Q1

The graph of $y = f(x)$ has a vertex at point A(-1, -2) and zero at x = -3

a) Write $f(x)$ in the form $a(x - x_1)(x - x_2)$

b) Show that $f(x) = \frac{1}{2}x^2 + x - 1\frac{1}{2}$

c) Function g is obtained by reflecting the graph $f(x)$ in x -axis followed by a translation of $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$. Show that $g(x) = -0.5x^2 - 2x - 2$

d) Express $g(x)$ in the vertex form

e) Sketch the graph of $f(x)$ and $g(x)$ and clearly label axis intercepts and the vertices

f) Solve the inequality $g(x) \geq 0$

g) Solve the inequality $f(x) > 0$

h) Find the maximum vertical separation between the curves which occurs between the points of the intersection of these two curves.

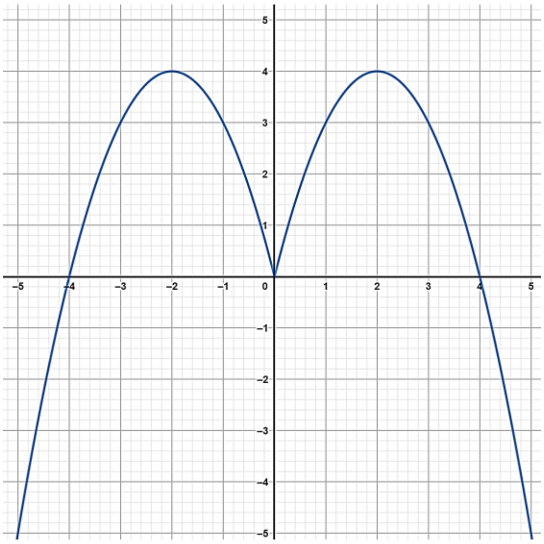
i) A line $y = mx - 2$ is a tangent to the function $f(x)$. Find the possible values of m

j) Graph the tangent lines on the graph.

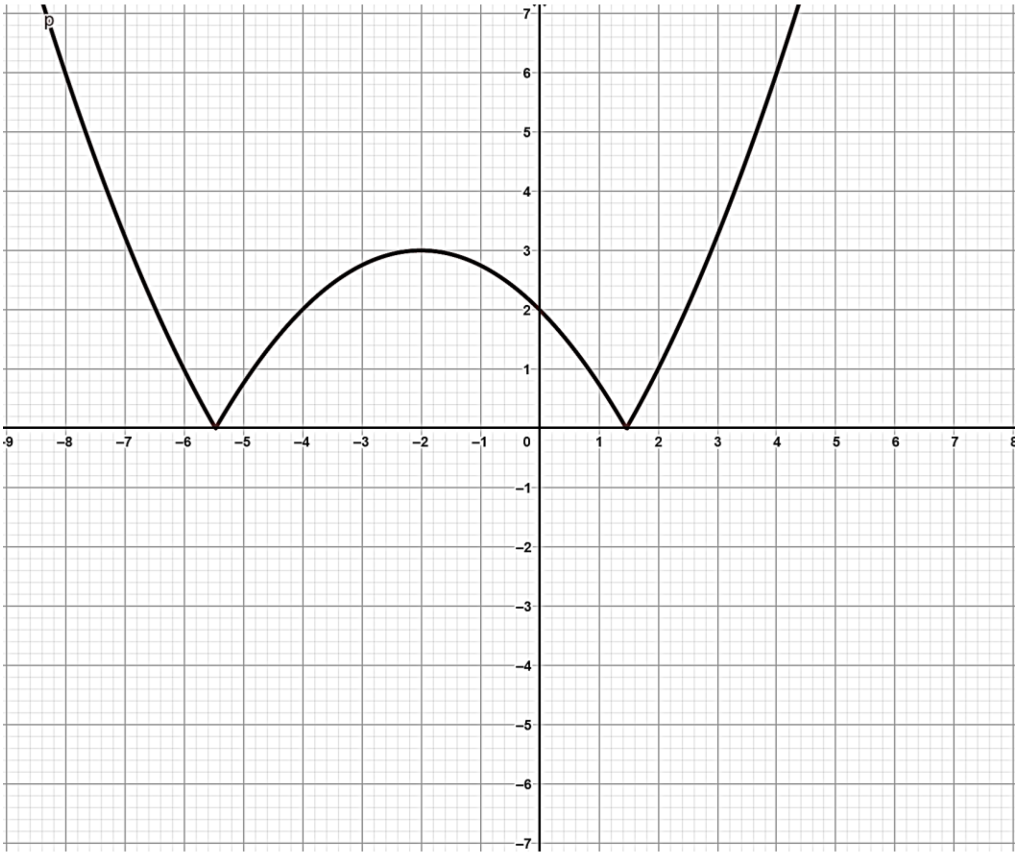
Q2

a) Find the sequence of transformations that transform $f(x) = x^2$ to the function $g(x)$ shown below.

b) Write the equation of $g(x)$



Q3 Find the sequence of transformations that transform $f(x) = x^2$ to the function $g(x)$ shown below.



