Mixed examination practice

Exam-style questions

1. The following data represents the results from a survey of students in the same tutor group:

U = {students in tutor group}, $n(U) = 50$

H = {History students}, G = {Geography students}

$n(H) = 16$, $n(G) = 24$, $n(H \cap G) = 2$.

(a) Explain in words what the region $(H \cup G)'$ represents.

(b) State the value of $n(H \cap G)'$.

(c) Draw a Venn diagram to represent the data from above, indicating the number of students in each region of the diagram.

2. A survey was carried out on 40 students about when they did their homework over the weekend. The results are shown below:

F = {homework done on Friday night}

S = {homework done on Saturday/Sunday}

$n(F) = 20$, $n(S) = 29$.

(a) State the value of $n(F \cap S)$.

(b) What does $(F \cup S)'$ mean? Explain why $n(F \cup S)' = 0$.

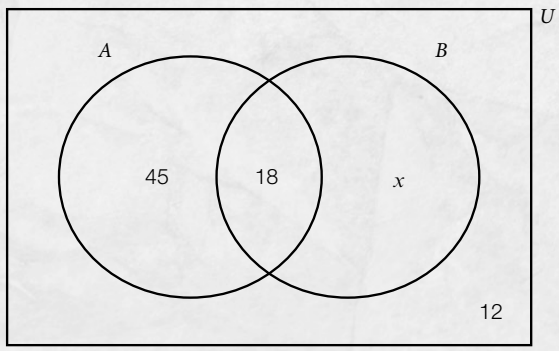
(c) Draw a Venn diagram to represent the data from above, indicating the number of students in each region of the diagram.

3. 100 Mathematics teachers who attended a conference were asked which programme of study they had taught in the last 18 months. Their responses are illustrated on the Venn diagram below:

U = {Mathematics teachers at conference}

A = {teachers who have taught on the Advanced Level programme}

B = {teachers who have taught on the IB Diploma programme}



(a) Describe the region denoted by x using:

(i) words (ii) set notation.

(b) State the value of x.

(c) Find: (i) $n((A \cap B)')$ (ii) $n((A \cup B)')$.

4. A survey which asked 120 people about what types of books and materials they read most recently in their local library provided the following results:

71 read fiction books (F),

54 read non-fiction books (N), including textbooks,

44 read reference books (R), including journals and newspapers,

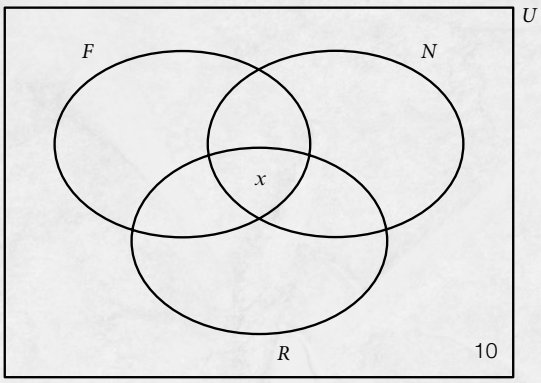
20 read both fiction and reference,

8 read reference and non-fiction but not fiction,

15 read fiction and non-fiction but not reference,

x people read all three types of materials,

10 read none of the types of materials mentioned above.



(a) Show that $n(F \cap R \cap N') = 20 - x$.

(b) Find, in terms of x, $n(N \cap R' \cap F')$.

(c) Complete the Venn diagram, indicating the number of people corresponding to each region.

(d) Hence, or otherwise, find the value of x.

