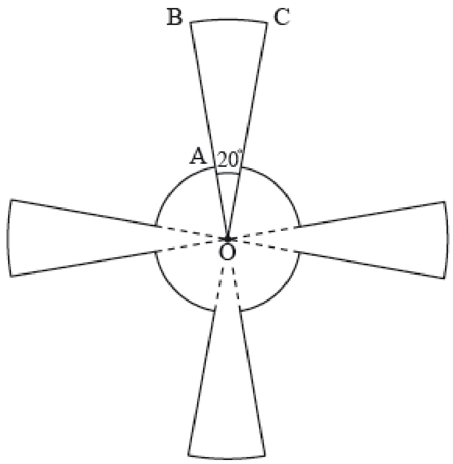
Arcs and sectors *[37 marks]*

1. This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\mathrm{OB} = 9\ \mathrm{cm}$ and four equal sectors of a smaller circle of radius $\mathrm{OA} = 3\ \mathrm{cm}$. *[4 marks]*
   The angle $\mathrm{BOC} = 20°$.

   Find the area of the pendant.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

area = (four sector areas radius 9) + (four sector areas radius 3) ***(M1)***

$= 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) + 4\left(\frac{1}{2}3^2\frac{7\pi}{18}\right)$ ***(A1)(A1)***

$= 18\pi + 7\pi$

$= 25\pi\left(= 78.5\text{ cm}^2\right)$ ***A1***

METHOD 2

area =

(area of circle radius 3) + (four sector areas radius 9) – (four sector areas radius 3) ***(M1)***

$\pi 3^2 + 4\left(\frac{1}{2}9^2\frac{\pi}{9}\right) - 4\left(\frac{1}{2}3^2\frac{\pi}{9}\right)$ ***(A1)(A1)***

Note: Award ***A1*** for the second term and ***A1*** for the third term.

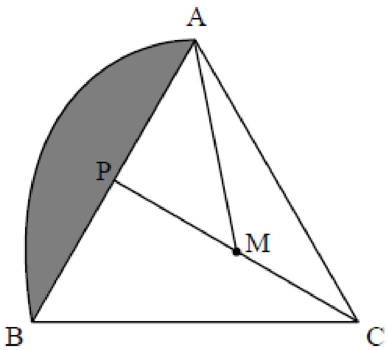
$= 9\pi + 18\pi - 2\pi$

$= 25\pi\left(= \ 78.5\text{ cm}^2\right)$ ***A1***

Note: Accept working in degrees.

[4 marks]

Consider the following diagram.



The sides of the equilateral triangle ABC have lengths 1 m. The midpoint of [AB] is denoted by P. The circular arc AB has centre, M, the midpoint of [CP].

2a. Find AM. *[3 marks]*

Markscheme

METHOD 1

PC $= \frac{\sqrt{3}}{2}$ or 0.8660 ***(M1)***

PM $= \frac{1}{2}$PC $= \frac{\sqrt{3}}{4}$ or 0.4330 **(A1)**

AM $= \sqrt{\frac{1}{4} + \frac{3}{16}}$

$= \frac{\sqrt{7}}{4}$ or 0.661 (m) ***A1***

METHOD 2

using the cosine rule

AM$^2 = 1^2 + \left(\frac{\sqrt{3}}{4}\right)^2 - 2 \times \frac{\sqrt{3}}{4} \times \cos(30°)$ ***M1A1***

AM $= \frac{\sqrt{7}}{4}$ or 0.661 (m) ***A1***

[3 marks]

2b. Find $\mathrm{A}\hat{\mathrm{M}}\mathrm{P}$ in radians. *[2 marks]*

Markscheme

$\tan(A\hat{M}P) = \frac{2}{\sqrt{3}}$ or equivalent ***(M1)***

= 0.857 ***A1***

[2 marks]

2c. Find the area of the shaded region. *[3 marks]*

Markscheme

EITHER

$\frac{1}{2}AM^2\left(2A\hat{M}P - \sin\left(2A\hat{M}P\right)\right)$ ***(M1)A1***

OR

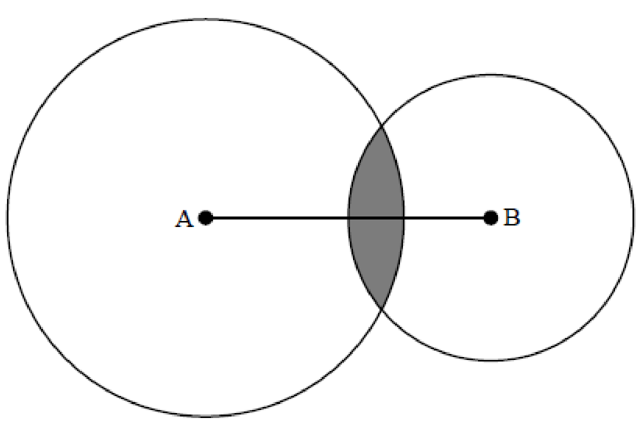
$\frac{1}{2}AM^2 \times 2A\hat{M}P - = \frac{\sqrt{3}}{8}$ ***(M1)A1***

= 0.158(m^2) ***A1***

Note: Award ***M1*** for attempting to calculate area of a sector minus area of a triangle.