Q4

A sum of twice one number and three times the other is 24. Find the numbers knowing that their product is greatest possible. Show your method clearly.

Q5

Show algebraically - using quadratics' theory - that among all rectangles of a fixed perimeter p, the one with the largest area is a square.

Q6

The equation $2kx^2 + (k-2)x + (2k-1) = 0$ has two distinct negative roots.

Find the set of possible values of k

Q7

The quadratic equation $kx^2 + (k-8)x + (1-k) = 0$, $k \neq 0$, has one root which is two more than the other. Find k and the two roots.

Q8

The roots of $2x^2 + 5x - 9 = 0$ are α and β.

Find **all** quadratic equations with roots α^2 and β^2

Q9

Consider the functions

$$f(x) = x^2 - 4$$

$$g(x) = \frac{1}{2}x^2 - 2x$$

a) Clearly describe the sequence of geometric transformations that transform $y = x^2$ into $y = g(x)$
b) Find the solutions of the equation $f(x) = g(x)$
c) Find the coordinates of points of intersection the graphs $y = f(x)$ and $y = g(x)$

Q10

Point A lies on the positive parts of the x- axis. Point B lies on the positive part of the y-axis.
The sum of the distances of points A and B to the origin is 2.
Let M be the midpoint of line segment AB.
Find the smallest distance of M from the origin.

Q11

Consider line given by $y = -\frac{1}{2}x + 5$.

Point A on the line has x coordinate equal to s. Find the value of s for which the distance from the origin is the smallest. Find the smallest distance.

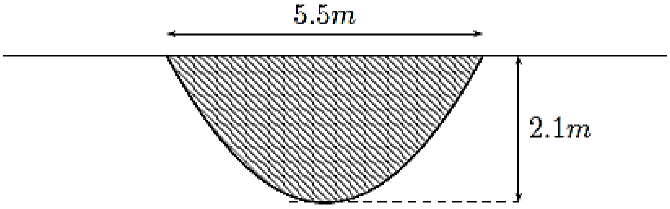
Q12

The equation $(m + 1)x^2 + 2(m + 2)x + 2(m - 1) = 0$ has two distinct positive roots.

Find the set of possible values of k

Q13

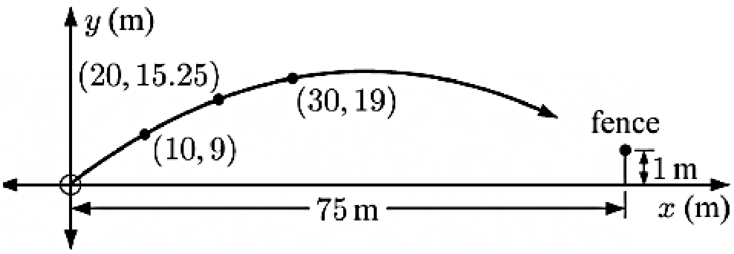
The diagram below shows the boundary of the cross-section of a water channel. Its shape is a parabola.



The width of the channer at the widest place is 5.5 metres while the largest depth is 2.1 metres. What is the width of the channel on the depth of 1.5 metres?

Q14

A player hits the ball in the air with the path shown below.



a) Find the model connecting y and x
b) Find the maximum height reached by the ball
c) The 1 metre boundary fence is 75 metres away. Will the ball clear the boundary fence?

Q15

A tunnel's cross-section is in the shape of a parabola. Its height measured at the highest point is 5 metres and its width measured at the bottom is 4 metres (see the diagram). A truck is 3.4 metres high and 2.5 metres wide.

(i) Can the truck pass through the tunnel? Justify your answer.

(ii) What is the greatest possible height of a truck 2.5 metres wide that can pass through the tunnel? Give your answer correct to 1cm.