5. 100 students were asked which resources they used during their revision for their final examination in Mathematics. The three main resources were:

C = {CDs, videos, etc.}

P = {printed materials, including textbooks, etc.}

W = {web/internet resources}

The number of students representing the corresponding regions are:

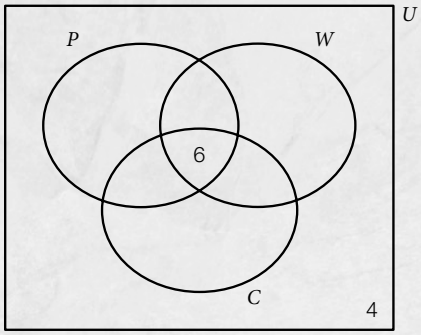
$n(P) = 51$ $\quad$ $n(C \cap W) = 13$

$n(W) = 32$ $\quad$ $n(C \cap P \cap W) = 6$

$n(C) = 63$ $\quad$ $n[(C \cup P)' \cap W] = 6$

$n(P \cap C) = 24$ $\quad$ $n(C \cup P \cup W)' = 4$

Complete the Venn diagram below using the information given above.

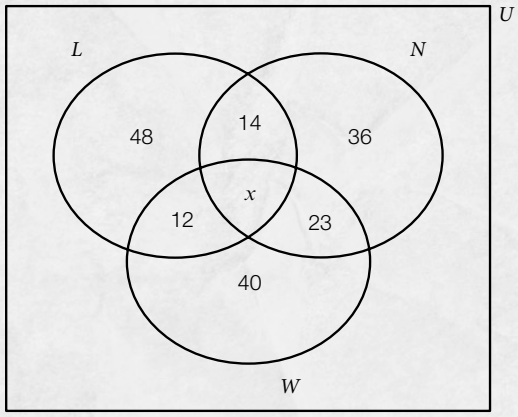


6. The principals of 180 colleges were asked where they advertised for teachers to fill vacant positions. Their responses are illustrated on the Venn diagram below, where:

L = {local newspapers}

N = {national newspapers}

W = {web/internet}



(a) Given that $n(L \cup N \cup W)' = 0$, determine the value of x.

(b) From the information in the Venn diagram, write down the number of principals who advertised:

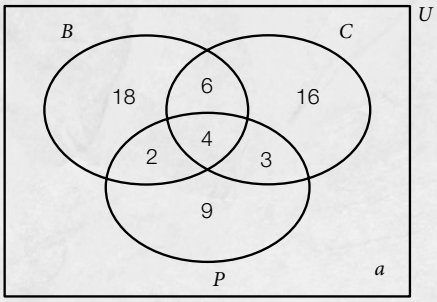
(i) in both local newspapers and on the internet

(ii) in local newspapers and/or on the internet

(iii) in both national newspapers and on the internet but not in local newspapers

(iv) in local and/or national newspapers but not on the internet.

7. The following Venn diagram shows the number of students who study Biology (B), Chemistry (C) and Physics (P) in a college.



(a) Find:

(i) $n(B \cap C \cap P)$ (ii) $n(C \cup P)$.

Given that $n(U) = 100$, find:

(b) the value of a

(c) $n(B')$.

8. The Mathematics Enrichment Club in a school runs sessions on Mondays (M), Wednesdays (W) and Fridays (F). A number of students were asked which of the enrichment sessions they had attended during the previous week.

8 students attended on Monday only,

6 students attended on Monday and Wednesday but not on Friday,

7 students attended on Monday and Friday but not on Wednesday,

3 students attended on Wednesday and Friday but not on Monday,

20 students did not attend any of the sessions.

(a) Illustrate the above information on a Venn diagram.

Given that:

25 students attended on Monday,

24 students attended on Wednesday,

35 students attended on Friday,

(b) find the number of students who attended all three sessions during the week

(c) find the total number of students in the group

(d) hence complete the Venn diagram from part (a).

9. 300 tourists were asked which attractions they had seen while in London.

Most of them had seen Buckingham Palace (B), Trafalgar Square (T) and Westminster Abbey (W).

25 people had seen all three attractions,

52 people had seen both Trafalgar Square and Westminster Abbey,

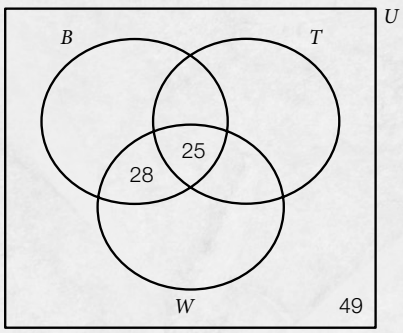
28 people had seen Buckingham Palace and Westminster Abbey but not Trafalgar Square,

88 people had seen exactly two of the three attractions,

211 people had seen Buckingham Palace or Trafalgar Square,

199 people had seen Trafalgar Square or Westminster Abbey,

49 people had seen other attractions, but none of the three places listed above.



