Binomial Theorem 3IB *[39 marks]*

1. In the expansion of $(x+k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63. *[5 marks]*
   Find the possible values of k.

2. Consider the expansion of $(3+x^2)^{n+1}$, where $n \in \mathbb{Z}^+$. *[5 marks]*
   Given that the coefficient of x^4 is $20\,412$, find the value of n.

3. Consider the expansion of $\left(3x^2 - \frac{k}{x}\right)^9$, where $k > 0$. *[6 marks]*
   The coefficient of the term in x^6 is 6048. Find the value of k.

4. Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2} - \frac{x}{2}\right)^9$. *[6 marks]*

5. Consider the expansion of $(2+x)^n$, where $n \geqslant 3$ and $n \in \mathbb{Z}$. *[6 marks]*
   The coefficient of x^3 is four times the coefficient of x^2. Find the value of n.

6. The coefficient of x^2 in the expansion of $\left(\frac{1}{x} + 5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a+5x)^7,\ a \in \mathbb{R}$. Find the value of a. *[6 marks]*

7. Consider the expansion of $\left(8x^3 - \frac{1}{2x}\right)^n$ where $n \in \mathbb{Z}^+$. Determine all possible values of n for which the expansion has a non-zero constant term. *[5 marks]*