Q126

Let $f(x) = 2(x-h_1)^2 + k_1$ and $g(x) = (x-h_2)^2 + k_2$, where $h_1, h_2, k_1, k_2 \in \mathbb{R}$. The vertex of the graph of f is at $(m, -m^2)$ and the vertex of the graph of g is at $(-m, -m)$, where $0 < m < 1$. The graphs of f and g intersect at exactly one point. Find the value of m.

Q17

The graph of $f(x) = ax^2 + 2x + c$ has axis of symmetry $x = 2$. Point A (-4, -10) lies on $f(x)$.

k) Find a and c

l) Write $f(x)$ in the form $a(x - x_1)(x - x_2)$

m) Write $f(x)$ in the form in the vertex form.

n) Function g is obtained by transforming the graph $f(x)$ by the following transformations

1) Horizontal dilation by factor 0.5
2) Reflection in the x- axis
3) Translation of $\binom{1}{6}$.

Find the coordinates of the vertex of $g(x)$.

o) Show that the equation of $g(x) = 2x^2 - 8x + 6$

p) Sketch the graph of $f(x)$ and $g(x)$ in the same plane (math grid)

q) Solve the inequality $f(x) > g(x)$

r) Find the maximum vertical separation between the curves which occurs between the points of the intersection of these two curves.

s) A line $y = mx + 8$ is a tangent to the function $f(x)$. Find the possible values of m

t) Graph the tangent line(s) on the graph as in h)

u) Sketch on the same graph $|g(x)|$

v) The graph of $g(x)$ may be obtained from the graph of $h(x) = 6x^2 - 2$ by the following sequence of transformations:

1) stretch in the $y-$direction, followed by
2) a translation by vector $\binom{p}{q}$

Find the values of a, p, q.

l) Find the solutions of the equation $f(x) = h(x)$ and give your answers in the simplest exact form

18

The equation $kx^2 + 4kx - 1 = k$ has two real, distinct roots.

(a) Find the set of possible values for k.

(b) Consider the case when $k = 3$. The roots of the equation can be expressed in the form $p \pm \frac{q}{\sqrt{3}}$, where $p, q \in \mathbb{Z}$. Find the value of p and of q.

19

Let $f(x) = mx^2 - 6mx + 16$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = 2mx - 16$ meets the graph of f at exactly one point.

(a) Show that $m = 2$.

The function f can be expressed in the form $f(x) = 2(x - p)(x - q)$, where $p, q \in \mathbb{R}$ and $p < q$.

(b) Find the value of p and the value of q.

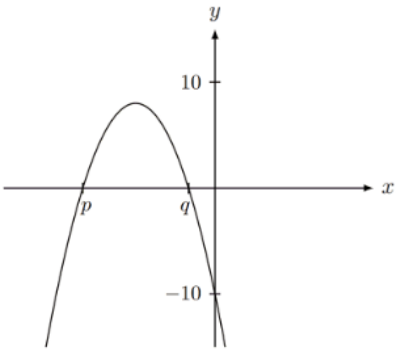
The function f can also be expressed in the form $f(x) = 2(x - h)^2 + k$, where $h, k \in \mathbb{R}$.

(c) Find the value of h and the value of k.

(d) Hence find the values of x where the graph of f is both negative and increasing.

20

Let $f(x) = a(x+1)(x+5)$, for $x \in \mathbb{R}$, where $a \in \mathbb{Z}$. The following diagram shows part of the graph of f.



The graph of f has x-intercepts at $(p, 0)$ and $(q, 0)$, and a y-intercept at $(0, -10)$.

(a) (i) Write down the value of p and the value of q.

(ii) Find the value of a.

(b) Find the equation of the axis of symmetry.

(c) Find the coordinates of the vertex.

The graph of a function g is obtained from the graph of f by a reflection in the y-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. The point $P(-2, 6)$ on the graph of f is mapped to point Q on the graph of g.

(d) Find the coordinates of Q.

21

Let $f(x) = 2(x-1)^2 - 8$, for $x \in \mathbb{R}$.

(a) Show that $f(x) = 2x^2 - 4x - 6$.

(b) For the graph of f:

(i) write down the coordinates of the vertex;

(ii) write down the y-intercept;

(iii) find both x-intercepts.

(c) Hence sketch the graph of f.

Let $g(x) = 6x^2$, for $x \in \mathbb{R}$.

The graph of f may be obtained from the graph of g using the following two transformations:

a compression of scale factor a in the y-direction, followed by

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.