[3 marks]

3. Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region. *[6 marks]*



Markscheme

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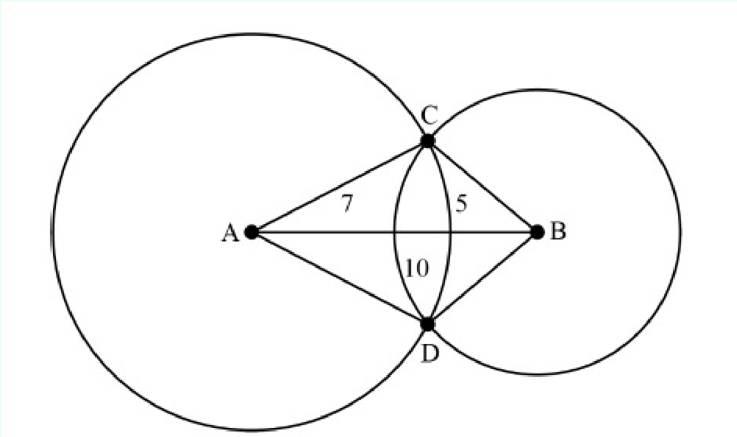


use of cosine rule **(M1)**

CÂB = arccos $\left(\frac{49+100-25}{2\times7\times10}\right) = 0.48276\ldots (= 27.660\ldots^{\circ})$ **(A1)**

$\mathrm{C}\overset{\wedge}{\mathrm{B}}\mathrm{A}$ = arccos $\left(\frac{25+100-49}{2\times5\times10}\right) = 0.70748\ldots (= 40.535\ldots^{\circ})$ **(A1)**

attempt to subtract triangle area from sector area **(M1)**

area $= \frac{1}{2}\times 49\left(2\mathrm{C}\overset{\wedge}{\mathrm{A}}\mathrm{B} - \sin 2\mathrm{C}\overset{\wedge}{\mathrm{A}}\mathrm{B}\right) + \frac{1}{2}\times 25\left(2\mathrm{C}\overset{\wedge}{\mathrm{B}}\mathrm{A} - \sin 2\mathrm{C}\overset{\wedge}{\mathrm{B}}\mathrm{A}\right)$

= 3.5079... + 5.3385... **(A1)**

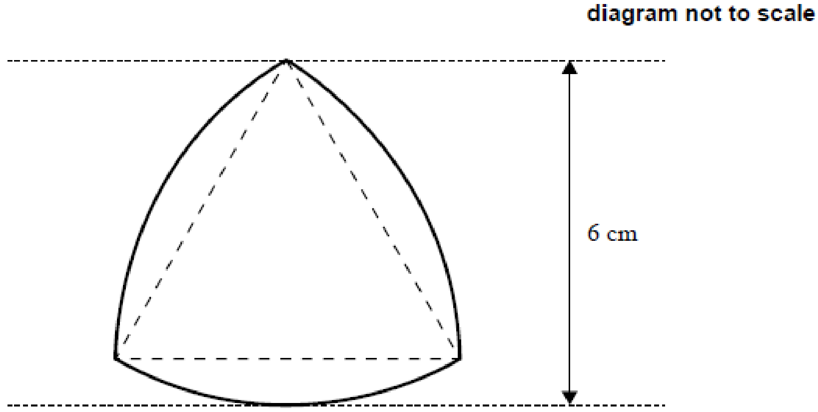
Note: Award this ***A1*** for either of these two values.

= 8.85 (km^2) ***A1***

Note: Accept all answers that round to 8.8 or 8.9.

[6 marks]

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.



