Binomial Theorem 3IB *[39 marks]*

1. In the expansion of $(x+k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63. *[5 marks]*

   Find the possible values of k.

Markscheme

EITHER

attempt to use the binomial expansion of $(x+k)^7$ ***(M1)***

${}^7C_0 x^7 k^0 + {}^7C_1 x^6 k^1 + {}^7C_2 x^5 k^2 + \ldots$ (or ${}^7C_0 k^7 x^0 + {}^7C_1 k^5 x^1 + {}^7C_2 k^5 x^2 + \ldots$)

identifying the correct term ${}^7C_2 x^5 k^2$ (or ${}^7C_5 k^2 x^5$) ***(A1)***

OR

attempt to use the general term ${}^7C_r x^r k^{7-r}$ (or ${}^7C_r k^r x^{7-r}$) ***(M1)***

$r = 2$ (or $r = 5$) ***(A1)***

THEN

${}^7C_2 = 21$ (or ${}^7C_5 = 21$ (seen anywhere) ***(A1)***

$21x^5k^2 = 63x^5 \left(21k^2 = 63, k^2 = 3\right)$ ***A1***

$k = \pm\sqrt{3}$ ***A1***

Note: If working shown, award ***M1A1A1A1A0*** for $k = \sqrt{3}$.

[5 marks]

2. Consider the expansion of $\left(3+x^2\right)^{n+1}$, where $n \in \mathbb{Z}^+$. *[5 marks]*

   Given that the coefficient of x^4 is $20\,412$, find the value of n.

Markscheme

METHOD 1

product of a binomial coefficient, a power of 3 (and a power of x^2) seen ***(M1)***

evidence of correct term chosen ***(A1)***

$^{n+1}C_2 \times 3^{n+1-2} \times (x^2)^2 \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4\right)$ **OR** $n - r = 1$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

EITHER

$^{n+1}C_2 \times 3^{n-1} = 20412$ ***(A1)***

OR

$^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$ ***(A1)***

THEN

$n = 7$ ***A1***

METHOD 2

$3^{n+1}\left(1 + \frac{x^2}{3}\right)^{n+1}$

product of a binomial coefficient, and a power of $\frac{x^2}{3}$ **OR** $\frac{1}{3}$ seen ***(M1)***

evidence of correct term chosen ***(A1)***

$3^{n+1} \times \frac{n(n+1)}{2!} \times \left(\frac{x^2}{3}\right)^2 \left(= \frac{3^{n-1}}{2} n(n+1)x^4\right)$

equating their coefficient to 20412 or their term to $20412x^4$ ***(M1)***

$3^{n-1} \times \frac{n(n+1)}{2} = 20412$ ***(A1)***

$n = 7$ ***A1***

[5 marks]

3. Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$. *[6 marks]*

The coefficient of the term in x^6 is 6048. Find the value of k.

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for r). ***(M1)***

eg

$$\binom{9}{r}(3x^2)^{9-r}\left(-\frac{k}{x}\right)^r,\ (3x^2)^9+\binom{9}{1}(3x^2)^8\left(-\frac{k}{x}\right)^1+\binom{9}{2}(3x^2)^7\left(-\frac{k}{x}\right)^2+\ldots$$

valid attempt to identify correct term ***(M1)***

eg $2(9-r)-r=6,\ (x^2)^r(x^{-1})^{9-r}=x^6$

identifying correct term (may be indicated in expansion) ***(A1)***

eg $r=4,\ r=5$

correct term or coefficient in binominal expansion ***(A1)***

eg $\binom{9}{4}(3x^2)^5\left(-\frac{k}{x}\right)^4,\ 126(243x^{10})\left(\frac{k^4}{x^4}\right),\ 30618k^4$

correct equation in k ***(A1)***

eg $\binom{9}{4}(243)(k^4)x^6=6048x^6,\ 30618k^4=6048$

$k=\frac{2}{3}$ (exact) 0.667 ***A1 N3***

Note: Do not award ***A1*** if additional answers given.