(d) Find the values of a, h and k.

22

Let $f(x) = 2x^2 - 8x + 6$, for $x \in \mathbb{R}$.

(a) Write down the value of $f(0)$. [1]

(b) Solve the equation $f(x) = 0$. [3]

The function f can be written in the form $f(x) = a(x-h)^2 + k$.

(c) Find the values of a, h and k. [5]

(d) For the graph of f, write down:

(i) the coordinates of the vertex;

(ii) the equation of the axis of symmetry. [2]

The graph of a function g is obtained from the graph of f by a reflection in the x-axis, followed by a translation by the vector $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

(e) Find $g(x)$, giving your answer in the form $g(x) = px^2 + qx + r$. [4]

23

Determine a full geometric description of the following transformations:

a from $f(x) = x^2$ to $g(x) = \left(\frac{x}{2}\right)^2 - 3$

b from $f(x) = 2(x - 3)^2 + 4$ to $g(x) = x^2$

c from $f(x) = x^2$ to $g(x) = 4(x - 1)^2 + 2$

d from $f(x) = 2(x - 3)^2 + 4$ to $g(x) = 2 - (x - 2)^2$.

25

Given the function $f(x) = 11 - (x - 2)^2$ in the domain $2 \le x \le 5$.

a **i** Sketch the graph of $f(x)$ in the given domain.

ii Determine the range of $f(x)$ in the given domain.

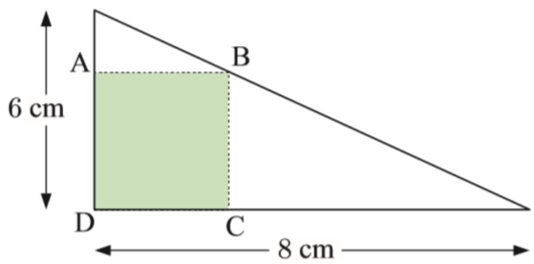
iii The graph of $f(x)$ can be obtained from the graph of $y = x^2$ using a sequence of transformations. Write down a full geometric description of the transformations.

b The function $f(x)$ undergoes the following sequence of transformations in order to become the function $g(x)$: first a translation 3 units to the right and 1 unit down followed by a stretch by scale factor $\frac{1}{2}$ parallel to the y-axis.

Determine an expression for the function $g(x)$.

26

Infinitely many rectangles may be inscribed within the right angled triangle shown alongside. One of them is illustrated.



a Let AB $= x$ cm and BC $= y$ cm.
Use similar triangles to find y in terms of x.

b Find the dimensions of rectangle ABCD of maximum area.

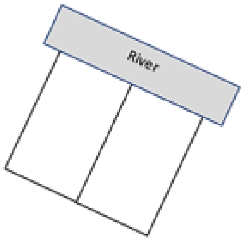
27

A plot of land is in the shape of two adjacent and identical rectangles. Both rectangles are adjacent from one side to the river. The plot is to be surrounded by the fencing.

Additional line segment of fencing is to be built to split the plot into two rectangles.

The fancing is not to be build along the river.

The situation is illustrated on this diagram.

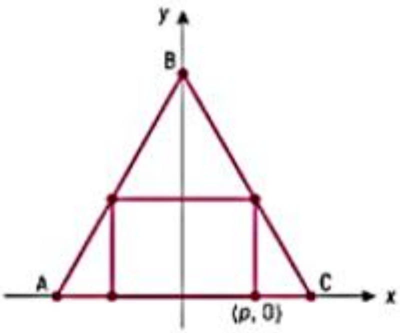


The total length of fencing is 270 metres.

Find the dimensions of the plot of land given that its area is largest possible.

28

A rectangle is inscribed in isosceles triangle ABC as shown in the diagram.



The altitude of triangle ABC from B to side AC is 7 cm and AC = 8 cm. The coordinates of one of the vertices of the inscribed rectangle are $(p, 0)$.

- a Write down the coordinates of points A, B and C.
- b Find the equation of the line passing through points B and C.
- c Find the dimensions of the rectangle inscribed in the triangle, in terms of p.
- d Write down an expression for the area of the inscribed rectangle in terms of p.
- e Find the dimensions of the rectangle with maximum possible area.
- f Find the maximum possible area of the inscribed rectangle.

Sikora

Q1. (i) Describe clearly the sequence of geometric transformations that transform the graph of $y = f(x)$ into the graph of $y = g(x)$. [1]

(ii) Sketch the graph of $y = g(x)$ showing clearly the position of at least three points.