For this shape, calculate

4a. the perimeter. *[2 marks]*

Markscheme

each arc has length $r\theta = 6 \times \frac{\pi}{3} = 2\pi \ (= 6.283\ldots)$ ***(M1)***

perimeter is therefore $6\pi \ (= 18.8)$ (cm) ***A1***

[2 marks]

4b. the area. *[5 marks]*

Markscheme

area of sector, s, is $\frac{1}{2}r^2\theta = 18 \times \frac{\pi}{3} = 6\pi \ (= 18.84\ldots)$ ***(A1)***

area of triangle, t, is $\frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3} \ (= 15.58\ldots)$ ***(M1)(A1)***

Note: area of segment, k, is 3.261… implies area of triangle

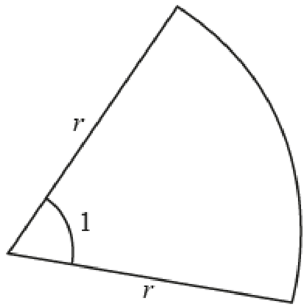
finding $3s - 2t$ or $3k + t$ or similar

area $= 3s - 2t = 18\pi - 18\sqrt{3} \ (= 25.4)$ (cm^2) ***(M1)A1***

[5 marks]

5. A sector of a circle with radius r cm , where $r > 0$, is shown on the following diagram. *[4 marks]*

The sector has an angle of 1 radian at the centre.



Let the area of the sector be A cm^2 and the perimeter be P cm. Given that $A = P$, find the value of r.

Markscheme

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$A = P$

use of the correct formula for area and arc length ***(M1)***

perimeter is $r\theta + 2r$ ***(A1)***

Note: ***A1*** independent of previous ***M1***.

$\frac{1}{2}r^2(1) = r(1) + 2r$ ***A1***

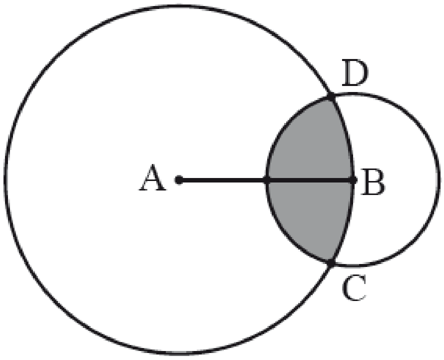
$r^2 - 6r = 0$

$r = 6$ (as $r > 0$) ***A1***

Note: Do not award final ***A1*** if $r = 0$ is included.

[4 marks]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

6a. Find an expression for the shaded area in terms of α, θ and r. *[3 marks]*

Markscheme

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$A = 2(\alpha - \sin\alpha)r^2 + \frac{1}{2}(\theta - \sin\theta)r^2$ ***M1A1A1***

Note: Award ***M1A1A1*** for alternative correct expressions *eg*.
$A = 4\left(\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)r^2 + \frac{1}{2}\theta r^2$.

[3 marks]

6b. Show that $\alpha = 4\arcsin\frac{1}{4}$. *[2 marks]*

Markscheme

METHOD 1

consider for example triangle ADM where M is the midpoint of BD ***M1***

$\sin\frac{\alpha}{4} = \frac{1}{4}$ ***A1***

$\frac{\alpha}{4} = \arcsin\frac{1}{4}$

$\alpha = 4\arcsin\frac{1}{4}$ ***AG***

METHOD 2

attempting to use the cosine rule (to obtain $1 - \cos\frac{\alpha}{2} = \frac{1}{8}$) ***M1***

$\sin\frac{\alpha}{4} = \frac{1}{4}$ (obtained from $\sin\frac{\alpha}{4} = \sqrt{\frac{1-\cos\frac{\alpha}{2}}{2}}$) ***A1***

$\frac{\alpha}{4} = \arcsin\frac{1}{4}$

$\alpha = 4\arcsin\frac{1}{4}$ ***AG***

METHOD 3

$\sin\left(\frac{\pi}{2} - \frac{\alpha}{4}\right) = 2\sin\frac{\alpha}{2}$ where $\frac{\theta}{2} = \frac{\pi}{2} - \frac{\alpha}{4}$

$\cos\frac{\alpha}{4} = 4\sin\frac{\alpha}{4}\cos\frac{\alpha}{4}$ ***M1***

Note: Award ***M1*** either for use of the double angle formula or the conversion from sine to cosine.

$\frac{1}{4} = \sin\frac{\alpha}{4}$ ***A1***

$\frac{\alpha}{4} = \arcsin\frac{1}{4}$

$\alpha = 4\arcsin\frac{1}{4}$ ***AG***

[2 marks]

6c. Hence find the value of r given that the shaded area is equal to 4. *[3 marks]*

Markscheme

(from triangle ADM), $\theta = \pi - \frac{\alpha}{2}$ $\left(= \pi - 2\arcsin\frac{1}{4} = 2\arcsin\frac{1}{4} = 2.6362\ldots\right)$
A1

attempting to solve $2(\alpha - \sin\alpha)r^2 + \frac{1}{2}(\theta - \sin\theta)r^2 = 4$

with $\alpha = 4\arcsin\frac{1}{4}$ and $\theta = \pi - \frac{\alpha}{2}$ $\left(= 2\arccos\frac{1}{4}\right)$ for r ***(M1)***

$r = 1.69$ ***A1***

[3 marks]