Use the information from above to complete the Venn diagram, indicating the number of people representing each of the regions.

10. 120 customers in a music shop were asked about the genre of music they had just bought from the shop. The three main genres were Classical (C), Folk (F) and Pop (P).

4 customers bought all three genres of music,

60 customers bought Pop or Folk music but not Classical,

59 customers bought Pop music,

30 customers bought at least two of the three genres of music,

7 customers bought Pop and Classical but not Folk music,

10 customers bought both Classical and Folk music,

25 customers bought none of these three genres of music.

Illustrate the information from above on a Venn diagram, indicating the number of customers for each region.

Past paper questions

1. At a certain school there are 90 students studying for their IB diploma. They are required to study at **least one** of the subjects: Physics, Biology or Chemistry.

50 students are studying Physics,

60 students are studying Biology,

55 students are studying Chemistry,

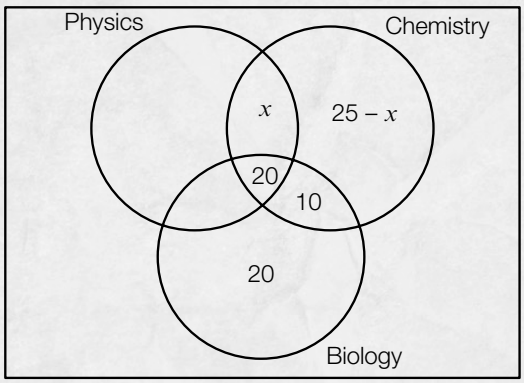
30 students are studying both Physics and Biology,

10 students are studying both Biology and Chemistry but not Physics,

20 students are studying all three subjects.

Let x represent the number of students who study both Physics and Chemistry but not Biology. Then $25 - x$ is the number who study Chemistry only.

The figure below shows some of this information and can be used for working.



(a) Express the number of students who study Physics only, in terms of x.

(b) Find x.

(c) Determine the number of students studying **at least two** of the subjects.

[Total 6 marks]

[May 2006, Paper 1, Question 13] (© *IB Organization 2006*)

2. A school offers three activities, basketball (B), choir (C) and drama (D). Every student must participate in at least one activity.

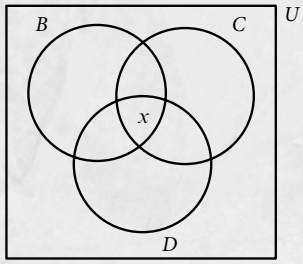
16 students play basketball only,

18 students play basketball and sing in the choir but do not do drama,

34 students play basketball and do drama but do not sing in the choir,

27 students are in the choir and do drama but do not play basketball,

(a) Enter the above information on the Venn diagram below. *[2 marks]*



99 of the students play basketball, 88 sing in the choir and 110 do drama.

(b) Calculate the number of students x participating in all three activities. *[1mark]*

(c) Calculate the total number of students in the school. *[3 marks]*

[Total 6 marks]

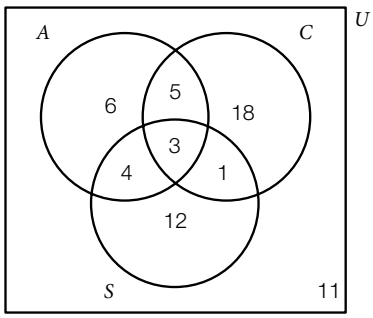
[Nov 2007, Paper 1, Question 8] (*© IB Organization 2007*)

Exercise 8.4

1. (a) $P \cap Q \cap R$
(b) $P \cap R$
(c) $(P \cup R)' \cap Q$
(d) $R \cap (P \cup Q)$