(1) $f(x) = x^2$ $\quad g(x) = \frac{1}{3}(x+3)^2 - 1$

(2) $f(x) = \sqrt{x}$ $\quad g(x) = \sqrt{-\frac{1}{2}x + 1}$

(3) $f(x) = \sqrt{x}$ $\quad g(x) = -\frac{1}{2}\sqrt{x-3} - 1$

(4) $f(x) = |x|$ $\quad g(x) = -3|x+1| + 2$

(5) $f(x) = \frac{1}{x}$ $\quad g(x) = \frac{-3}{x+2} - 1$

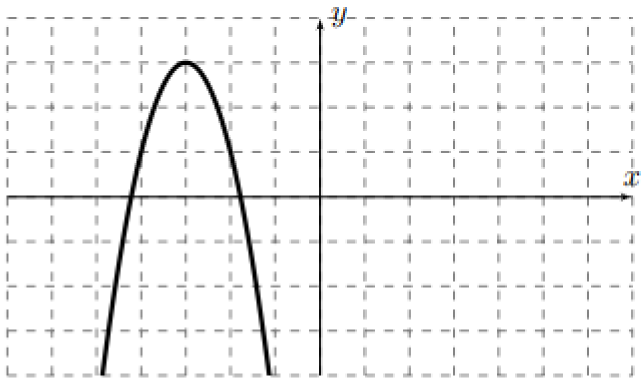
(6) $f(x) = \frac{1}{x}$ $\quad g(x) = \frac{1}{2x} + 2$

(7) $f(x) = -\frac{2}{3}x + 2$ $\quad g(x) = -\frac{2}{3}|x| + 2$

(8) $f(x) = x^2$ $\quad g(x) = |(x+3)^2 - 3|$

Q2. Write down an equation for each of the graphs shown.

(1)



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Q17. (i) Describe clearly the sequence of geometric transformations that transform the graph of $y = f(x)$ into the graph of $y = g(x)$.

(ii) Graph both functions in the same set of axes.

