$
$$3x + y + 2z = -2$$
$$-x + 2y + az = b$$

is known to have more than one solution. Find the value of a and the value of b.

(Total 5 marks)

19. A plane π has vector equation $\boldsymbol{r} = (-2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) + \lambda(2\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}) + \mu(6\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k})$.

(a) Show that the Cartesian equation of the plane π is $3x + 2y - 6z = 12$.

(6)

(b) The plane π meets the x, y and z axes at A, B and C respectively. Find the coordinates of A, B and C.

(3)

(c) Find the volume of the pyramid OABC.

(3)

(d) Find the angle between the plane π and the x-axis.

(4)

(e) **Hence**, or otherwise, find the distance from the origin to the plane π.

(2)

(f) Using your answers from (c) and (e), find the area of the triangle ABC.

(2)
(Total 20 marks)

20. A ray of light coming from the point $(-1, 3, 2)$ is travelling in the direction of vector $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ and meets the plane $\pi : x + 3y + 2z - 24 = 0$.

Find the angle that the ray of light makes with the plane.

(Total 6 marks)

21. (a) Show that lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect and find the coordinates of P, the point of intersection.

(8)

(b) Find the Cartesian equation of the plane *Π* that contains the two lines.

(6)

(c) The point Q(3, 4, 3) lies on *Π*. The line *L* passes through the midpoint of [PQ]. Point S is on *L* such that $\left|\overrightarrow{PS}\right|=\left|\overrightarrow{QS}\right|$ = 3, and the triangle PQS is normal to the plane *Π*. Given that there are two possible positions for S, find their coordinates.

(15)
(Total 29 marks)