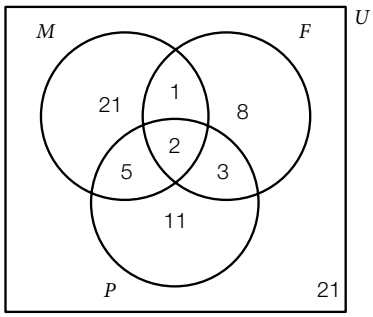
2. (a) {g}
(b) {a, e, f, h, j, k}
(c) {f, h, j}
(d) {c, f, g}

3. (a) and (b)



(c) 11

4. (a)

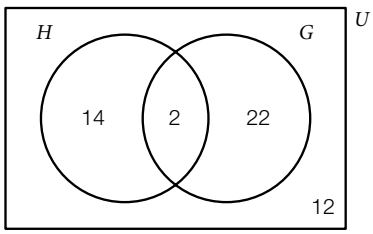


(b) 21

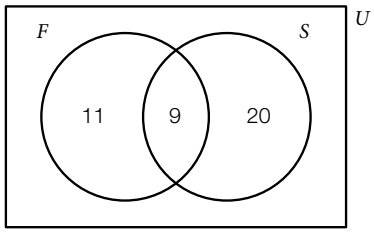
Mixed examination practice 8

Exam-style questions 8

1. (a) Neither History nor Geography
(b) 48
(c)

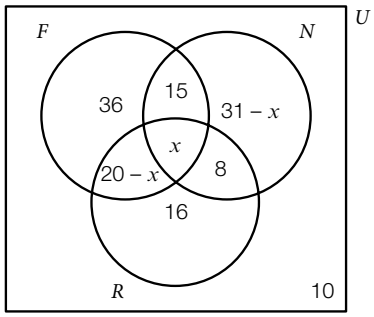


2. (a) 9
(b) Neither F nor S. Since $n(F \cup S) = 40$, $n(F \cup S)' = 0$
(c)



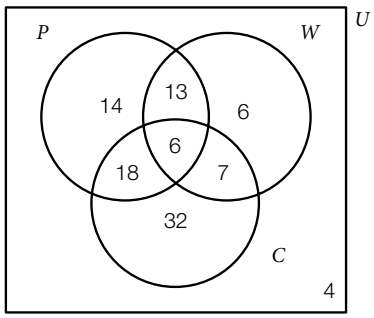
3. (a) (i) IB but not A Level
(ii) $B \cap A'$
(b) $x = 25$
(c) (i) 82 (ii) 12

4. (a) Given $n(F \cap R) = 20$. Given that x read all 3. So, the region, $(F \cap R \cap N') = 20 - x$
(b) $31 - x$
(c)



(d) $x = 16$

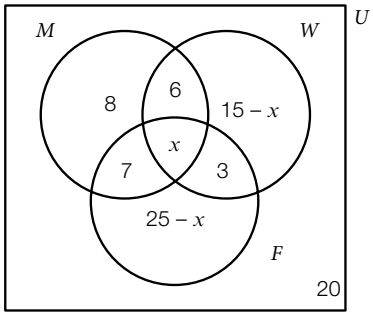
5.



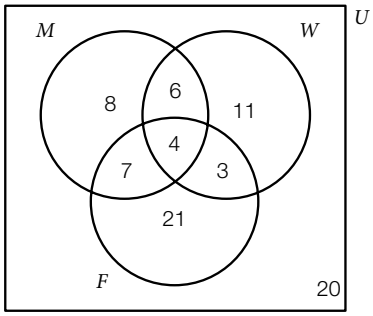
6. (a) $x = 7$
(b) (i) 19 (ii) 144
(iii) 23 (iv) 98

7. (a) (i) 4 (ii) 40
(b) 42 (c) 70

8. (a)



(b) $x = 4$ (c) 80
(d)



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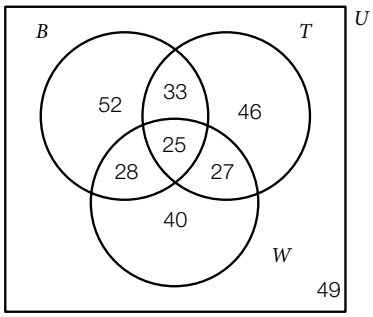