(1) $f(x) = x^2, \quad g(x) = 2(x-3)^2,$

(2) $f(x) = \frac{1}{x}, \quad g(x) = \frac{3}{x+1},$

(3) $f(x) = \sqrt{x}, \quad g(x) = -\sqrt{x+2},$

(4) $f(x) = x^3, \quad g(x) = (x-2)^3 - 2,$

(5) $f(x) = 3x + 2, \quad g(x) = 3|x| + 2,$

(6) $f(x) = |x|, \quad g(x) = -2|x-3|,$

(7) $f(x) = x^2, \quad g(x) = (2x)^2 + 1,$

(8) $f(x) = \frac{1}{x}, \quad g(x) = \frac{1}{3x} + 2,$

(9) $f(x) = \sqrt{x}, \quad g(x) = \sqrt{-x} - 1,$

(10) $f(x) = x^3, \quad g(x) = -x^3 - 3,$

(11) $f(x) = 2 - x, \quad g(x) = |2 - |x||,$

(12) $f(x) = |x|, \quad g(x) = 2|x+1| - 3,$

(13) $f(x) = x^2, \quad g(x) = -2(x+3)^2 + 2,$

(14) $f(x) = \frac{1}{x}, \quad g(x) = -\frac{2}{x+1} + 1,$

(15) $f(x) = \sqrt{x}, \quad g(x) = -2\sqrt{x-3} + 1,$

(16) $f(x) = x^3, \quad g(x) = -(x-2)^3 - 2,$

(17) $f(x) = x + 2, \quad g(x) = 3|x+2|,$

(18) $f(x) = |x|, \quad g(x) = |2x| - 1,$

(19) $f(x) = x^2, \quad g(x) = 3 - (x-3)^2,$

(20) $f(x) = \frac{1}{x}, \quad g(x) = \frac{2}{x-1} + 2,$

(21) $f(x) = \sqrt{x}, \quad g(x) = 2 - \sqrt{x-2},$

(22) $f(x) = x^3, \quad g(x) = \frac{1}{2}(x+1)^3 - 2,$

(23) $f(x) = 2x - 1, \quad g(x) = -|2x - 1|,$

(24) $f(x) = |x|, \quad g(x) = 3 - |\frac{x}{2}|,$

(25) $f(x) = x^2, \quad g(x) = \frac{1}{2}(x-3)^2 - 1,$

(26) $f(x) = x^2, \quad g(x) = 2 - \frac{1}{2}(x+3)^2,$

(27) $f(x) = x^2, \quad g(x) = 3(x+1)^2 - 3,$

(28) $f(x) = x^2, \quad g(x) = 4 - 2(x+2)^2,$

(29) $f(x) = \frac{1}{2}x + 1, \quad g(x) = \frac{1}{2}|x| + 1,$

(30) $f(x) = \frac{1}{2}x + 1, \quad g(x) = |\frac{1}{2}x + 1|,$

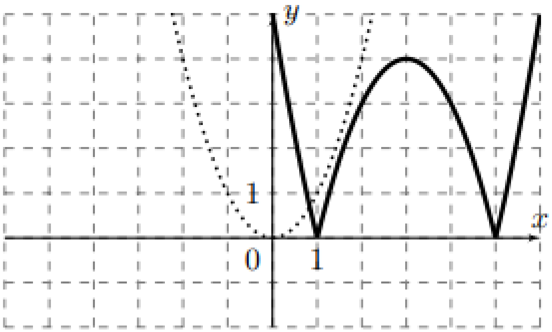
(31) $f(x) = x^2, \quad g(x) = 2(|x| - 2)^2 - 1,$

(32) $f(x) = x^2, \quad g(x) = |2(x+2)^2 - 4|.$

Q16. (i) Recognize the equations of the parent function (dotted line / curve).

(ii) State the single transformation or the sequence of transformations that have to be applied so that the parent function is transformed to the other one (solid line / curve).

(iii) Give the equation of the new function.



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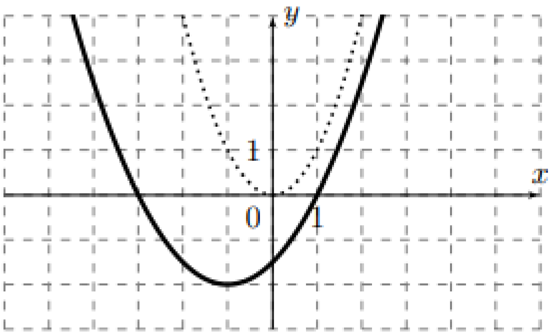

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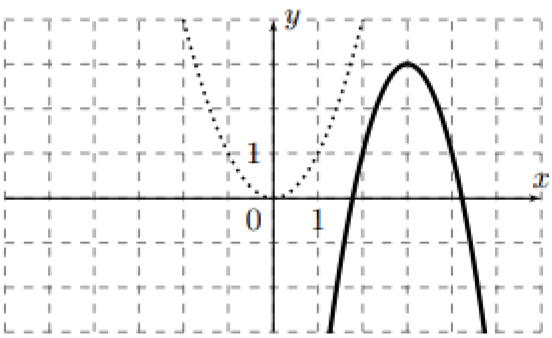

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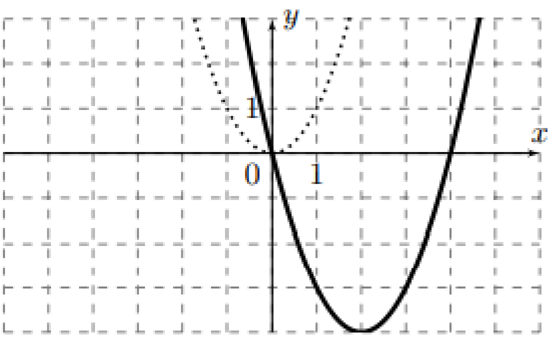


(2)

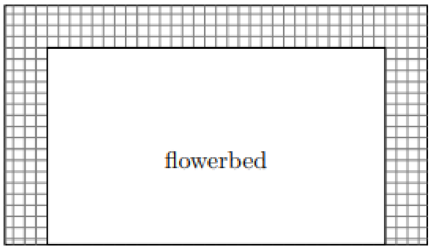
Q18. Find the equation of a parabola that has x-intercepts $(-4, 0)$ and $(2, 0)$ and passes through point $(-3, 5)$. Give answer in the form $y = ax^2 + bx + c$.

Q23. Find the equation of a parabola that has a vertex at $(-5,-1)$ and passes through point $(-3,3)$. Give answer in the form $y = ax^2 + bx + c$.

Q28. Find the equation of a parabola that passes through points $(-1,2)$ and $(2,-1)$ and is symmetrical in line $x = 1$. Give answer in the form $y = ax^2 + bx + c$.