[6 marks]

4. Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

use of Binomial expansion to find a term in either

$\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$, $\left(\frac{1}{3x^{7/3}}-\frac{x^{2/3}}{2}\right)^9$, $\left(\frac{1}{3}-\frac{x^3}{2}\right)^9$, $\left(\frac{1}{3x^3}-\frac{1}{2}\right)^9$ or $(2-3x^3)^9$

(M1)(A1)

Note: Award ***M1*** for a product of three terms including a binomial coefficient and powers of the two terms, and ***A1*** for a correct expression of a term in the expansion.

finding the powers required to be 2 and 7 ***(M1)(A1)***

constant term is ${}_9C_2 \times \left(\frac{1}{3}\right)^2 \times \left(-\frac{1}{2}\right)^7$ ***(M1)***

Note: Ignore all x's in student's expression.

therefore term independent of x is $-\frac{1}{32}$ $(= -0.03125)$ ***A1***

[6 marks]

5. Consider the expansion of $(2+x)^n$, where $n \geqslant 3$ and $n \in \mathbb{Z}$. *[6 marks]*

The coefficient of x^3 is four times the coefficient of x^2. Find the value of n.

Markscheme

attempt to find coefficients in binomial expansion ***(M1)***

coefficient of x^2 : $\binom{n}{2} \times 2^{n-2}$; coefficient of x^3 : $\binom{n}{3} \times 2^{n-3}$ ***A1A1***

Note: Condone terms given rather than coefficients. Terms may be seen in an equation such as that below.

$\binom{n}{3} \times 2^{n-3} = 4\binom{n}{2} \times 2^{n-2}$ ***(A1)***

attempt to solve equation using GDC or algebraically ***(M1)***

$\binom{n}{3} = 8\binom{n}{2}$

$\frac{n!}{3!(n-3)!} = \frac{8n!}{2!(n-2)!}$

$\frac{1}{3} = \frac{8}{n-2}$

$n = 26$ ***A1***

[6 marks]

6. The coefficient of x^2 in the expansion of $\left(\frac{1}{x} + 5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a + 5x)^7$, $a \in \mathbb{R}$. Find the value of a. *[6 marks]*

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

METHOD 1

${}^{8}C_{r}\left(\frac{1}{x}\right)^{8-r}(5x)^{r} = {}^{8}Cr(5)^{r}x^{2r-8}$ ***(M1)***

$r = 5$ ***(A1)***

${}^{8}C_{5} \times 5^{5} = {}^{7}C_{4}a^{3} \times 5^{4}$ ***M1A1***

$56 \times 5 = 35a^{3}$

$a^{3} = 8$ ***(A1)***

$a = 2$ ***A1***

METHOD 2

attempt to expand both binomials ***M1***

$175000x^{2}$ ***A1***

$21875a^{3}x^{4}$ ***A1***

$175000 = 21875a^{3}$ ***M1***

$a^{3} = 8$ ***(A1)***

$a = 2$ ***A1***

[6 marks]

7. Consider the expansion of $\left(8x^{3} - \frac{1}{2x}\right)^{n}$ where $n \in \mathbb{Z}^{+}$. Determine all possible values of n for which the expansion has a non-zero constant term. *[5 marks]*

Markscheme

EITHER

attempt to obtain the general term of the expansion

$T_{r+1} = {}_nC_r\left(8x^3\right)^{n-r}\left(-\frac{1}{2x}\right)^r$ OR $T_{r+1} = {}_nC_{n-r}\left(8x^3\right)^r\left(-\frac{1}{2x}\right)^{n-r}$ ***(M1)***

OR

recognize power of x starts at $3n$ and goes down by 4 each time ***(M1)***

THEN

recognizing the constant term when the power of x is zero (or equivalent) ***(M1)***

$r = \frac{3n}{4}$ or $n = \frac{4}{3}r$ or $3n - 4r = 0$ OR $3r - (n - r) = 0$ (or equivalent) ***A1***

r is a multiple of 3 $(r = 3, 6, 9, \ldots)$ or one correct value of n (seen anywhere) ***(A1)***

$n = 4k,\ k \in \mathbb{Z}^+$ ***A1***

Note: Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \ldots$
Do not accept $n = 4, 8, 12$

Note: Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \ldots$ and for recognizing that this pattern continues.

[5 marks]