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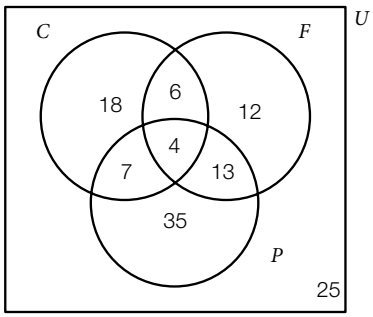
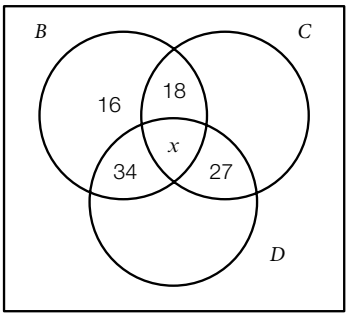


Past paper questions 8

1. (a) $20 - x$ (b) $x = 15$
(c) 55

2. (a)



(b) $x = 31$ (c) 156

Chapter 9

Exercise 9.1

1. (a) Proposition, true
(b) Not a proposition
(c) Proposition, false
(d) Proposition, true
(e) Proposition, true
(f) Not a proposition
(g) Proposition, true
(h) Not a proposition
(i) Proposition, true
(j) Not a proposition

Exercise 9.2

1. (a) $\neg p$ (b) $r \veebar p$
(c) $q \wedge r$ (d) $\neg r \wedge q$

2. (a) x is not a prime number.
(b) x is a prime number less than 100.
(c) Either x is not a prime number or it is a 2-digit number.
(d) Either x is not a prime number or it is not a 2-digit number.
(e) x is not a prime number less than 100.

3. (a) Jenny hates football and she does not watch Sky Sports.
(b) Jenny does not hate football and she watches Sky Sports.
(c) Either Jenny watches Sky Sports or she watches the Comedy Channel, but not both.

4. (a) $q \wedge p$ (b) $\neg q \wedge \neg r$
(c) $\neg p \vee \neg q$ (d) $\neg p \wedge \neg r$

Exercise 9.3

1. (a) Either Veejay attends football training or he passes his test, but not both.
(b) Veejay revises for his test and he does not attend football training.
(c) Veejay is not revising for his test and he attends football training.
(d) If Veejay revises for his test then he does not attend football training.
(e) If Veejay revises for his test then he passes his test.
(f) If Veejay does not revise for his test then he does not pass his test.

2. (a) $p \Rightarrow r$ (b) $q \Rightarrow \neg r$
(c) $\neg q \Rightarrow p$

3. (a) $p \Rightarrow q$ (b) $\neg q \Rightarrow \neg p$
(c) $p \Leftrightarrow q$

4. (a) If x is a quadrilateral and also a 2-D shape with a pair of parallel sides then x is a parallelogram. False.
(b) If x is a parallelogram then x is a quadrilateral. True.
(c) If x is a parallelogram then x is a 2-D shape with a pair of parallel sides. True.
(d) x is a quadrilateral and also a 2-D shape with a pair of parallel sides if and only if x is a parallelogram. False.

Exercise 9.4

1. (a) (i) $p \wedge q$ (ii) $\neg p \wedge \neg q$
(b) Either Donald did not pass his driving test or Debbie passed her driving test.
(c)

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

2. (a) (i) $\neg p \Rightarrow \neg q$
(ii) $q \Leftrightarrow p$
(b)

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Exercise 9.5

1. (a) $m \wedge (e \veebar s)$
(b) If a student does not choose Science then he/she chooses Economics.
(c)

e	s	$\neg e$	$\neg e \Rightarrow s$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

2. (a) If I go to the cinema then it is not the weekend.
(b)

p	q	$\neg p$	$q \Rightarrow \neg p$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

3.

p	q	r	$p \veebar q$	$r \Rightarrow (p \veebar q)$
T	T	T	F	F
T	T	F	F	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	F	T