Q30. Find the equation of a parabola that passes through points $(-4,3)$ and $(1,0.5)$ and is symmetrical in line $x = -2$. Give answer in the form $y = ax^2 + bx + c$.

Q53. A pavement 1.5 metres wide is built along three sides of a rectangular flowerbed. The total area of the pavement is $43.5m^2$. Find the dimensions of the flowerbed knowing that its surface area is largest possible.



Q54. An ant-hill is in the shape of a *paraboloid*, a solid whose base is a circle and each vertical cross-section through the centre of the base has a shape of a part of a parabola (with vertex up). The radius of the base is 80 centimetres. The height of the ant-hill is 1.6 metre. What would be the area the cross-section obtained if the ant-hill was cut with a horizontal plane on the level of 1 metre above the ground? Give your answer correct to $100cm^2$.

Q55. A jar with jam is in the shape of a cylinder with the height of 7.5 centimetres and the diameter of the base 8.5 centimetres. Paul placed the jar on a table. He wanted to cover the jar with a bowl turned upside down. The bowl is in the shape of a *paraboloid*, i.e. all of its vertical cross-sections passing through the centre of the base are parabolas. The circular base of the bowl has the diameter of 20 centimetres. Its height is 8 centimetres. It turned out that the bowl is too small to cover the jar. What is the distance between the brim of the bowl and the table? Give your answer correct to 1 milimetre.

Q56. A fountain jet is in the shape of a parabola. The water is spouted from the ground level and it reaches the height of 80 centimetres after travelling 30 centimetres horizontally. The maximum height that it reaches is 1 meter. Find the horizontal distance that the water covers till it reaches the maximum height.

Q57. A fountain jet is in the shape of a parabola. The water is spouted from the point placed 20 cm above the ground level and it reaches its maximum height of 1 metre. Then it goes back to the ground level after travelling 180 centimetres horizontally. Find the horizontal distance that the water covers till it reaches the maximum height.

Q58. A ditch has a cross-section in the shape of a parabola. Its maximum depth is 1.5 meters and so is its maximum width. A man tries to hide a chest in a ditch. The chest is a cuboid with the dimensions 2 metres by 1 metre by 85 centimetres. Is it possible to hide the chest in the ditch? How should it be placed?

Haese Chapter Review

Chapter 4

11 The roots of $2x^2 - 3x = 4$ are α and β. Find the simplest quadratic equation which has roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

12 One of the roots of $kx^2 + (1-3k)x + (k-6) = 0$ is the negative reciprocal of the other root. Find k and the two roots.

13 Solve for x:

a $2x^3 - 3x^2 - 9x + 10 = 0$ **b** $3x^3 = x(7x-2)$

c $x^3 + 60 = 23x + 2x^2$ **d** $x^2(x^2-3) = 64 - 6x^3 - 14x$

14 Use technology to solve:

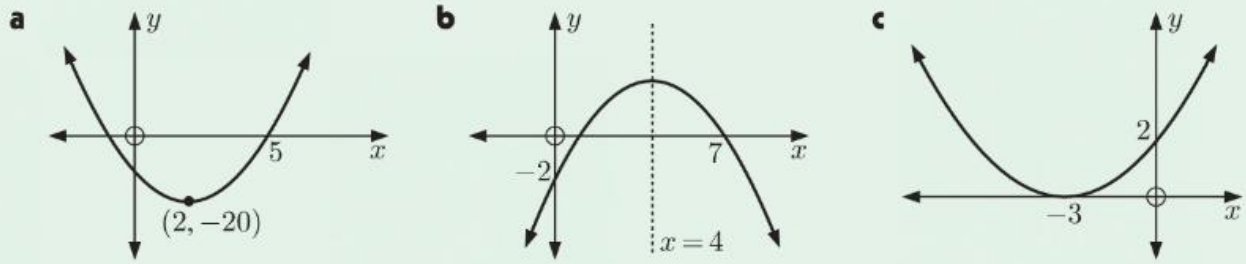
a $10 \times 2^{x-1} = 35$ **b** $\sqrt{x} = \dfrac{4}{x} - 1$ **c** $x^3 - \sqrt[3]{x} + 5 = 0$

10 Use the quadratic formula to explain why the sum of the solutions to the equation $ax^2 + bx + c = 0$, $a \neq 0$, is always $-\frac{b}{a}$.

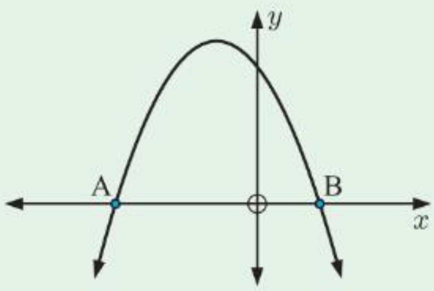
11 One of the roots of $2x^2 + kx + 12 = 0$ is three times the other. Find the possible values of k, and the two roots in each case.

Chapter 14

8 Find the equation of the quadratic with graph:



11 The graph shows the parabola $y = a(x+m)(x+n)$ where $m > n$.



a State the sign of:

i the discriminant Δ **ii** a.

b Find, in terms of m and n, the:

i coordinates of the x-intercepts A and B

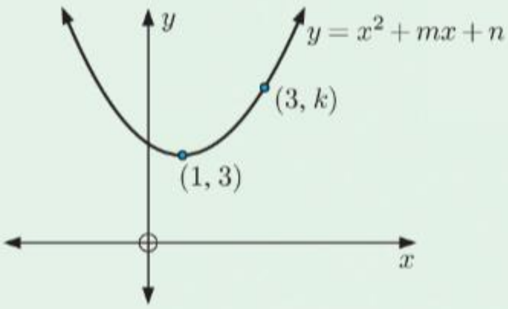
ii equation of the axis of symmetry.

19 Solve for x:

a $(3-x)(x+2) < 0$ **b** $x^2 - 4x - 5 \leqslant 0$ **c** $2x^2 + x > 10$

20 Find the values of k for which the function $f(x) = x^2 + kx + (3k-4)$:

a cuts the x-axis twice **b** touches the x-axis **c** misses the x-axis.

11 

Consider the graph of $y = x^2 + mx + n$.

a Determine the values of m and n.

b Hence find the value of k.

12 An open square-based box has capacity 120 mL. It is made from a square piece of tinplate with 4 cm squares cut from each of its corners. Find the dimensions of the original piece of tinplate.

13 Consider $y = -x^2 - 3x + 4$ and $y = x^2 + 5x + 4$.

a Solve for x: $-x^2 - 3x + 4 = x^2 + 5x + 4$.

b Sketch the curves on the same set of axes.

c Hence solve for x: $x^2 + 5x + 4 > -x^2 - 3x + 4$.

14 For each of the following quadratics:

i Write the quadratic in completed square form.

ii Write the quadratic in factored form.

iii Sketch the graph of the quadratic, identifying its axes intercepts, vertex, and axis of symmetry.

a $y = x^2 + 4x + 3$ **b** $y = x^2 + 2x - 3$

c $y = 2x^2 - 8x - 10$ **d** $y = -x^2 + 6x + 7$

17 A retailer sells sunglasses for \$45, and has 50 customers per day. From market research, the retailer discovers that for every \$1.50 increase in the price of the sunglasses, he will lose a customer per day.

Let $\$x$ be the price increase of the sunglasses.

a Show that the revenue collected by the retailer each day is

$R = (45 + x)\left(50 - \frac{x}{1.5}\right)$ dollars.

b Find the price the retailer should set for his sunglasses in order to maximise his daily revenue. How much revenue is made per day at this price?

20 Find the values of m for which the function $y = mx^2 + 5x + (m + 12)$:

a cuts the x-axis twice **b** touches the x-axis **c** misses the x-axis.

Chapter 16

6 The graph of the function $f(x) = (x + 1)^2 + 4$ is translated 2 units to the right and 4 units up.

a Find the function $g(x)$ corresponding to the translated graph.

b State the range of: **i** $f(x)$ **ii** $g(x)$

8 The graph of $f(x) = 3x^2 - x + 4$ is translated by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$. Write the equation of the image in the form $g(x) = ax^2 + bx + c$.

13 Graph on the same set of axes $y = x^2$, $y = \frac{1}{4}x^2$, and $y = \frac{1}{4}(x-2)^2 - 1$.
Describe a combination of transformations which transform $y = x^2$ to $y = \frac{1}{4}(x-2)^2 - 1$.

11 The quadratic function $f(x) = x^2 + bx + c$ is reflected in the y-axis, stretched horizontally with scale factor $\frac{3}{2}$, then translated through $\begin{pmatrix} -10 \\ 20 \end{pmatrix}$. The resulting quadratic function has the same x-intercepts as $f(x)$. Find b and c.